The Complete Set of Automatically Generated Angle Problems for Cyclic Hexagon and Pentagon

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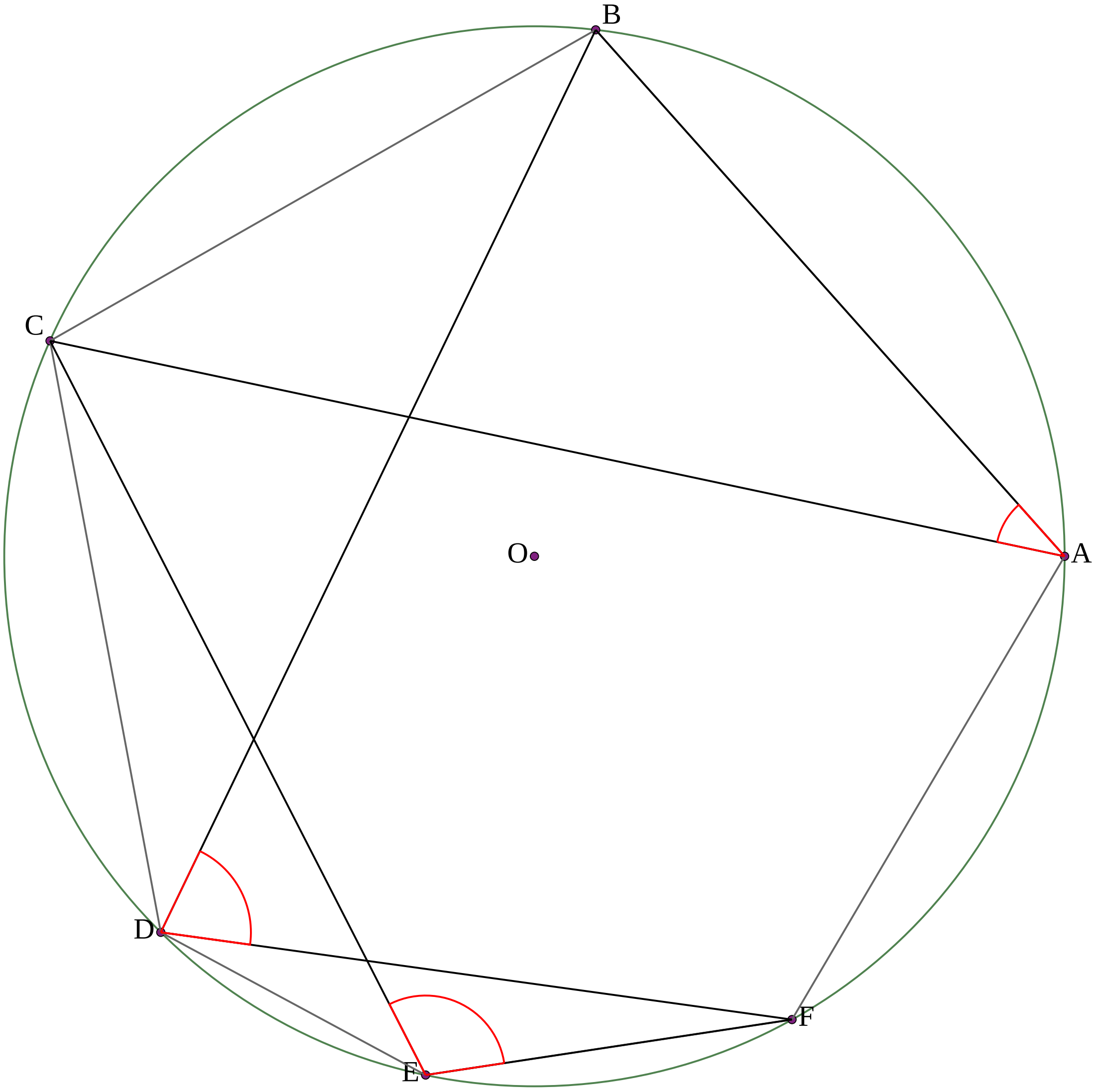
Automatically generated angle theorems are presented for hexagon and pentagon. These constitute the complete set of theorems relating three angles in these figures modulo rotation and reflection.

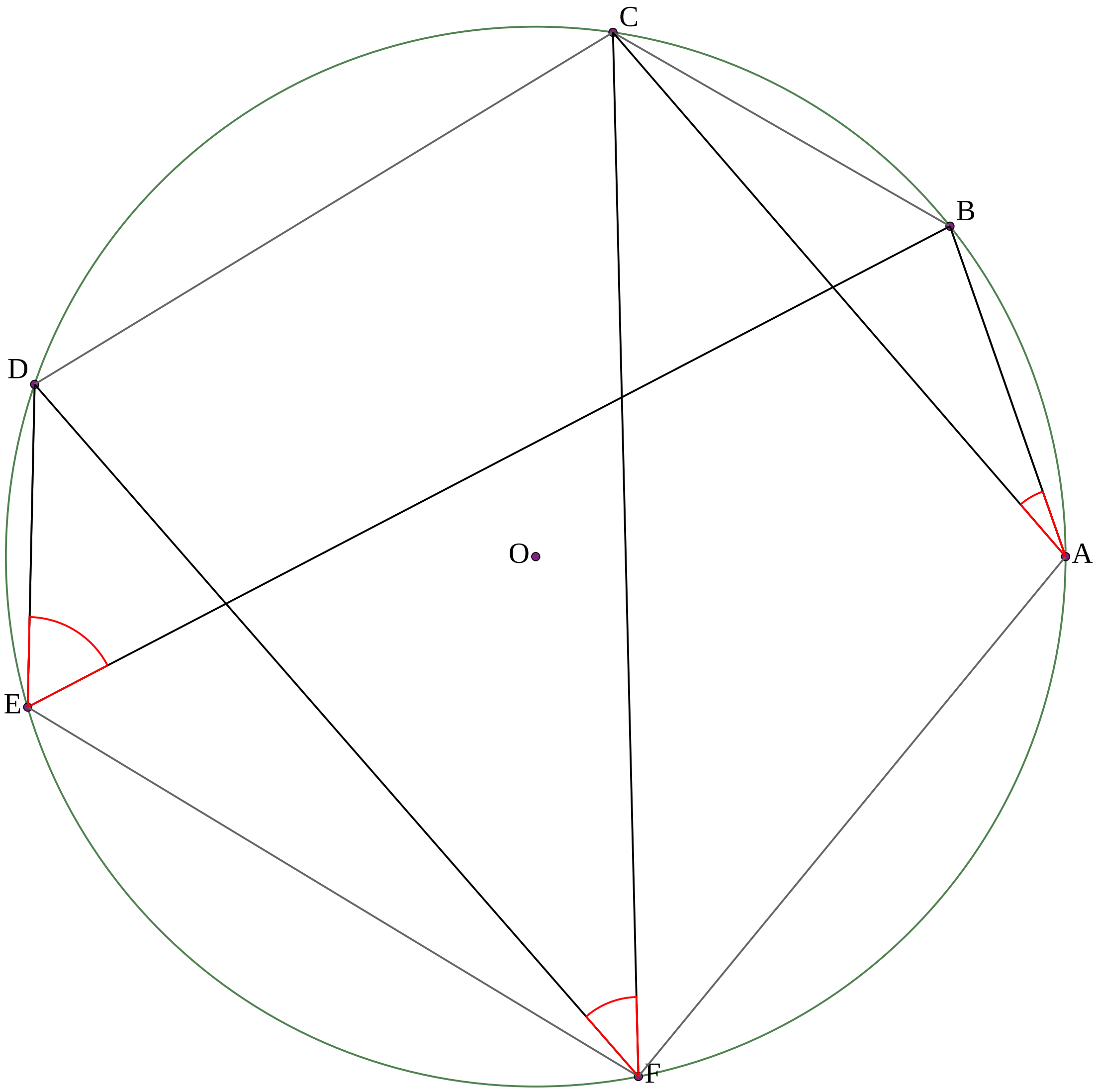
Automatically generated human-readable proofs for the theorems are also presented.

Example 1



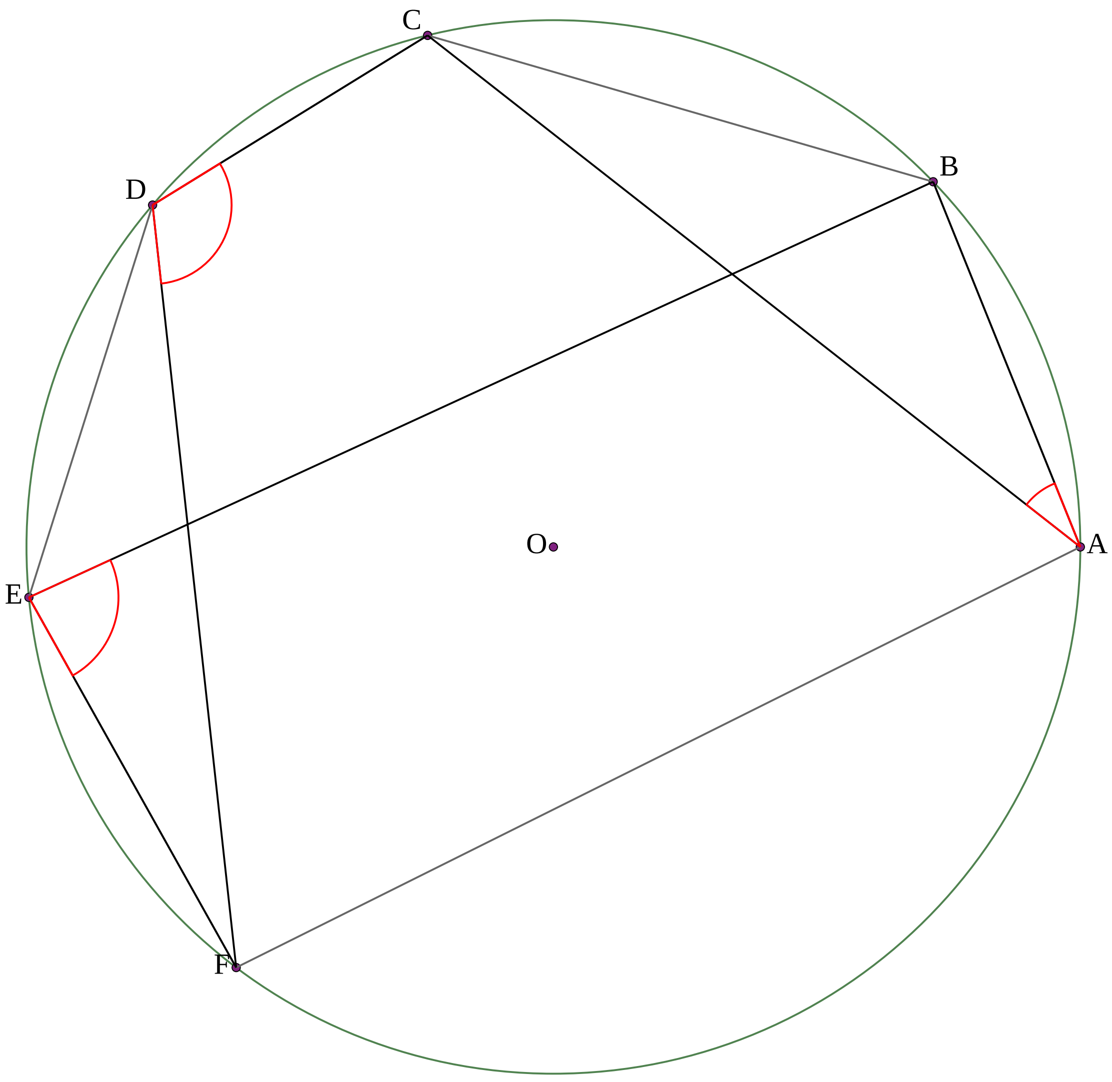
Let ABCDEF be a cyclic hexagon with center O.
Prove that CEF = BAC+BDF

Example 2



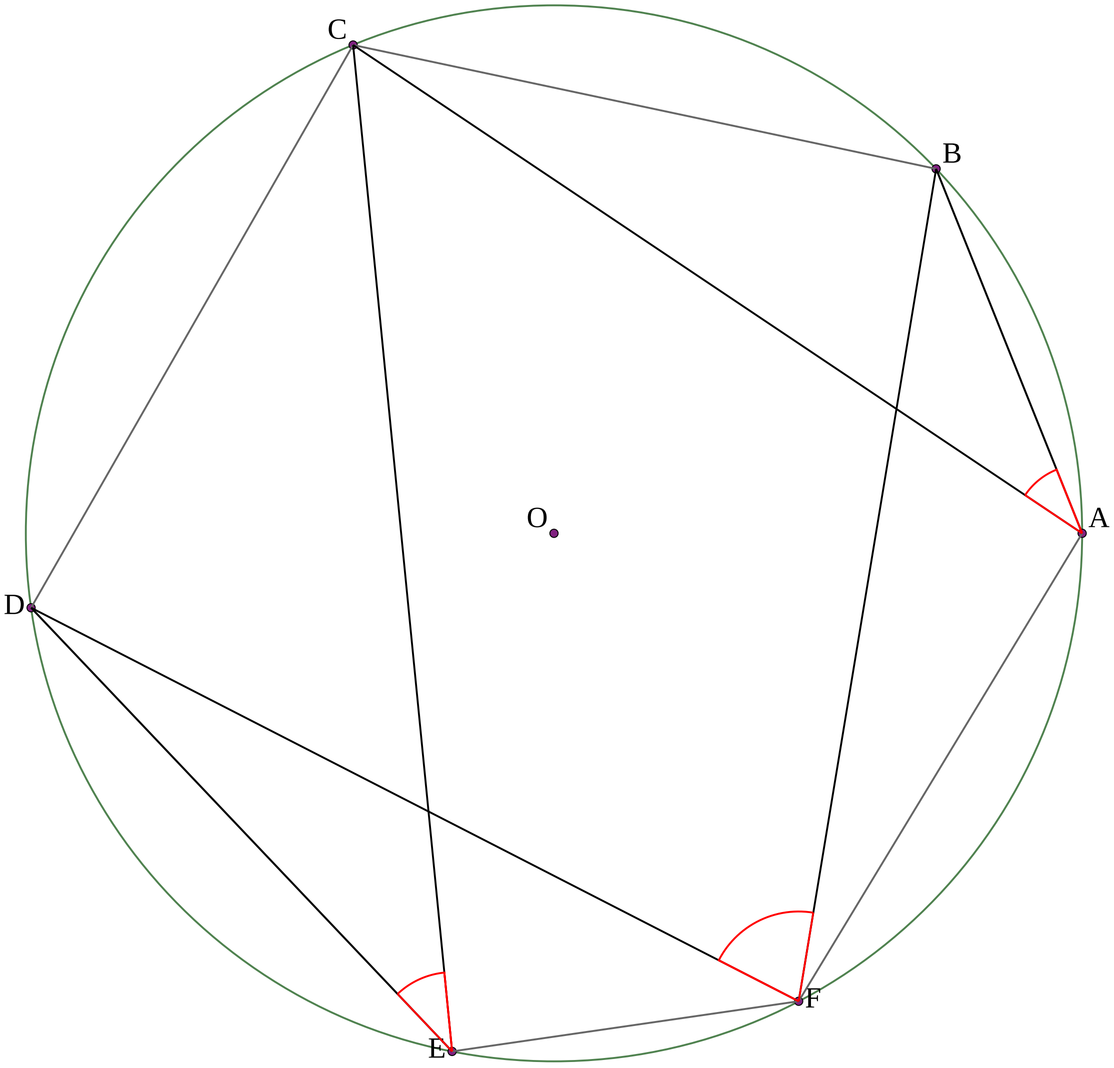
Let ABCDEF be a cyclic hexagon with center O.
Prove that BED = BAC+CFD

Example 3



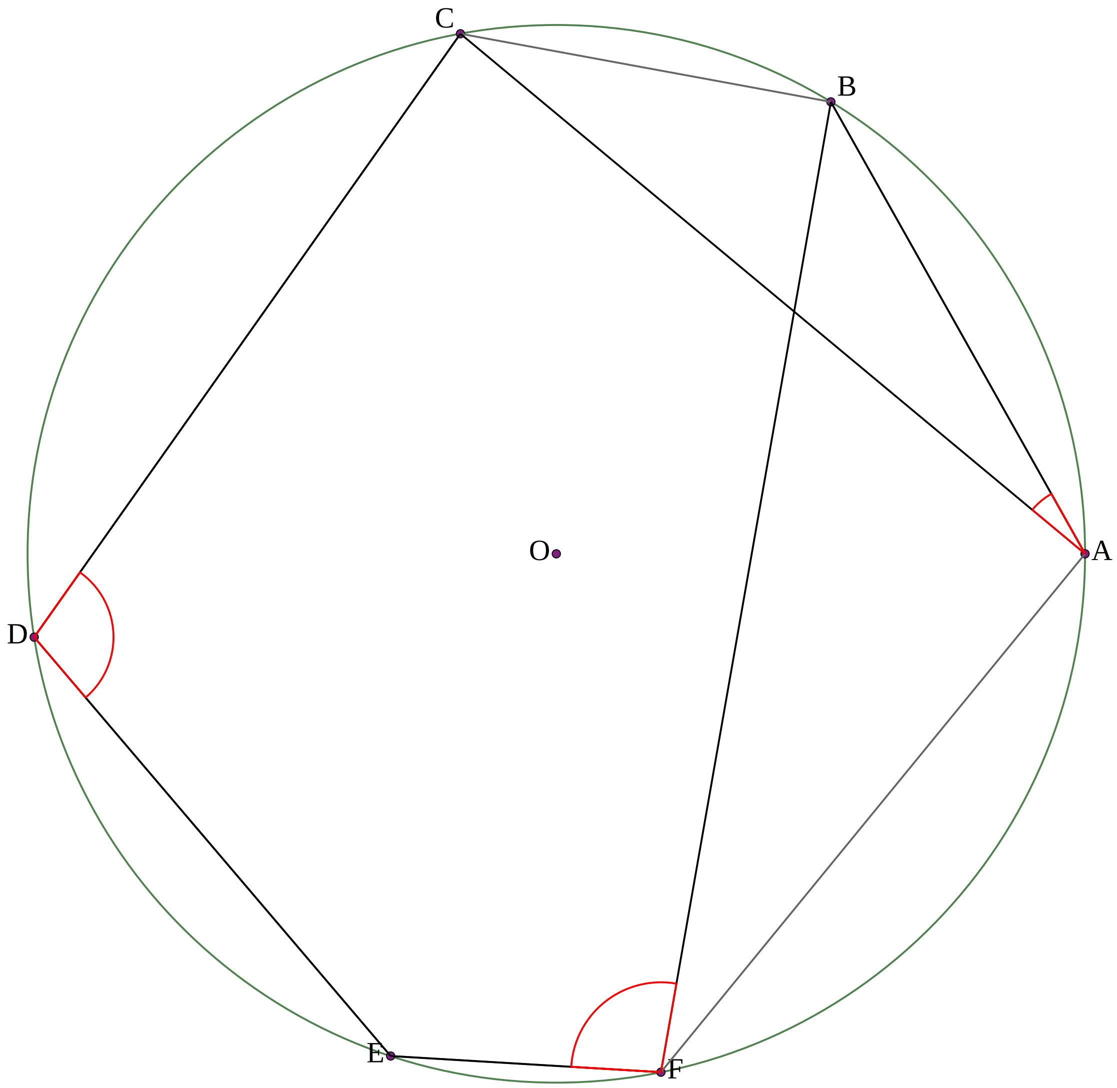
Let ABCDEF be a cyclic hexagon with center O.
Prove that CDF = BAC+BEF

Example 4



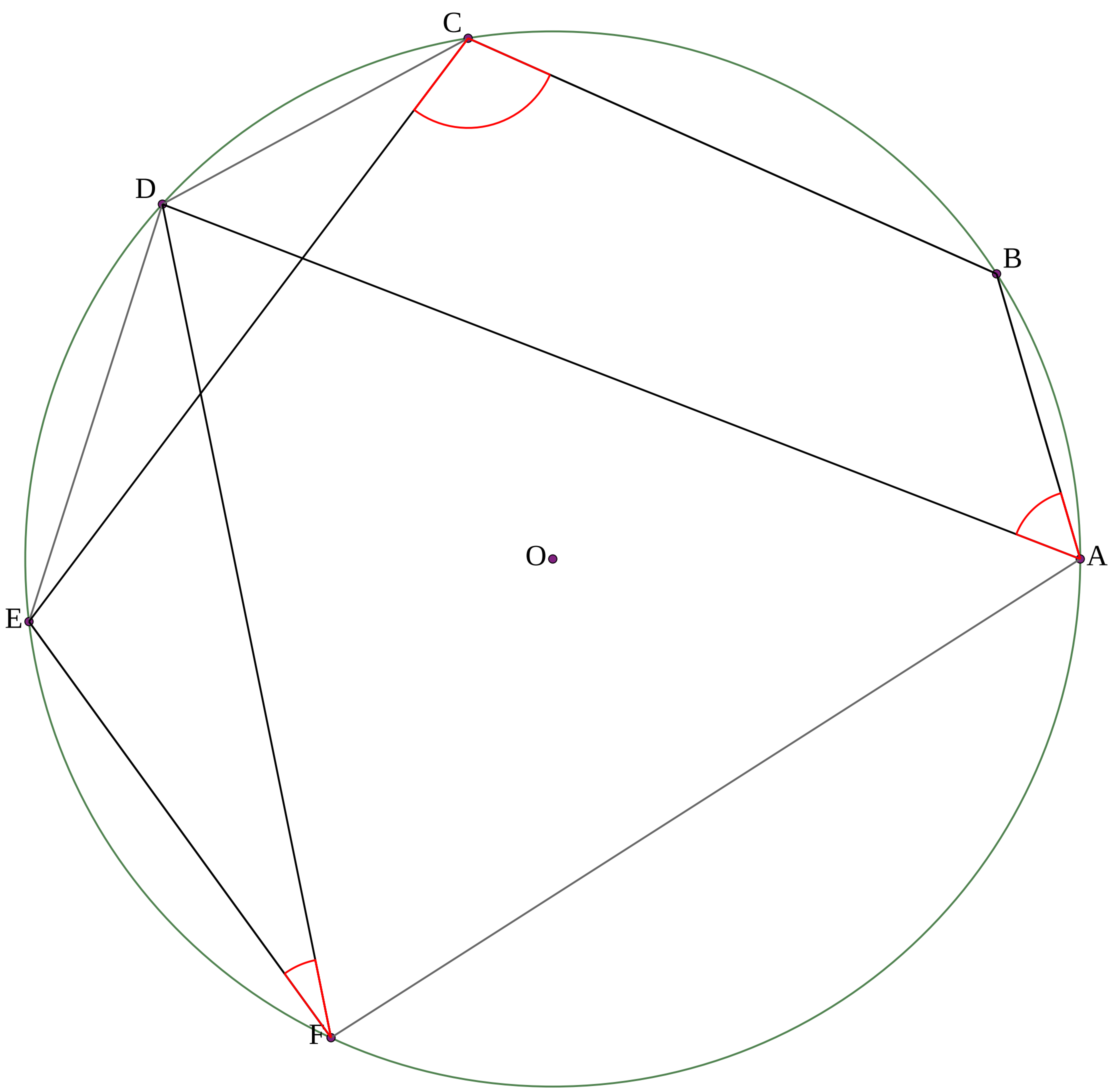
Let ABCDEF be a cyclic hexagon with center O.
Prove that BFD = BAC+CED

Example 5



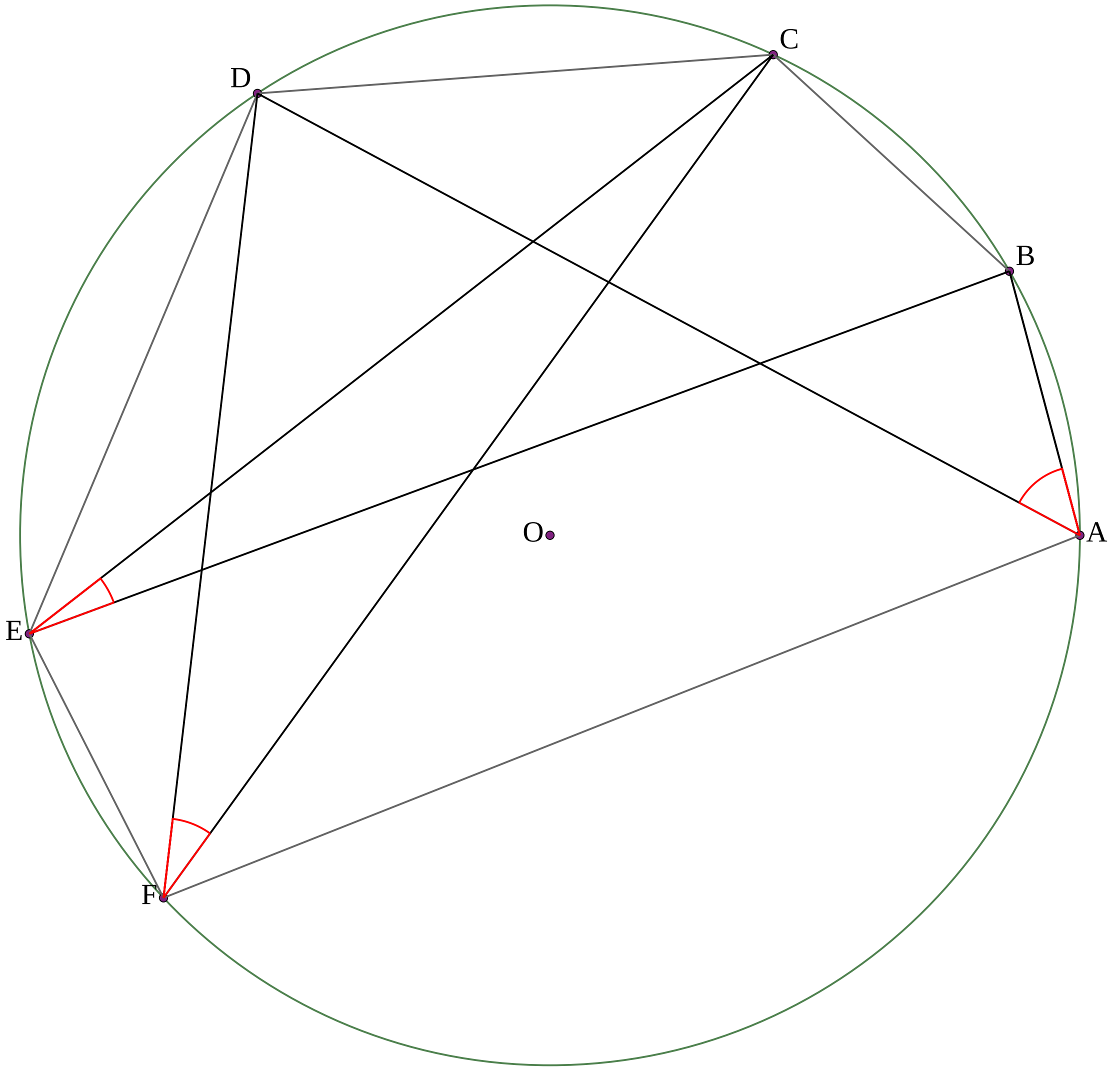
Let ABCDEF be a cyclic hexagon with center O.
Prove that CDE+BFE = BAC+180

Example 6



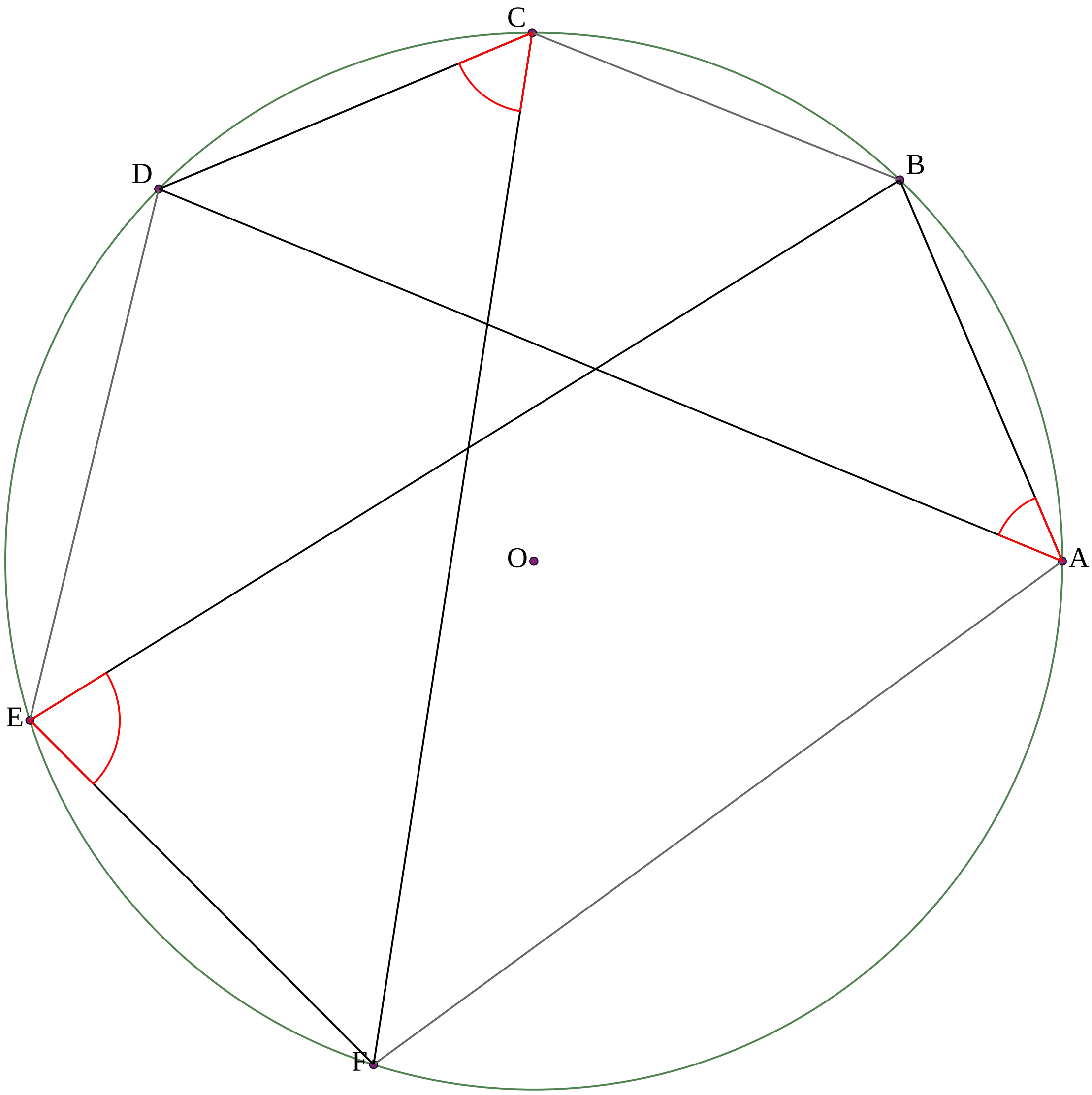
Let ABCDEF be a cyclic hexagon with center O.
Prove that BAD+DFE+BCE = 180

Example 7



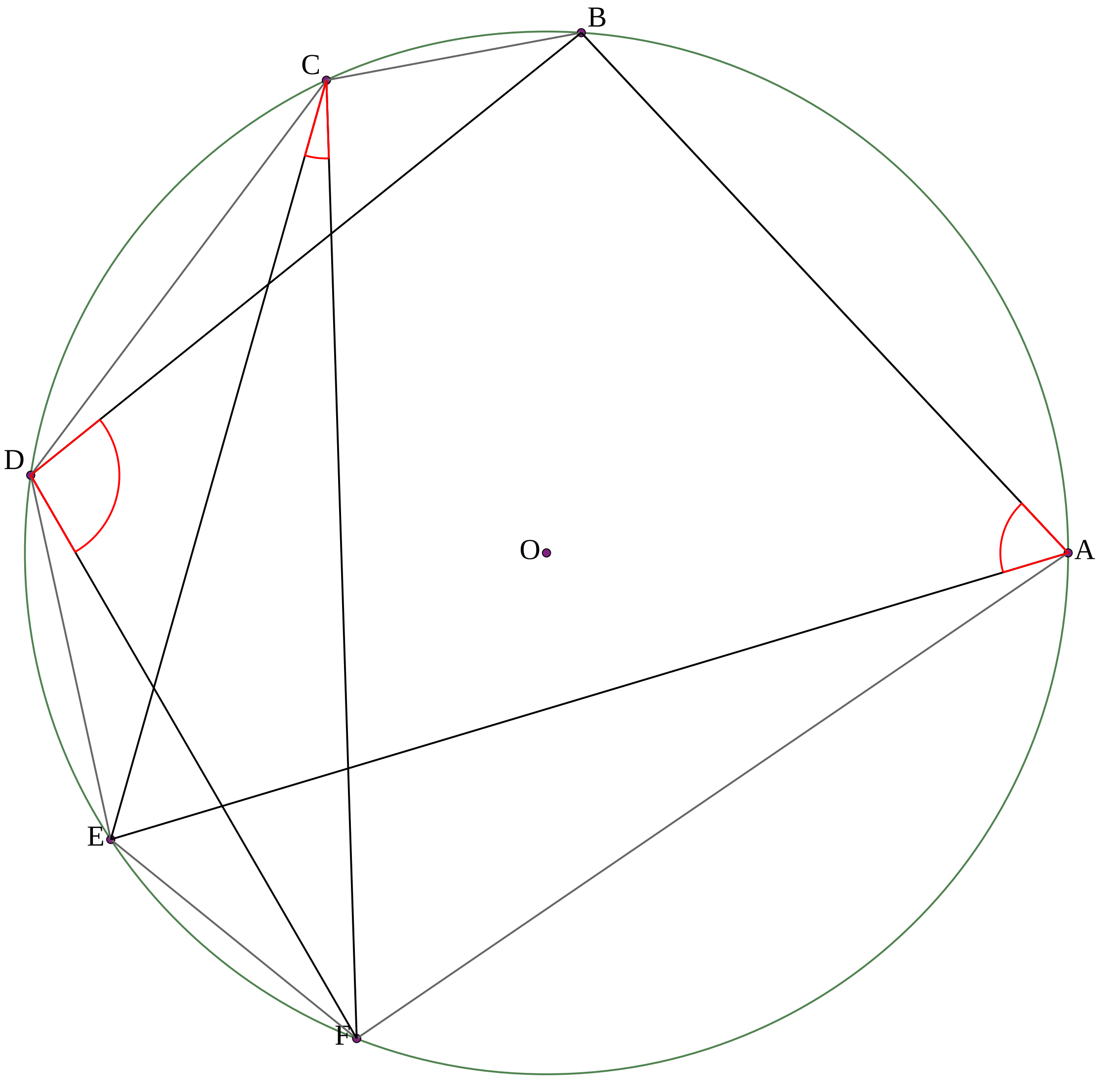
Let ABCDEF be a cyclic hexagon with center O.
Prove that CFD+BEC = BAD

Example 8



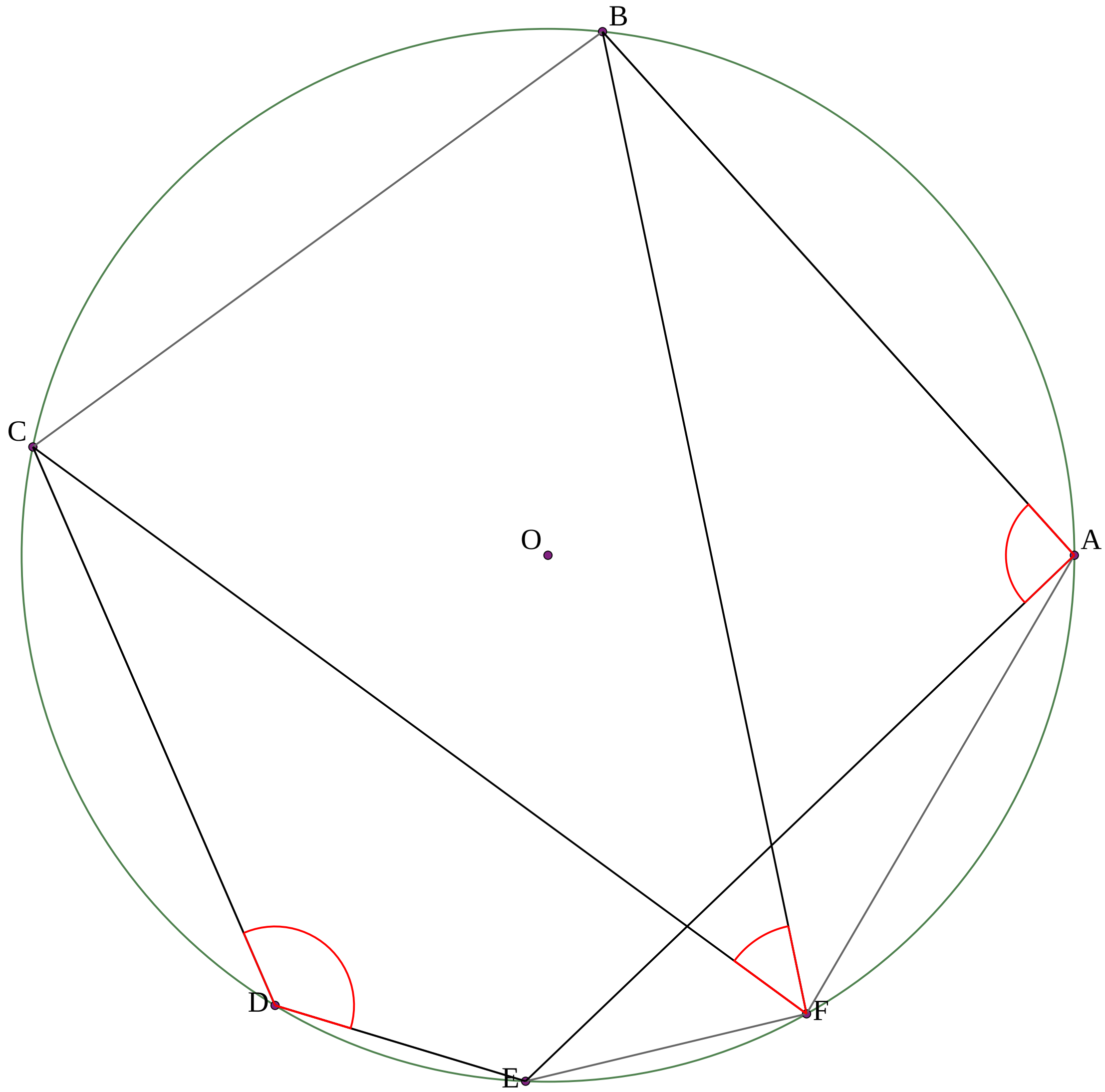
Let ABCDEF be a cyclic hexagon with center O.
Prove that BAD+DCF+BEF = 180

Example 9



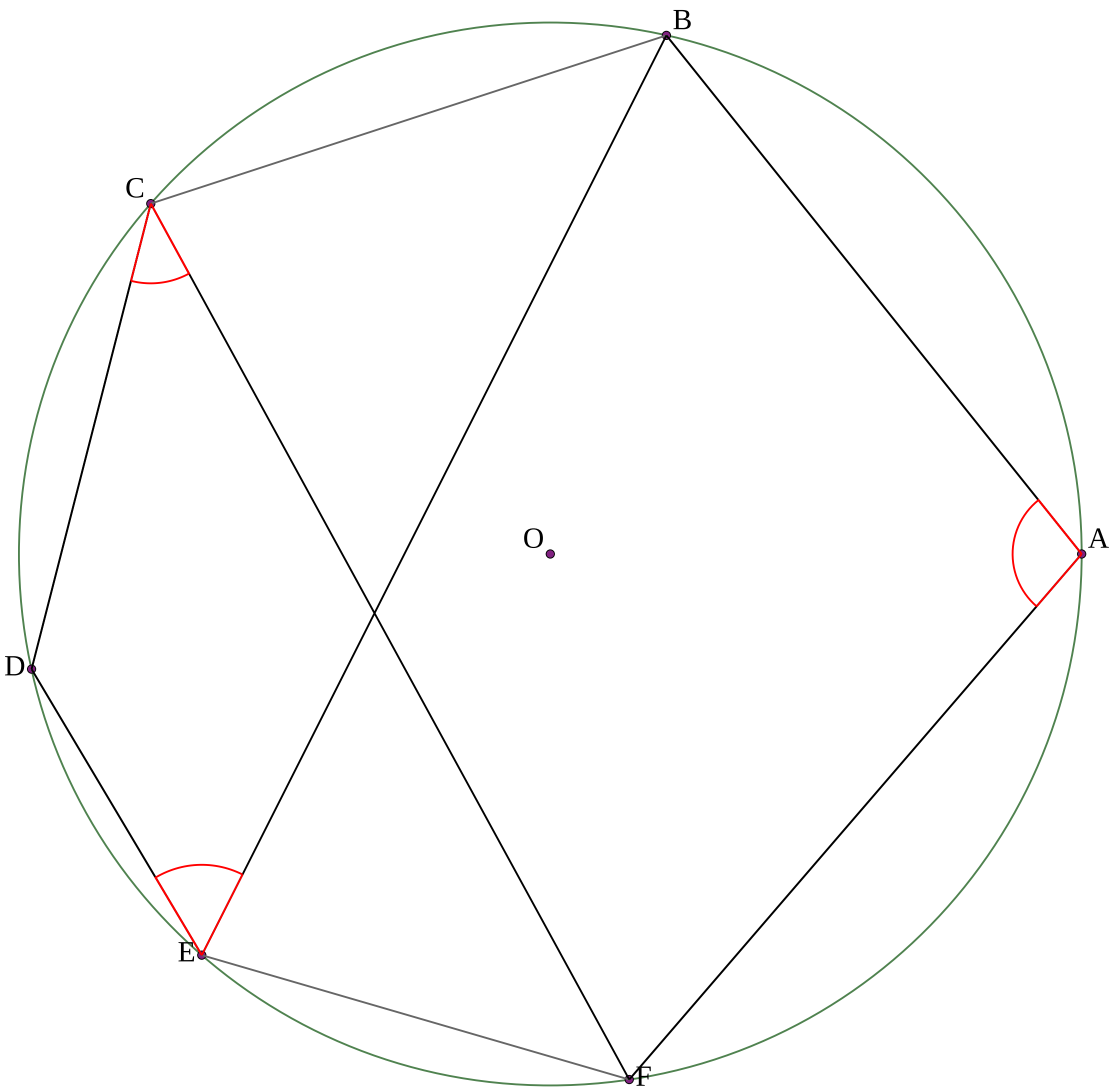
Let ABCDEF be a cyclic hexagon with center O.
Prove that BAE+ECF+BDF = 180

Example 10



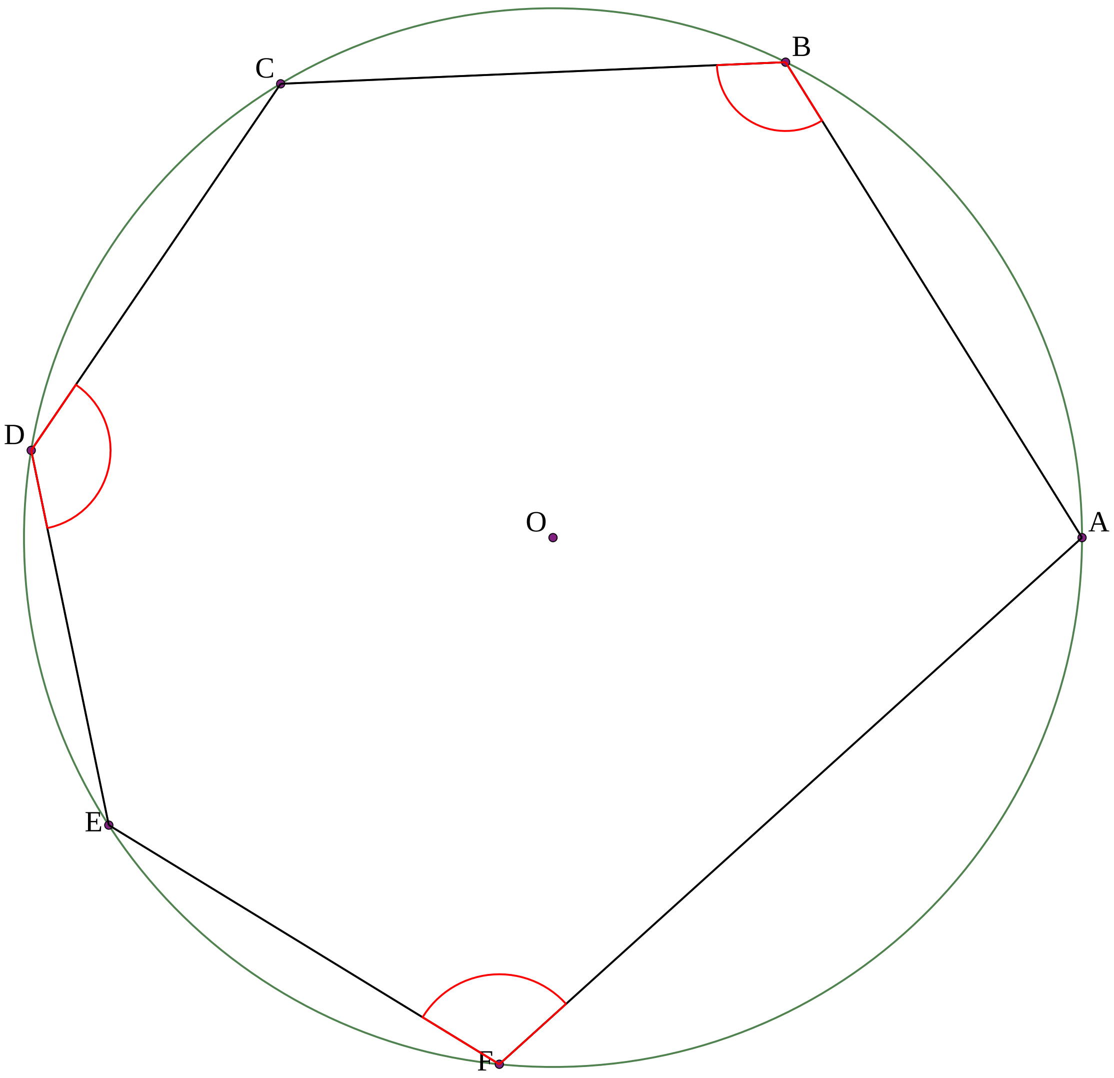
Let ABCDEF be a cyclic hexagon with center O.
Prove that BAE+CDE = BFC+180

Example 11



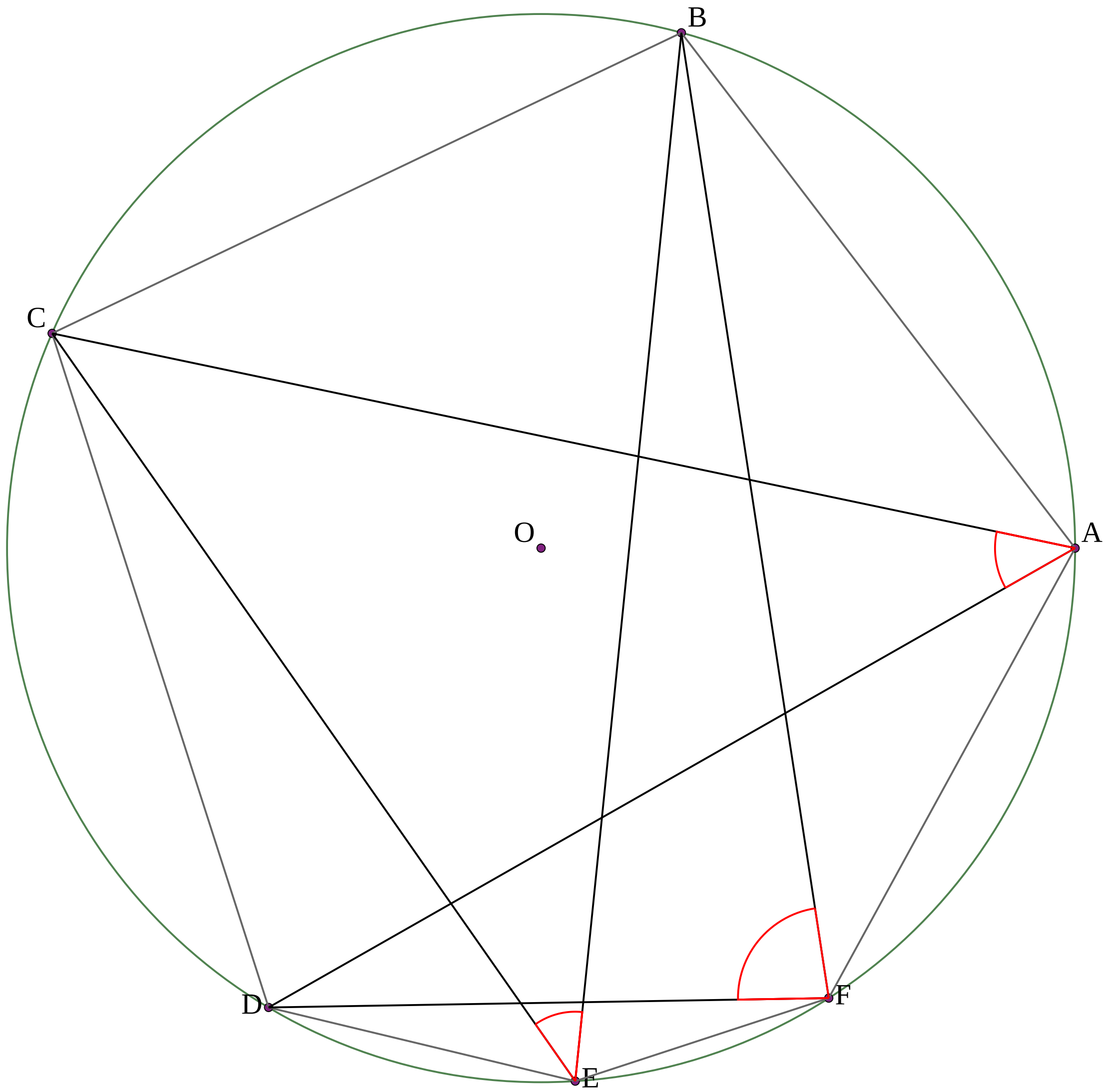
Let ABCDEF be a cyclic hexagon with center O.
Prove that DCF+BED = BAF

Example 12



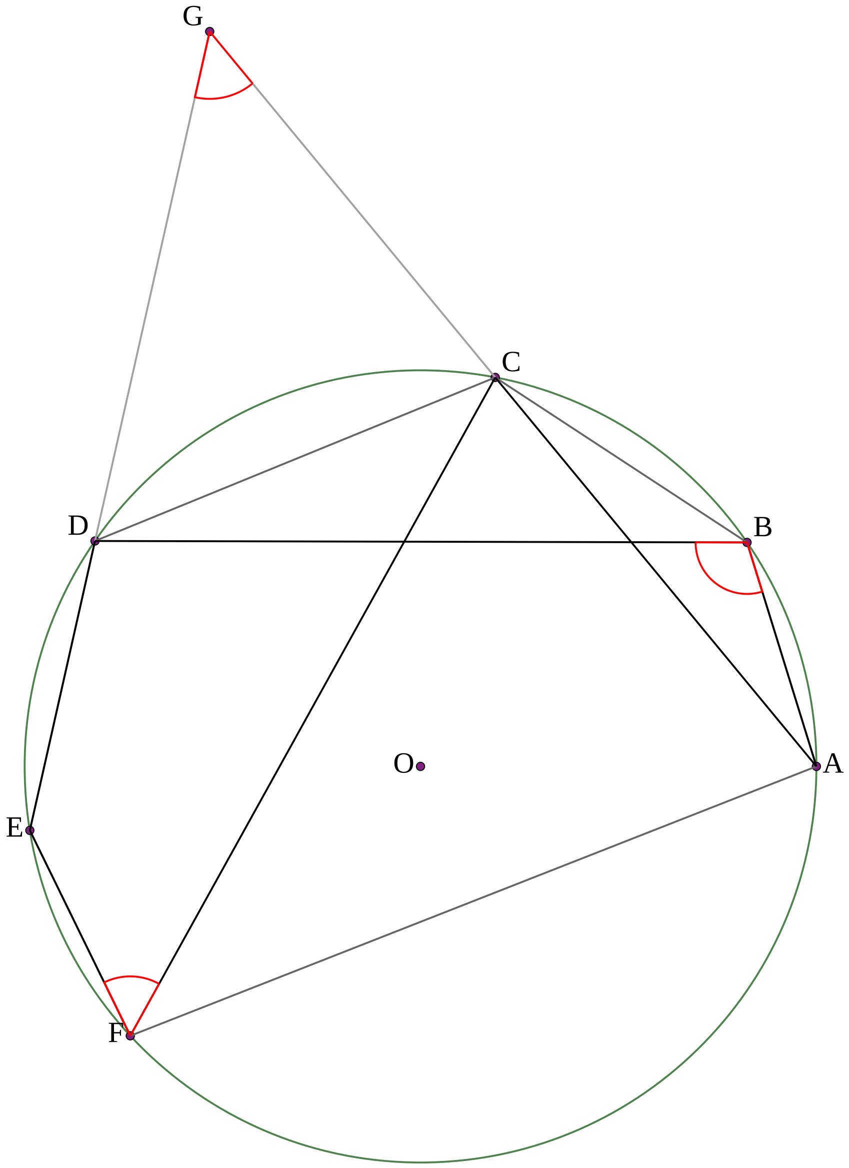
Let ABCDEF be a cyclic hexagon with center O.
Prove that ABC+CDE+AFE = 360

Example 13



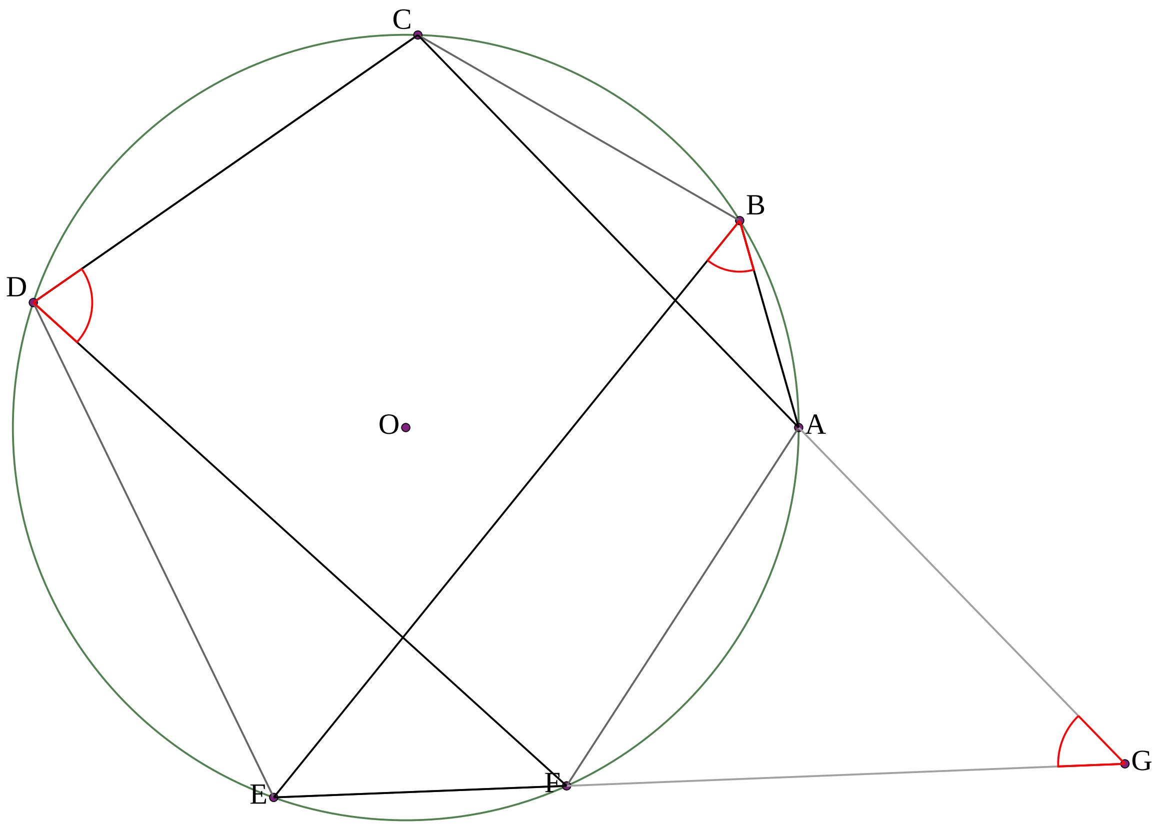
Let ABCDEF be a cyclic hexagon with center O.
Prove that BFD = BEC+CAD

Example 14



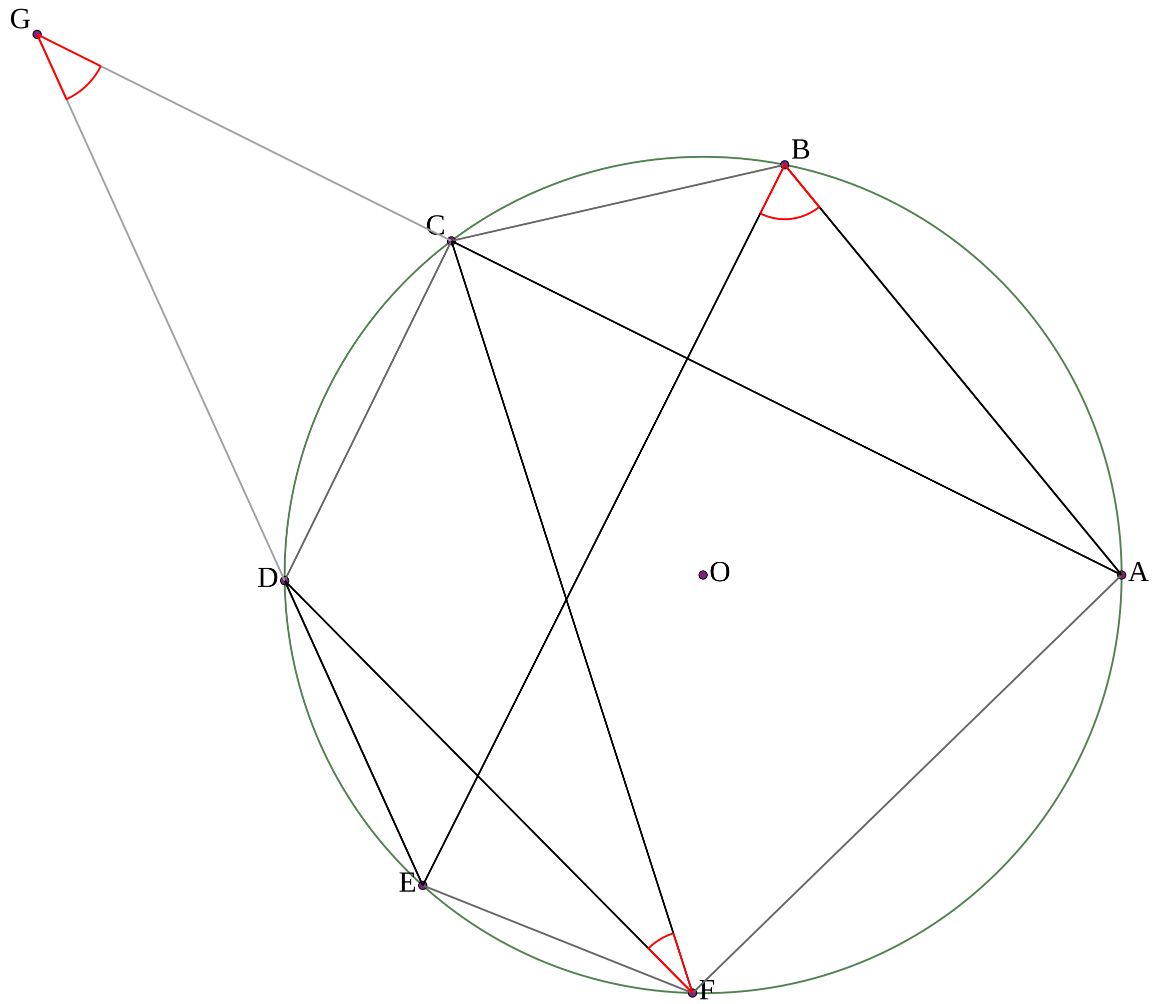
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DE and CA.
Prove that ABD = CFE+CGD

Example 15



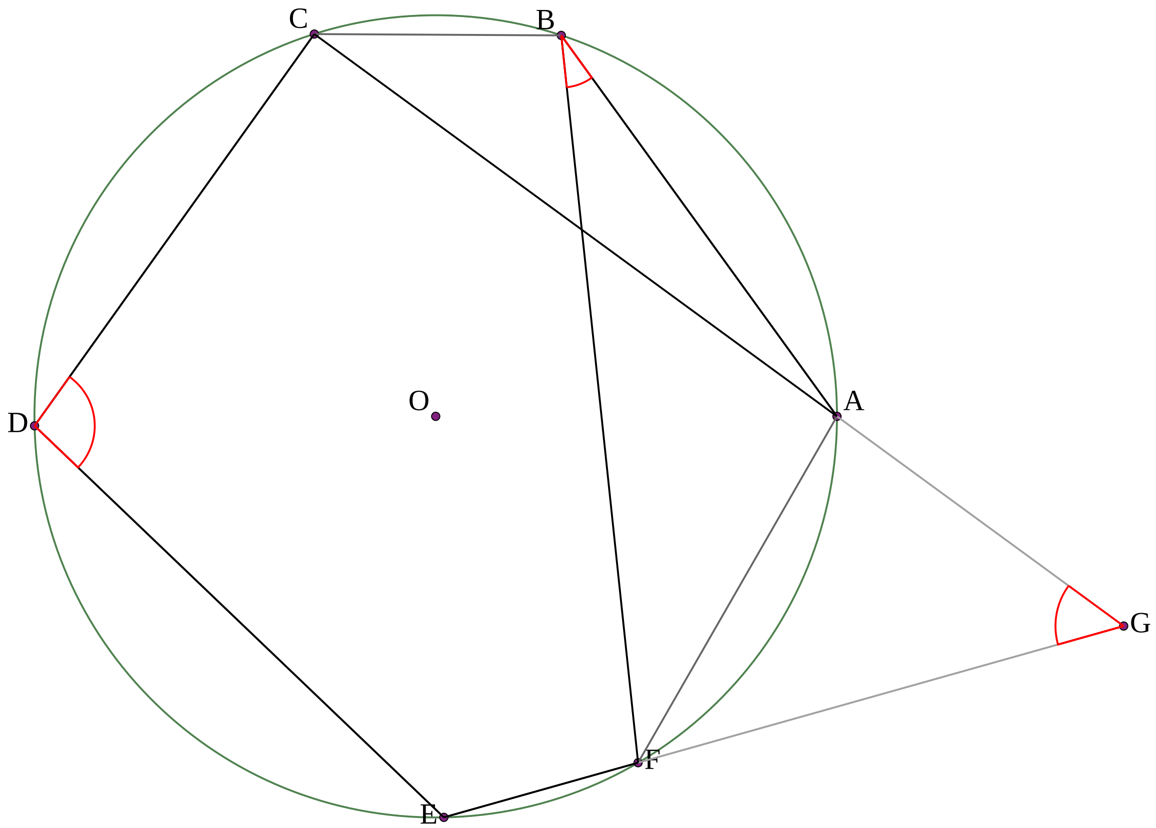
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and CA.
Prove that ABE+CDF+AGF = 180

Example 16



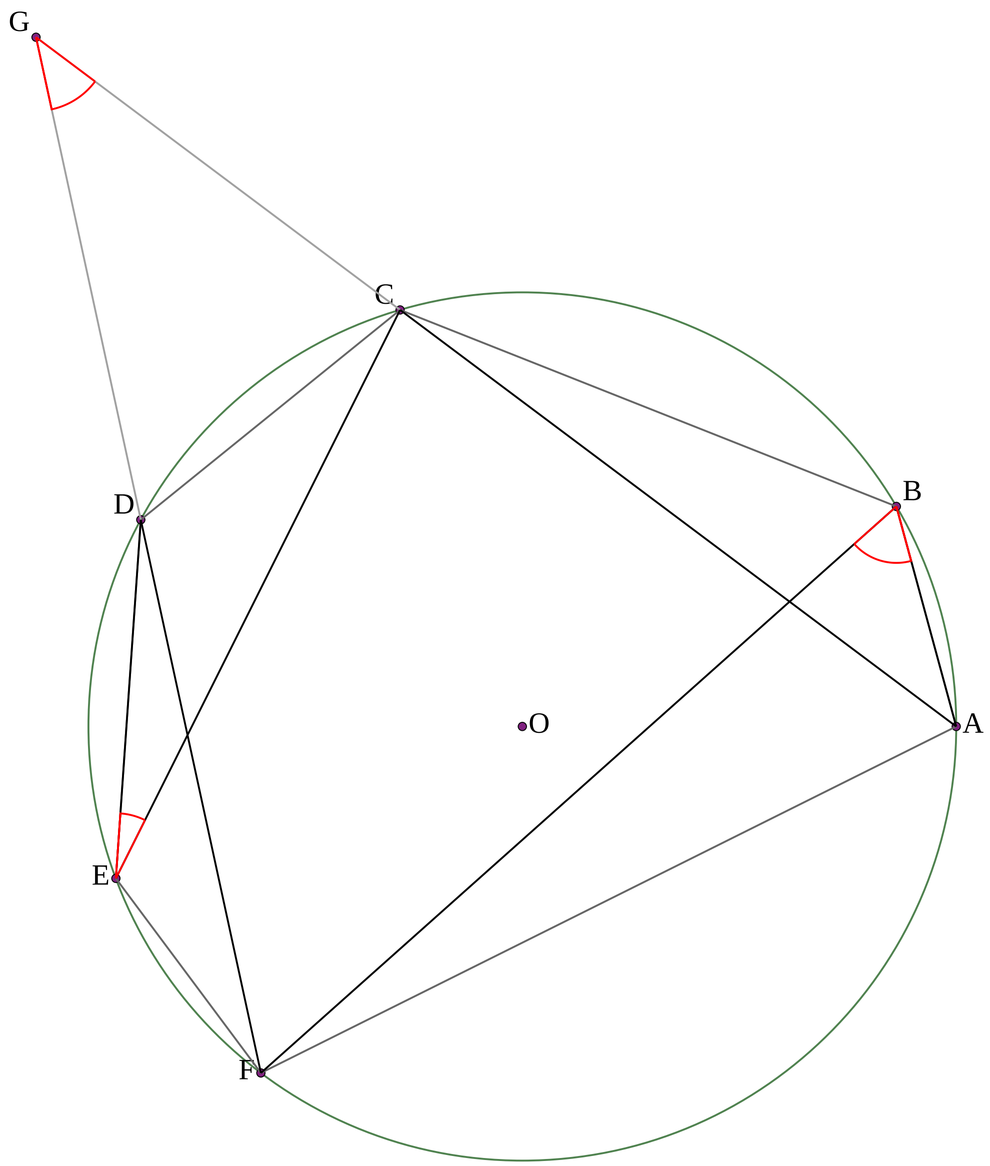
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and CA.
Prove that ABE = CFD+CGD

Example 17



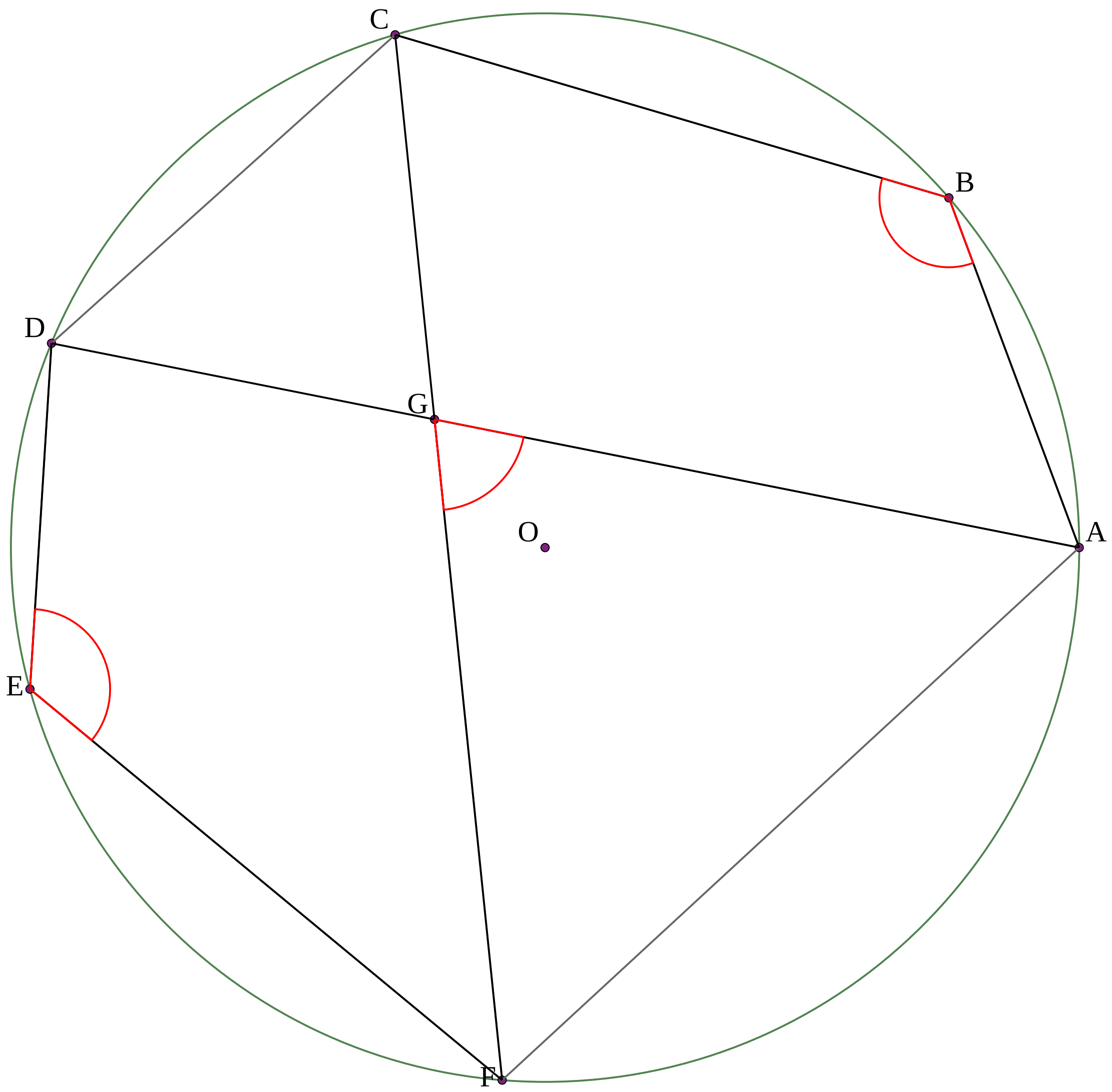
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FE and CA.
Prove that ABF+CDE+AGF = 180

Example 18



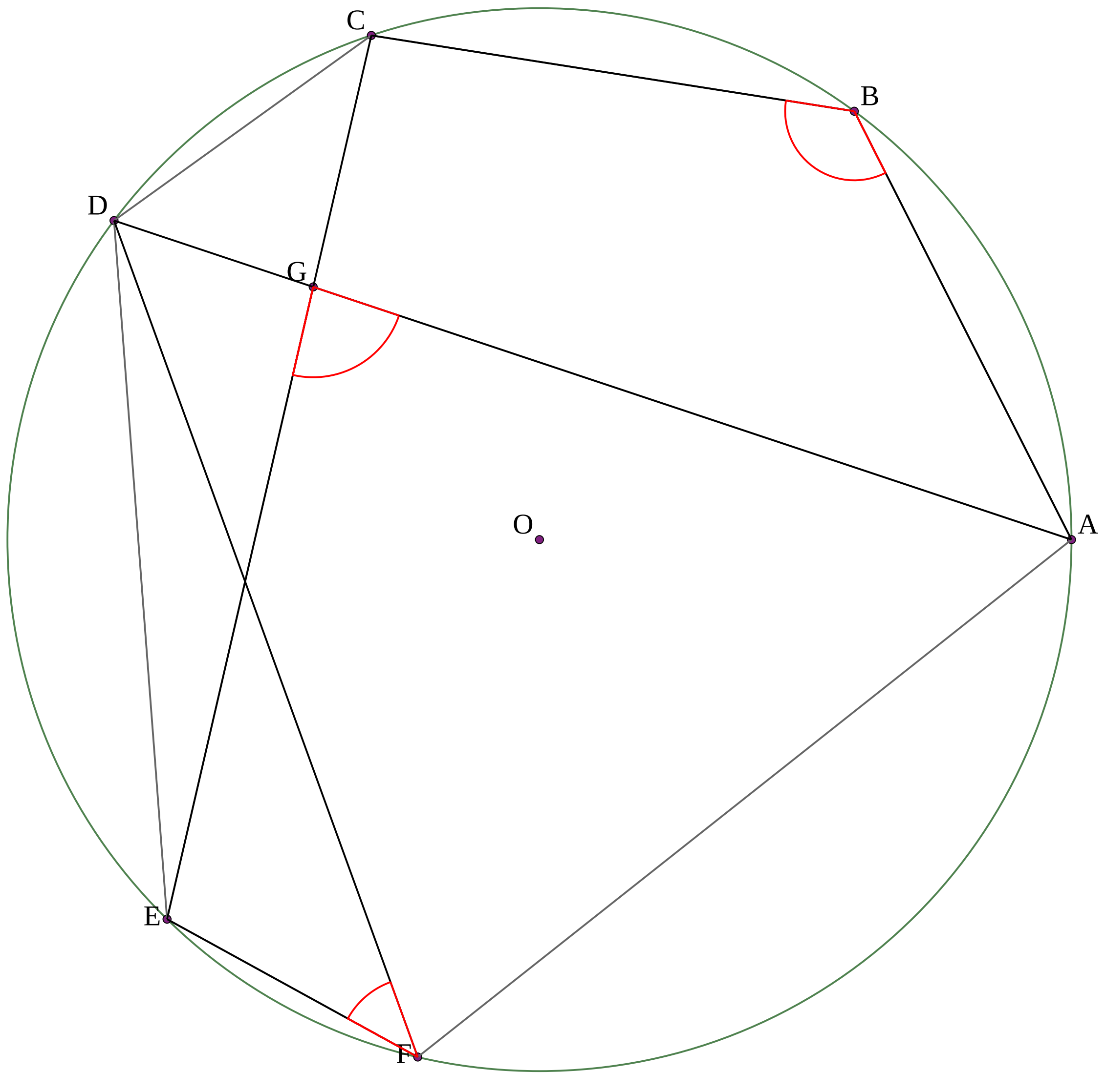
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and CA.
Prove that ABF = CED+CGD

Example 19



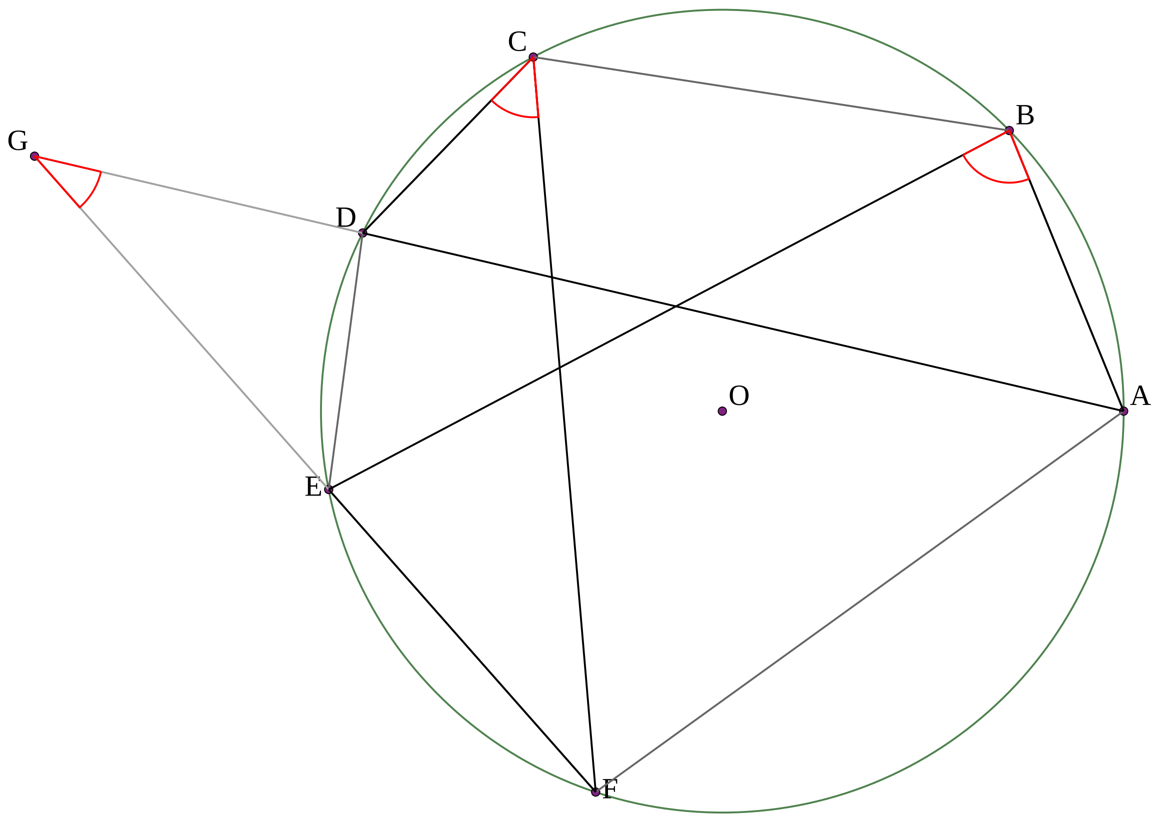
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CF and DA.
Prove that ABC+DEF = AGF+180

Example 20



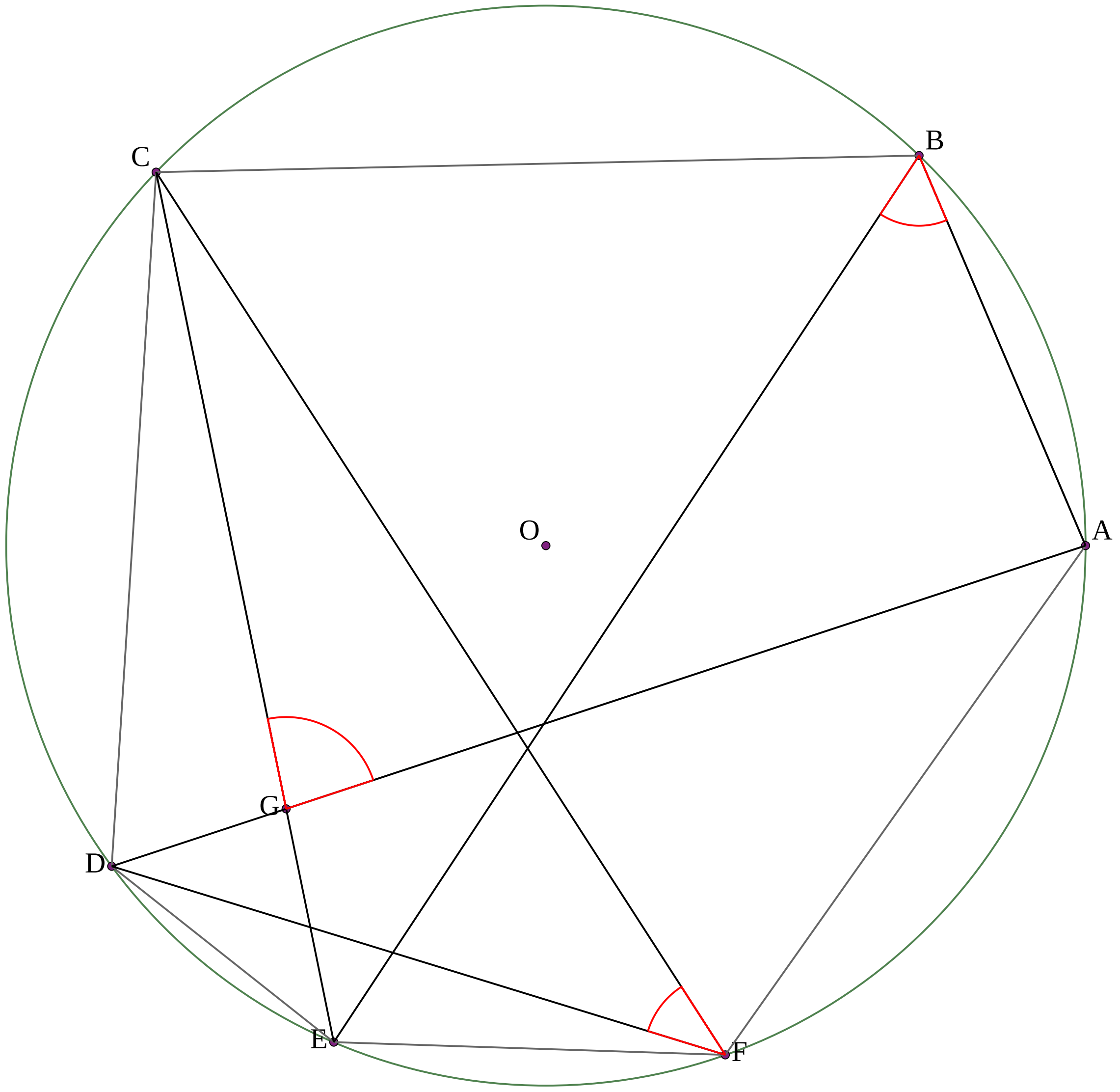
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CE and DA.
Prove that ABC = DFE+AGE

Example 21



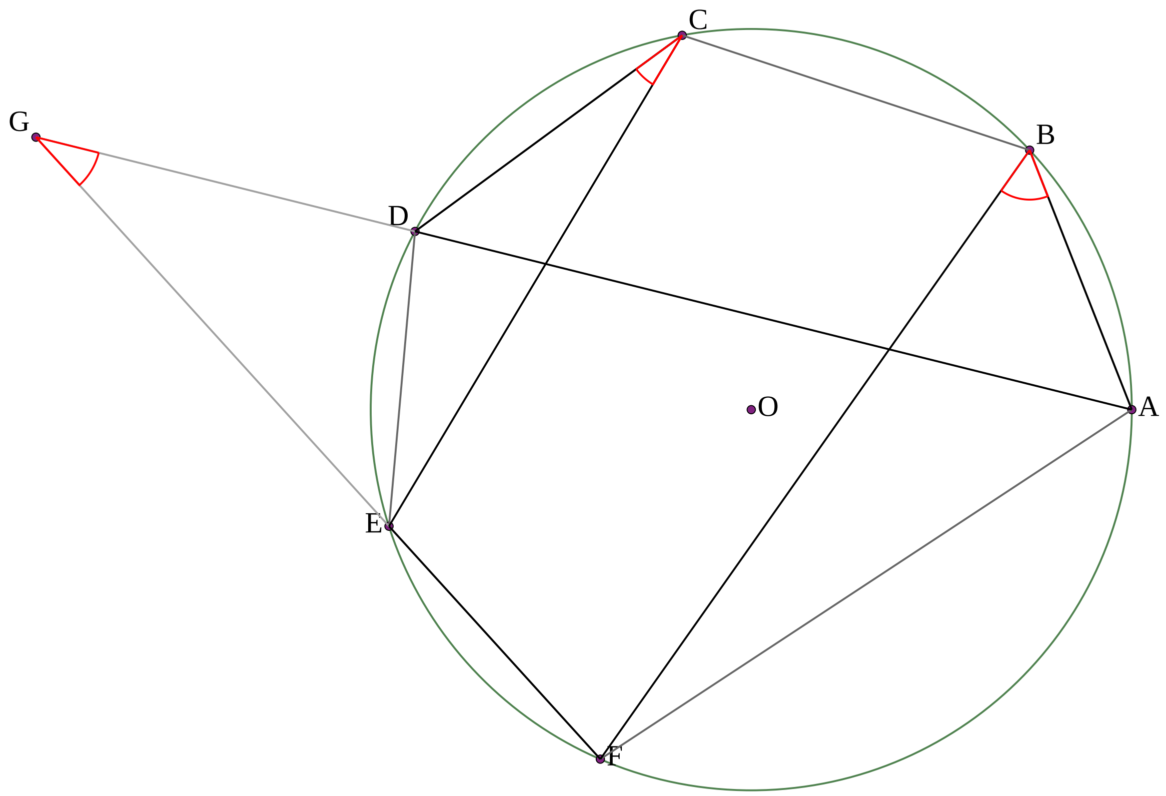
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and DA.
Prove that ABE = DCF+DGE

Example 22



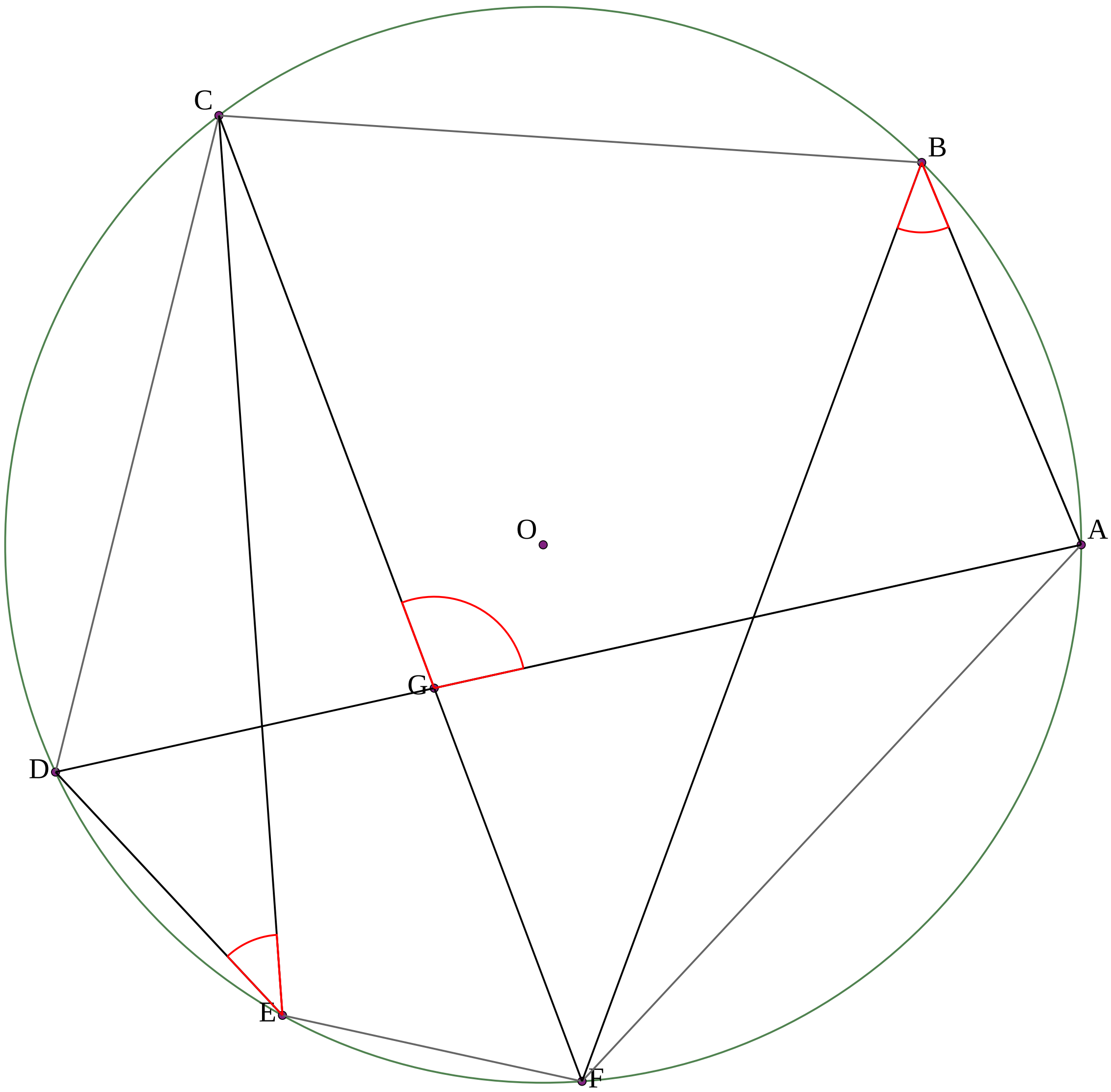
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and DA.
Prove that ABE+CFD+AGC = 180

Example 23



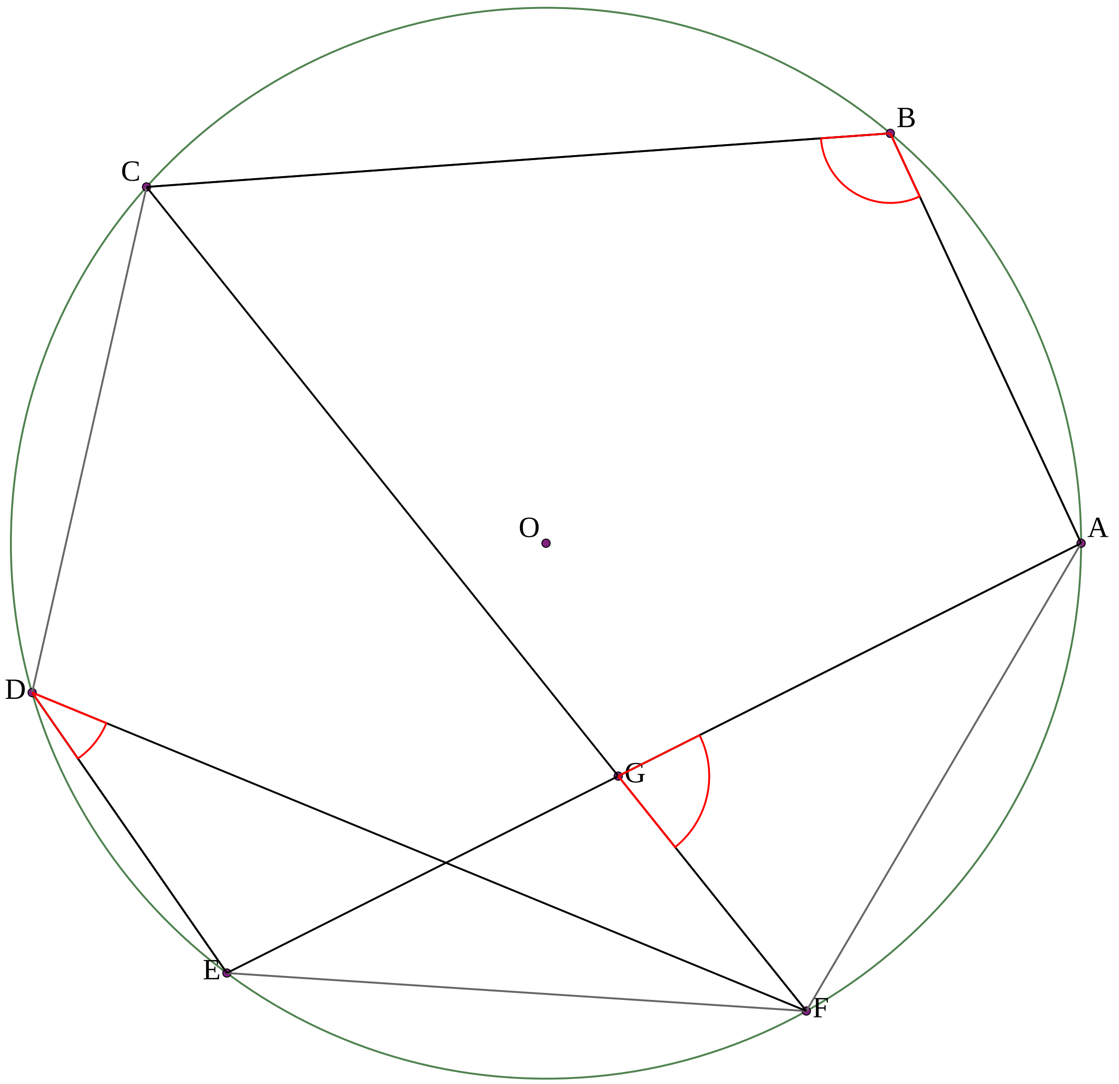
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FE and DA.
Prove that ABF = DCE+DGE

Example 24



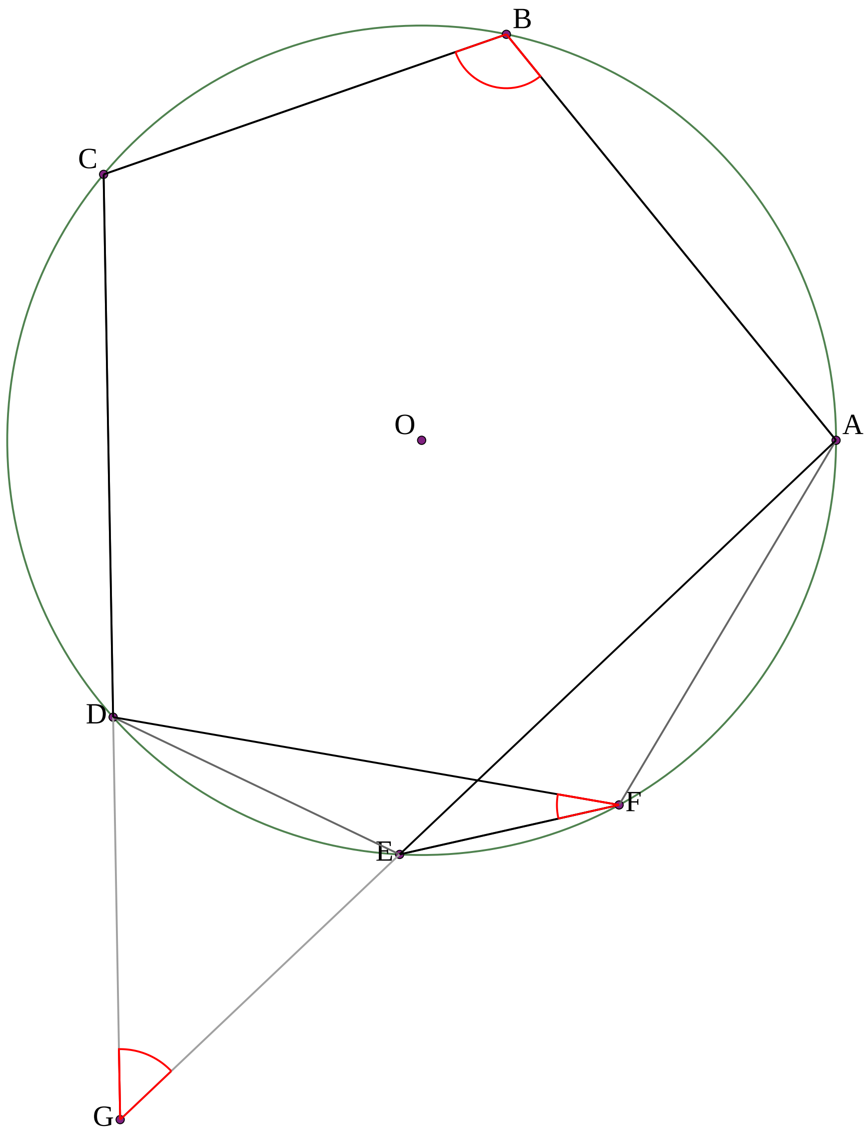
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FC and DA.
Prove that ABF+CED+AGC = 180

Example 25



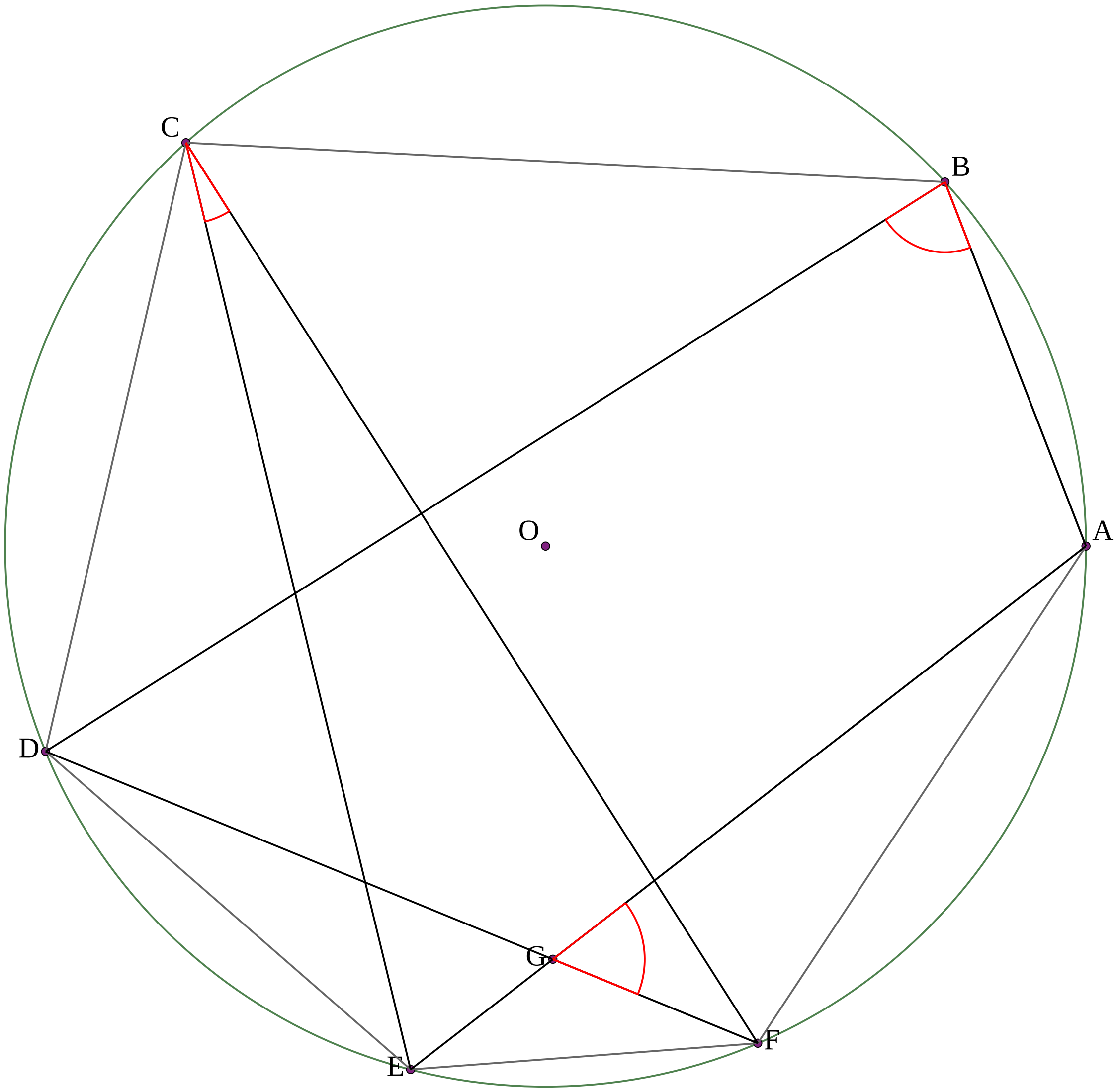
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CF and EA.
Prove that ABC = EDF+AGF

Example 26



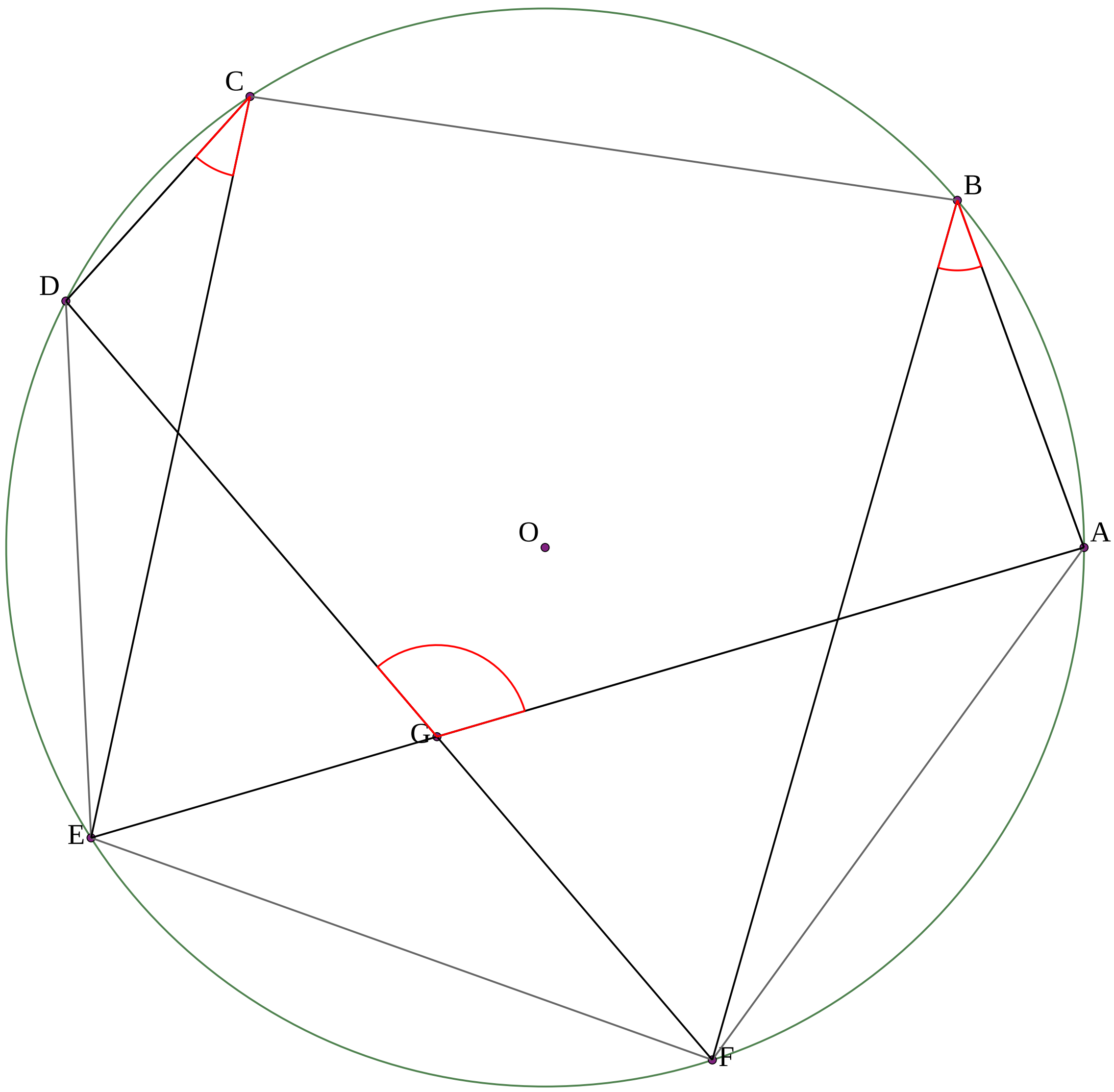
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CD and EA.
Prove that ABC+DFE+DGE = 180

Example 27



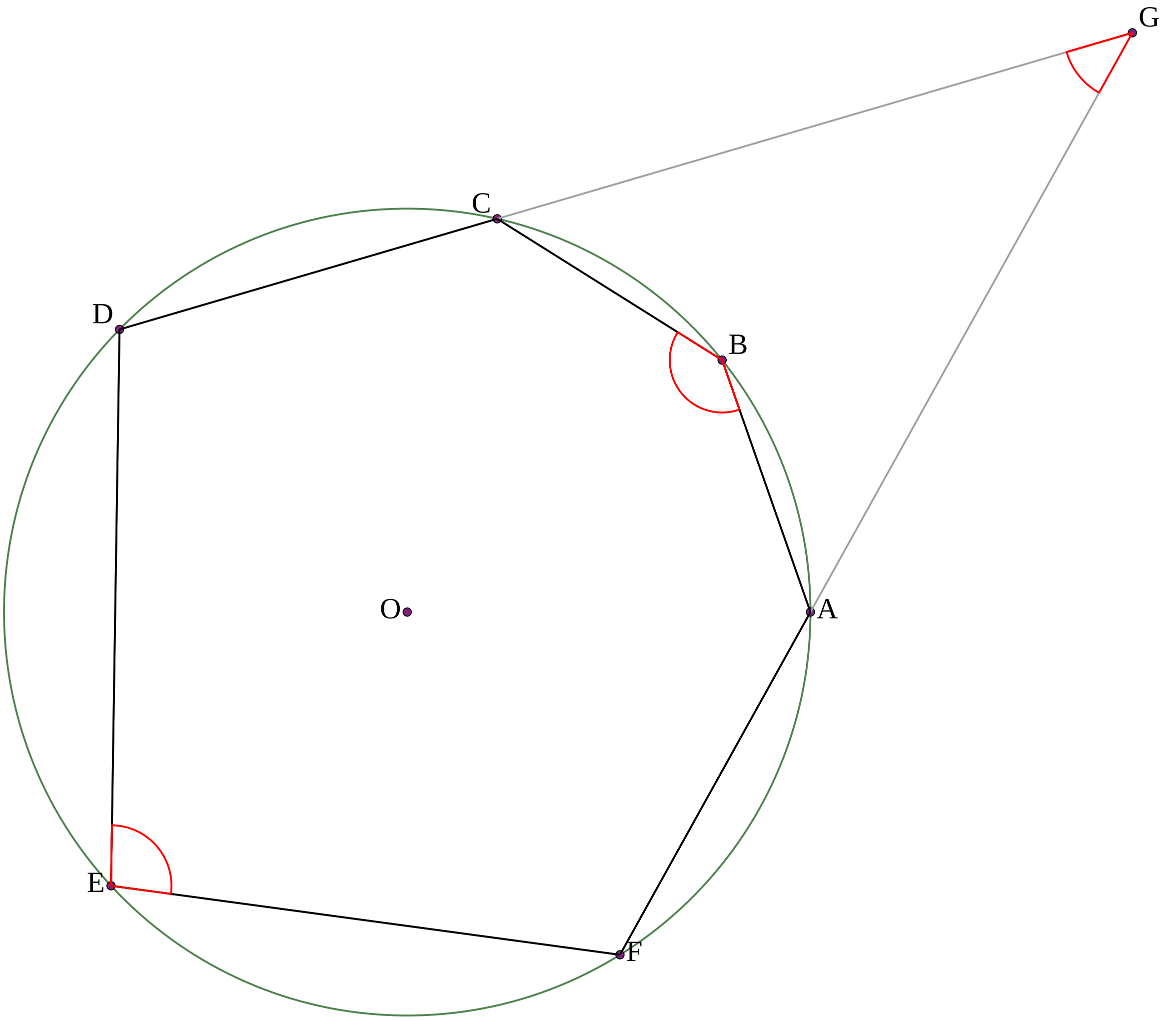
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DF and EA.
Prove that ABD = ECF+AGF

Example 28



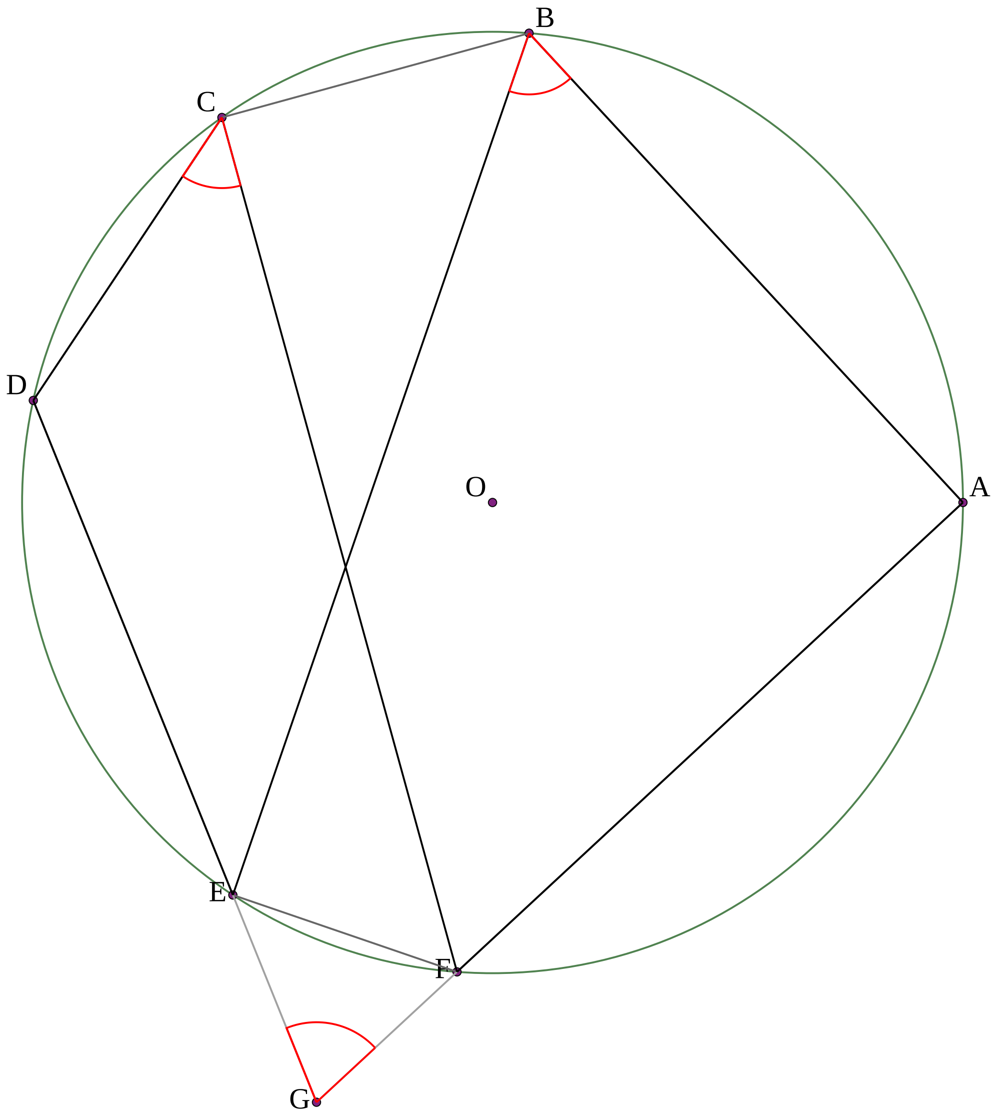
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and EA.
Prove that ABF+DCE+AGD = 180

Example 29



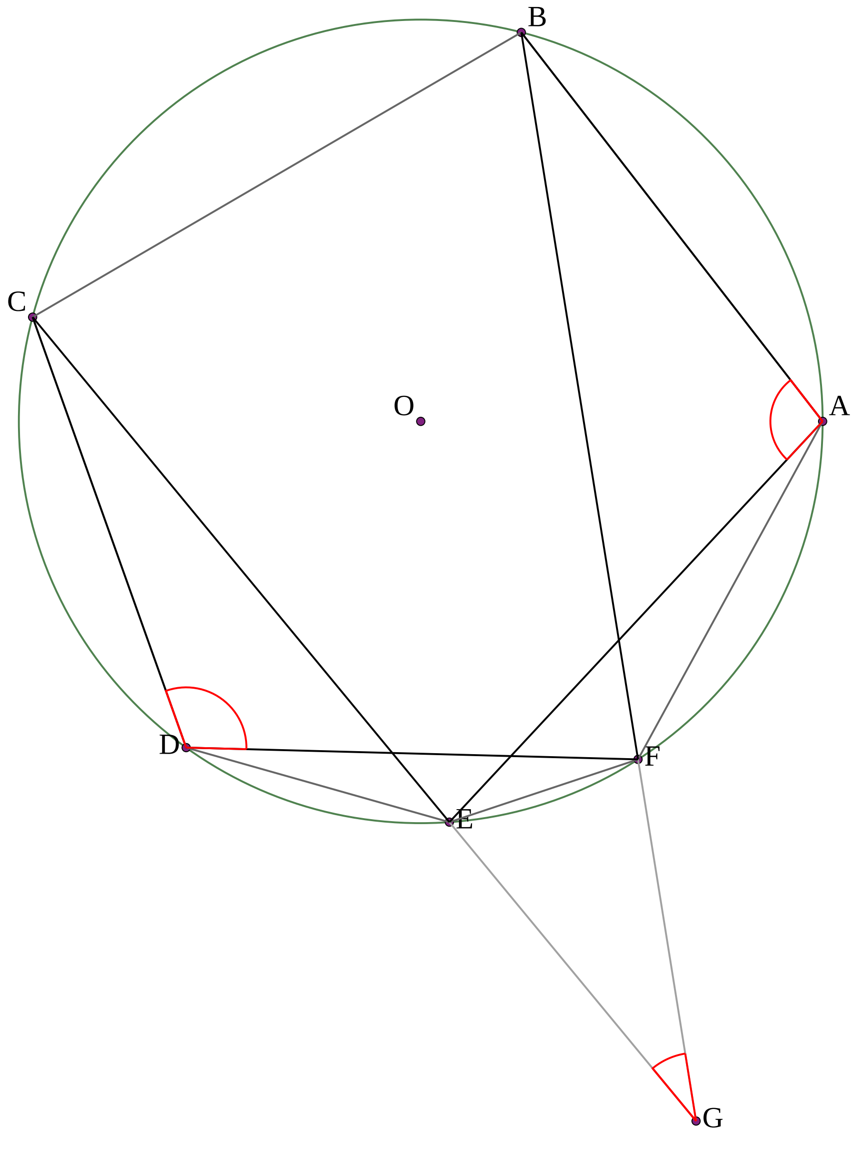
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CD and FA.
Prove that ABC = DEF+AGC

Example 30



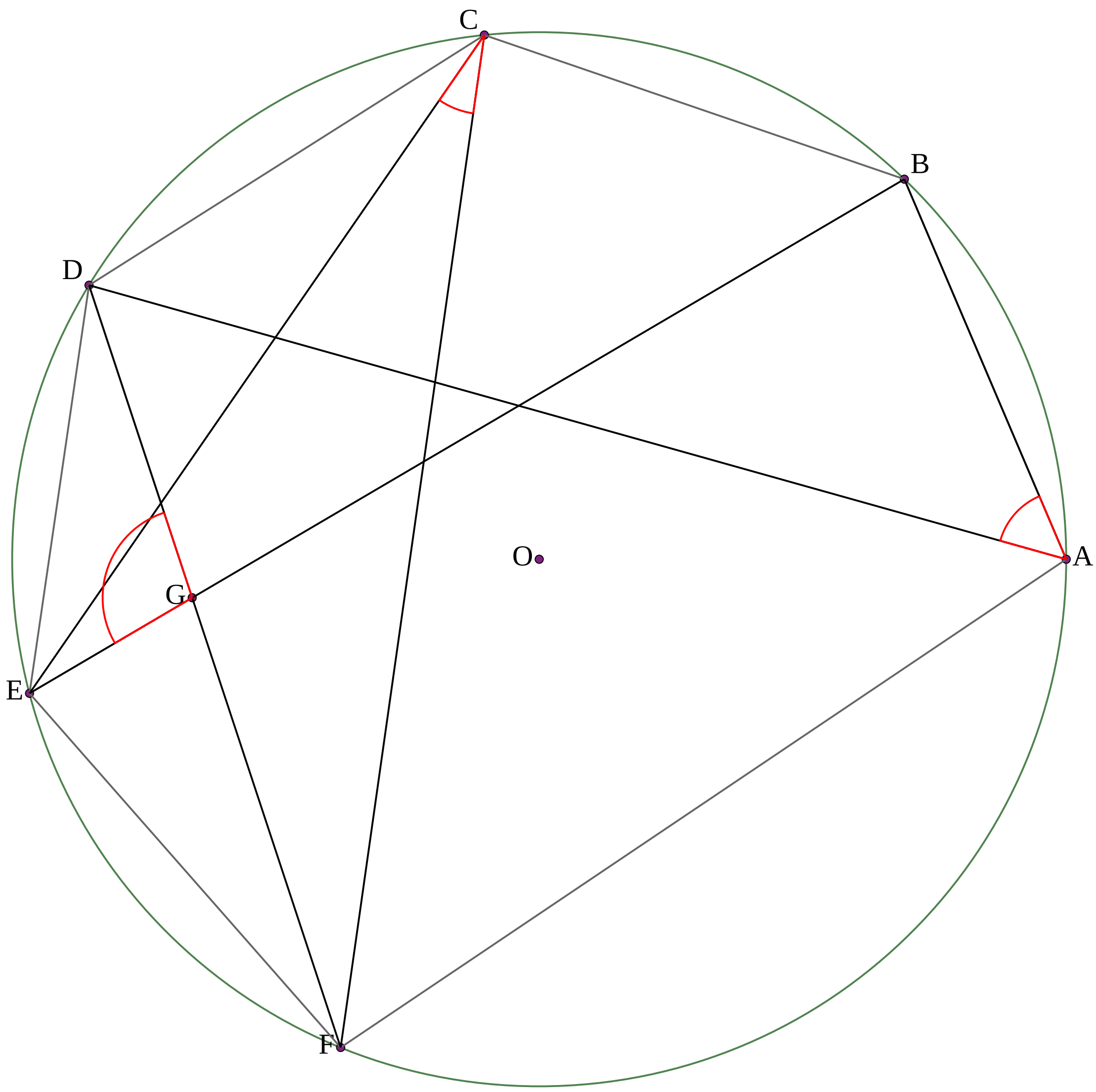
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and FA.
Prove that ABE+DCF+EGF = 180

Example 31



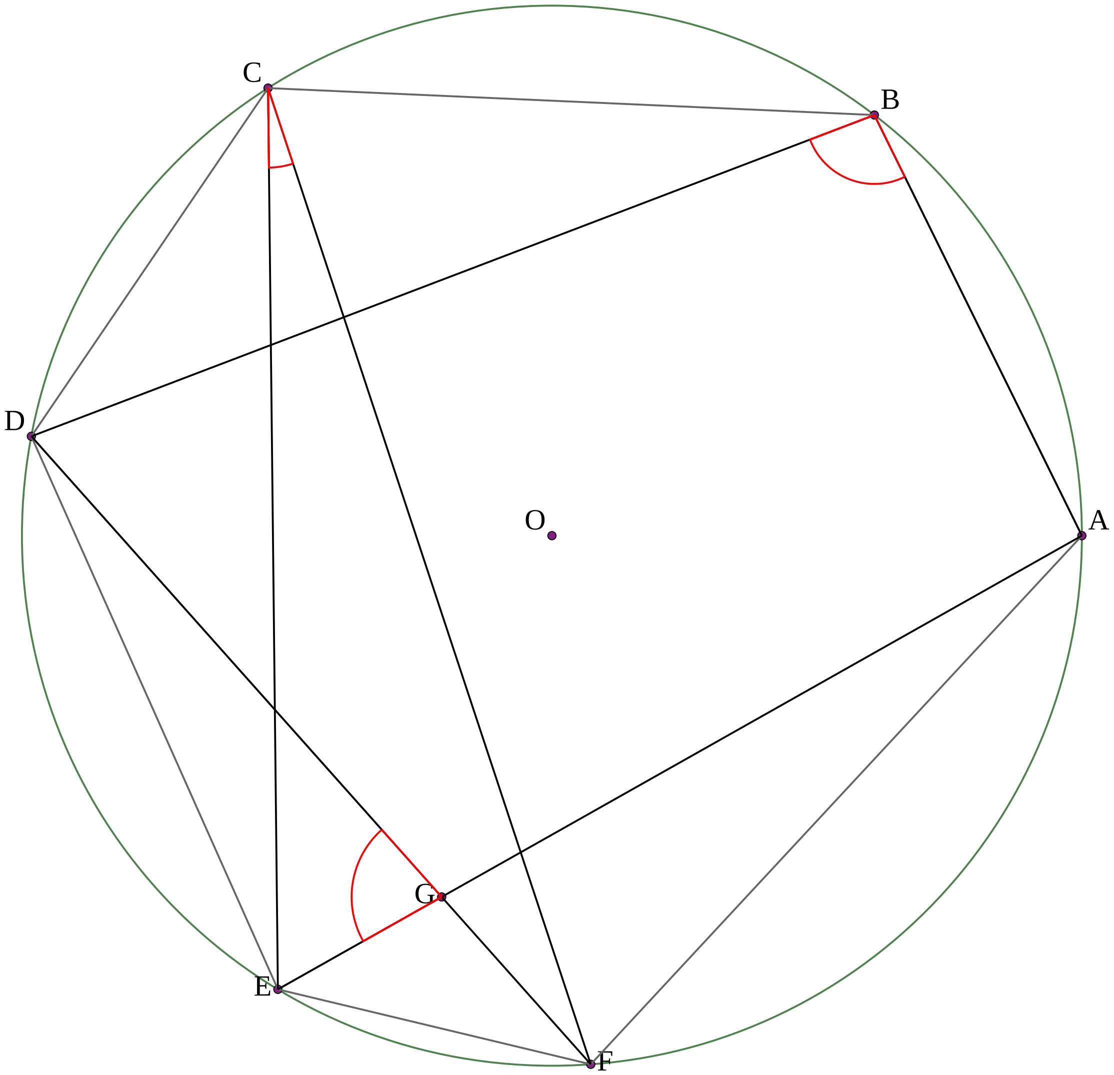
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and FB.
Prove that BAE+CDF = EGF+180

Example 32



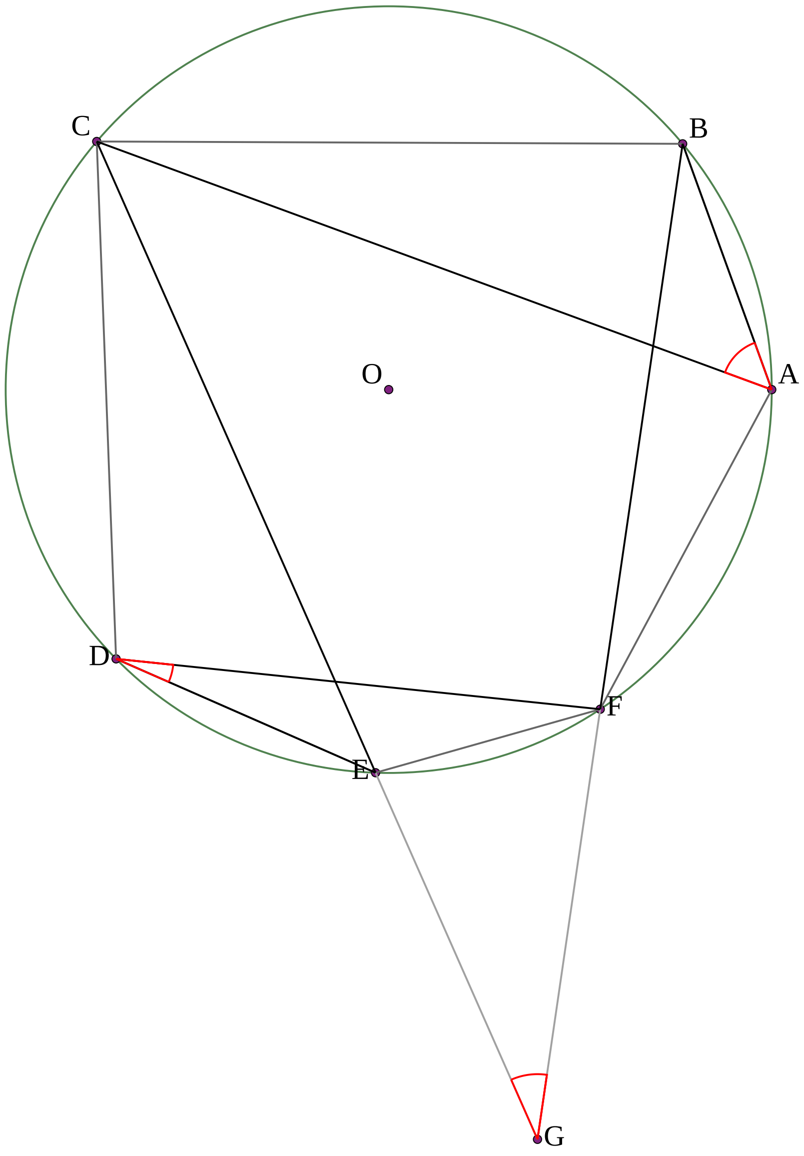
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BE and FD.
Prove that BAD+ECF+DGE = 180

Example 33



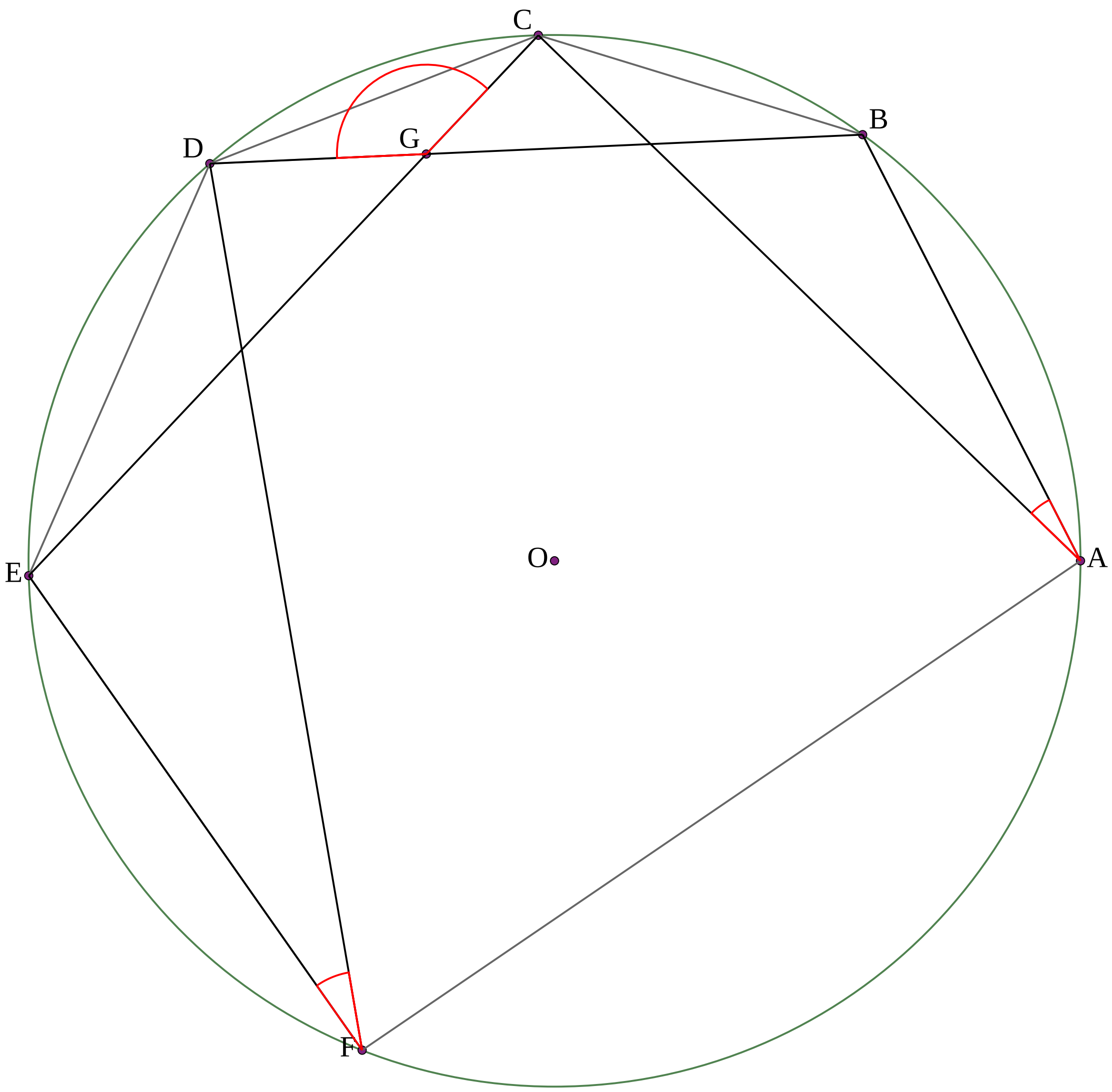
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AE and FD.
Prove that ECF+DGE = ABD

Example 34



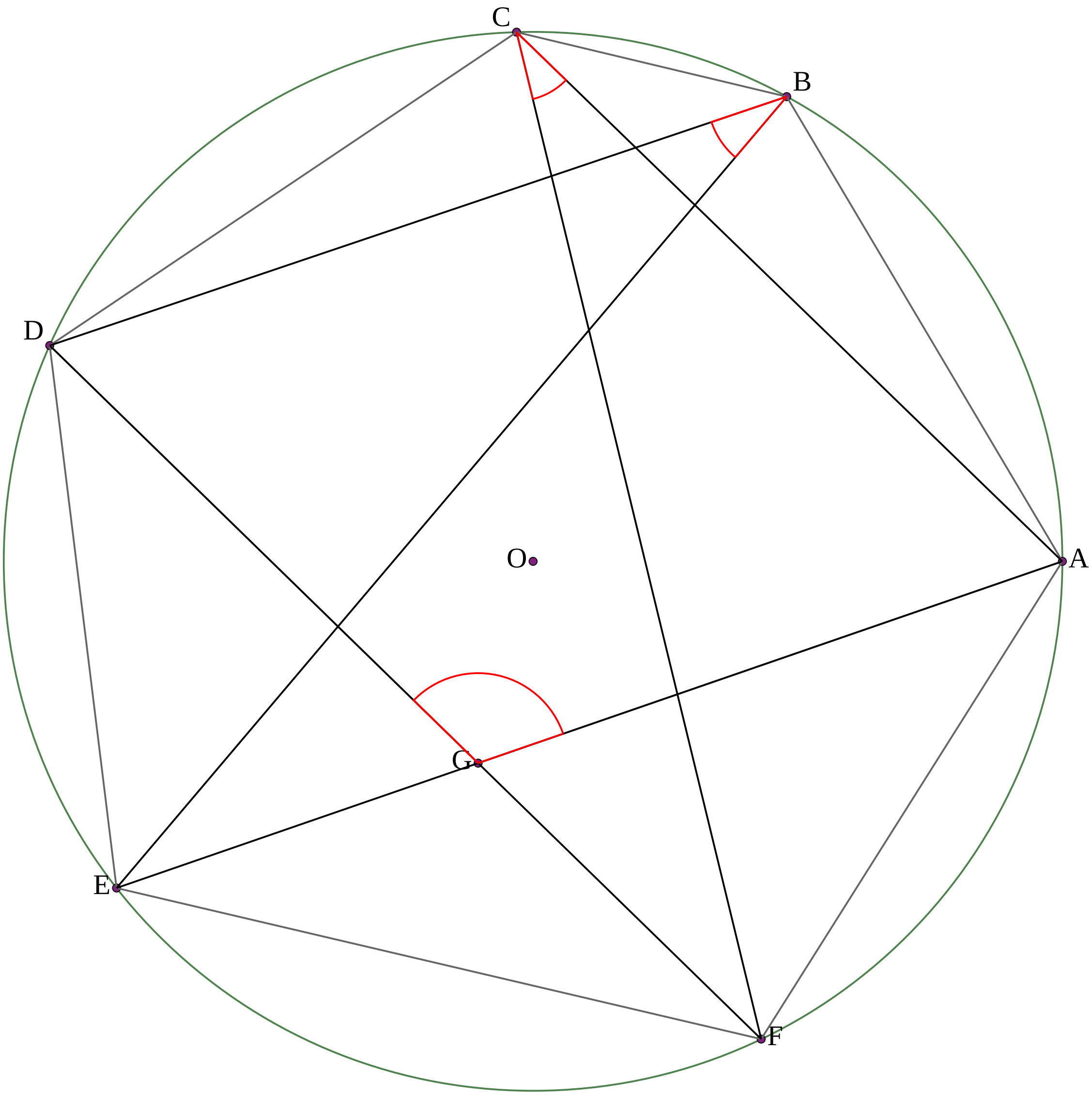
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BF and EC.
Prove that BAC = EDF+EGF

Example 35



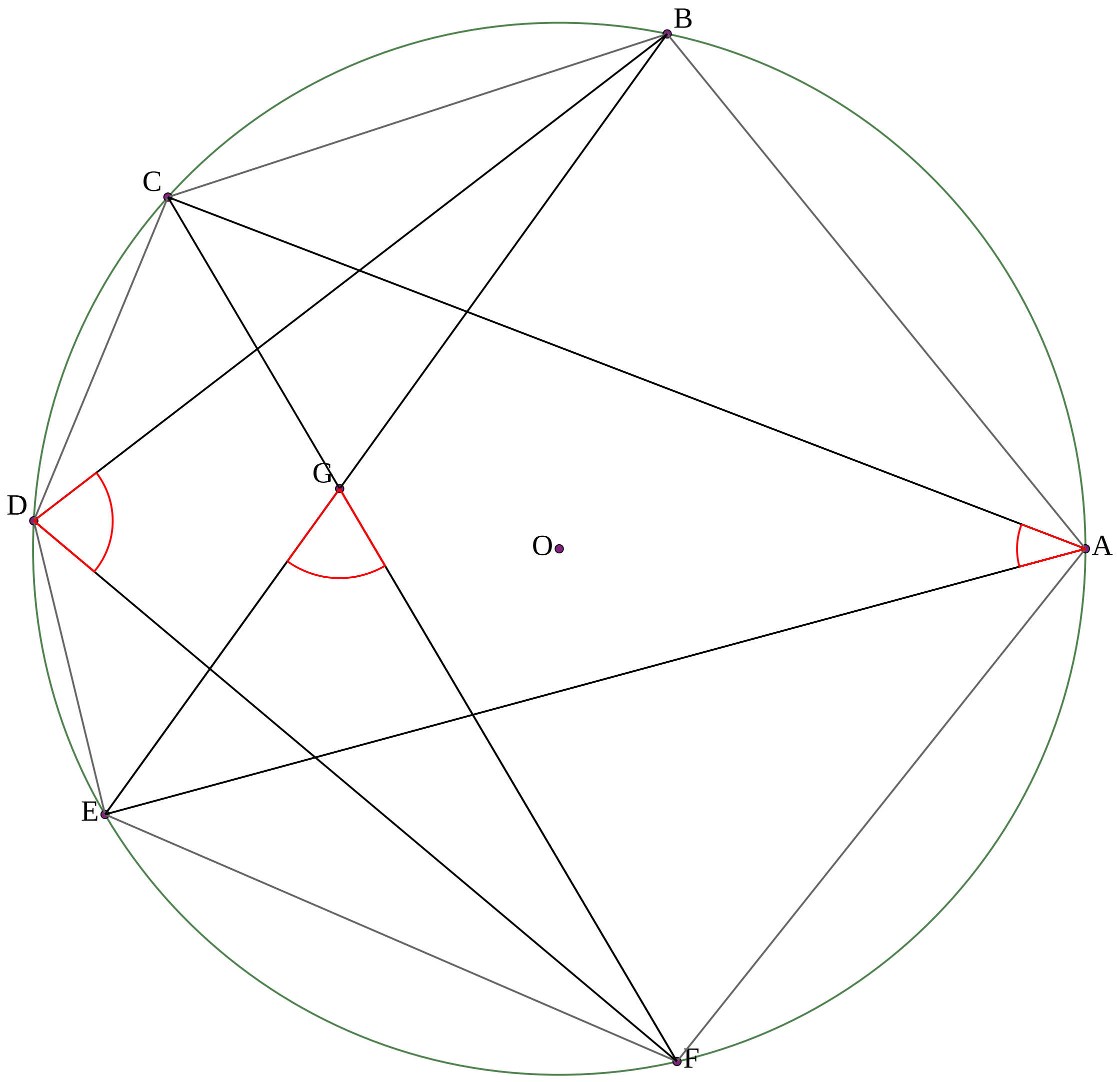
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BD and EC.
Prove that BAC+DFE+CGD = 180

Example 36



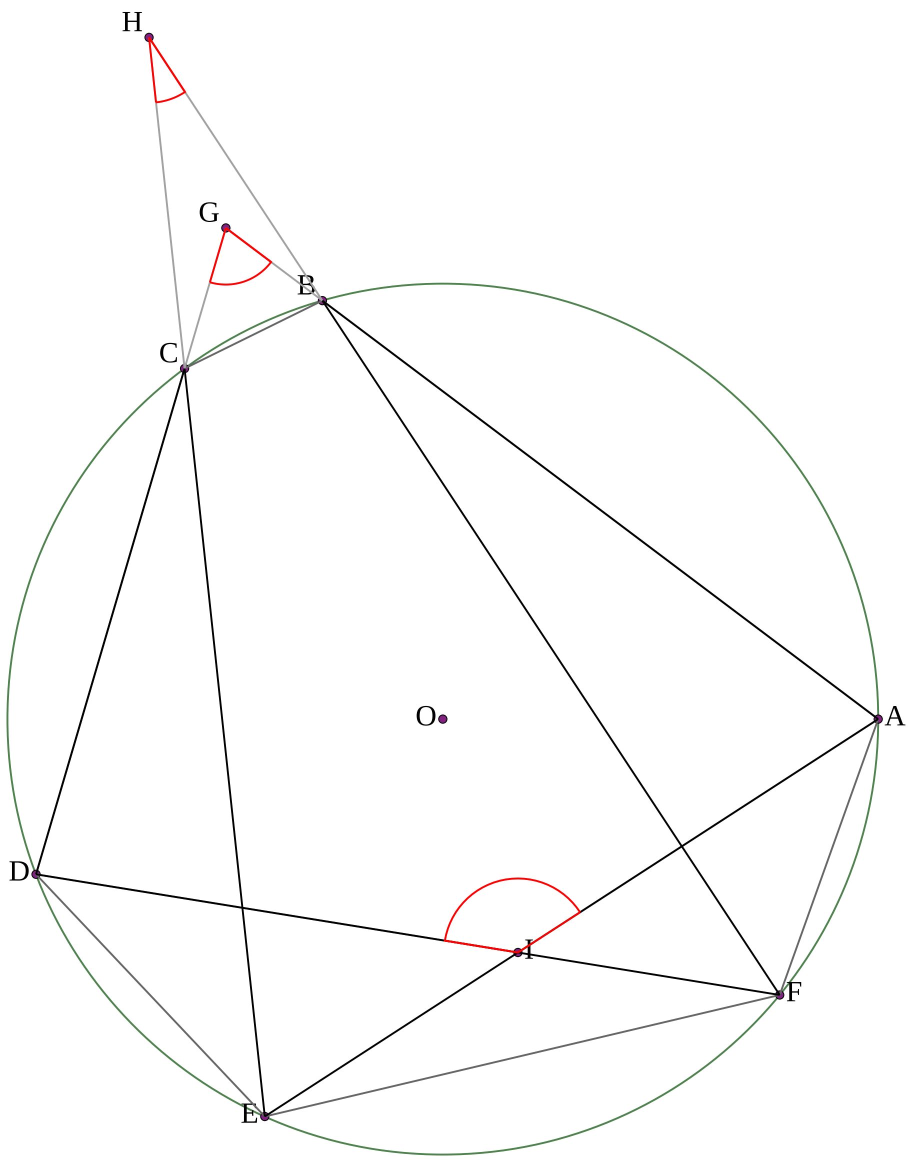
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and EA.
Prove that ACF+DBE+AGD = 180

Example 37



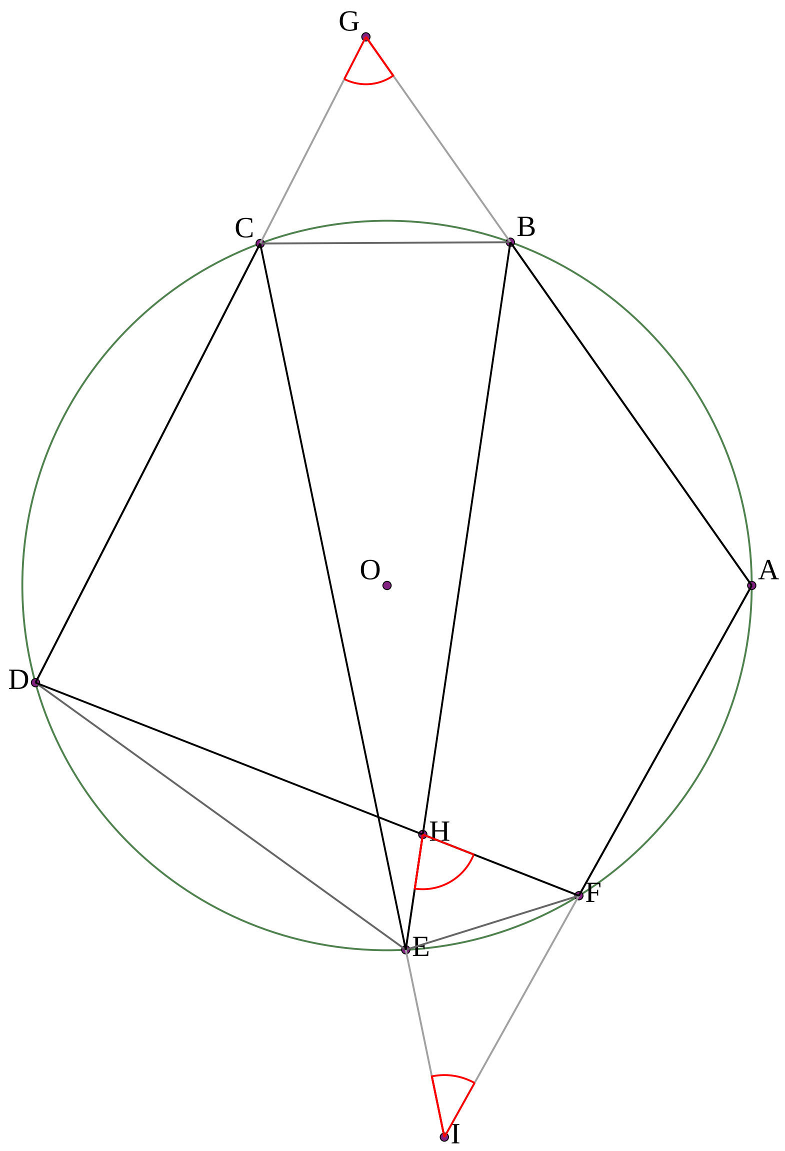
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CF and BE.
Prove that CAE+BDF+EGF = 180

Example 38



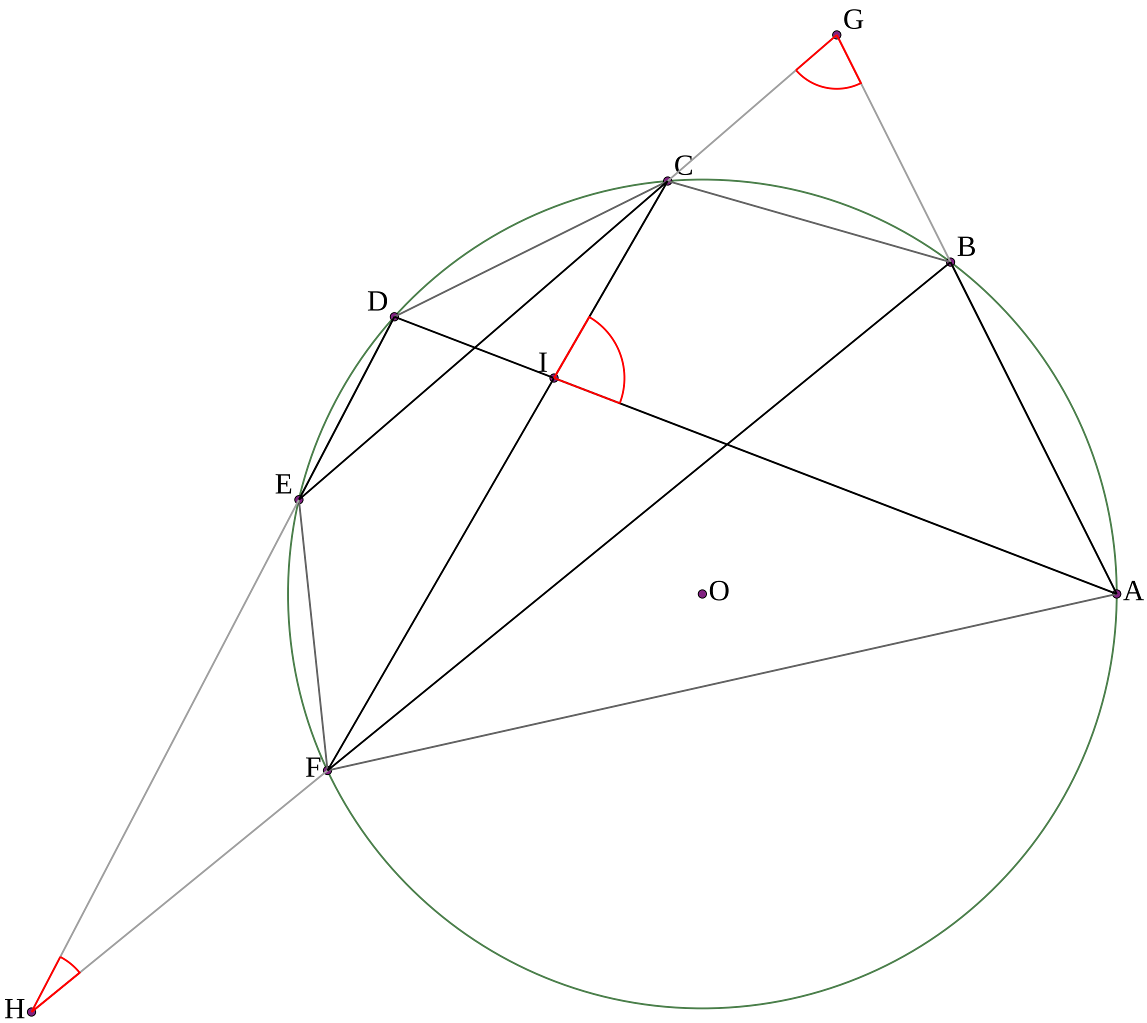
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and DC. Let H be the intersection of BF and CE. Let I be the intersection of FD and EA.
Prove that BGC+AID = BHC+180

Example 39



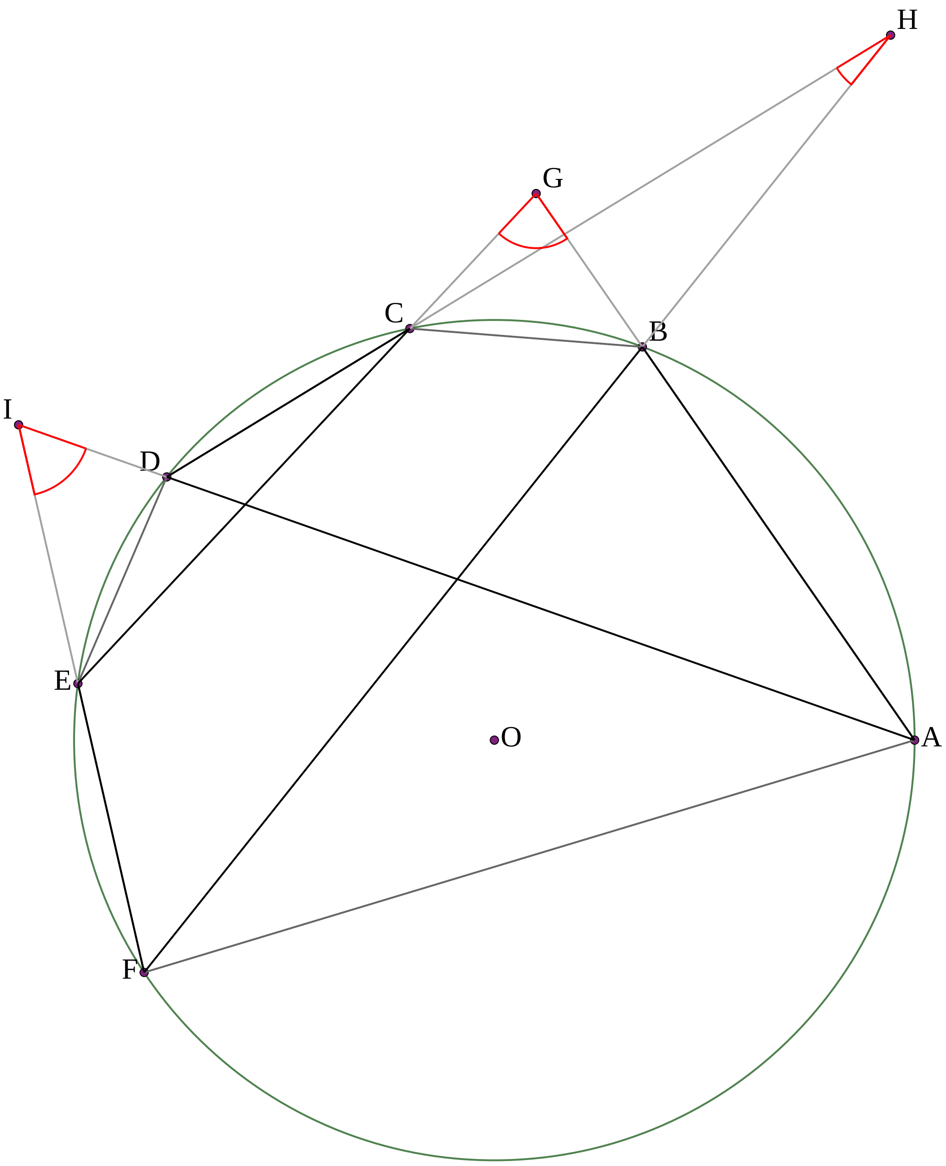
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CD. Let H be the intersection of BE and DF. Let I be the intersection of EC and FA.
Prove that BGC+EHF+EIF = 180

Example 40



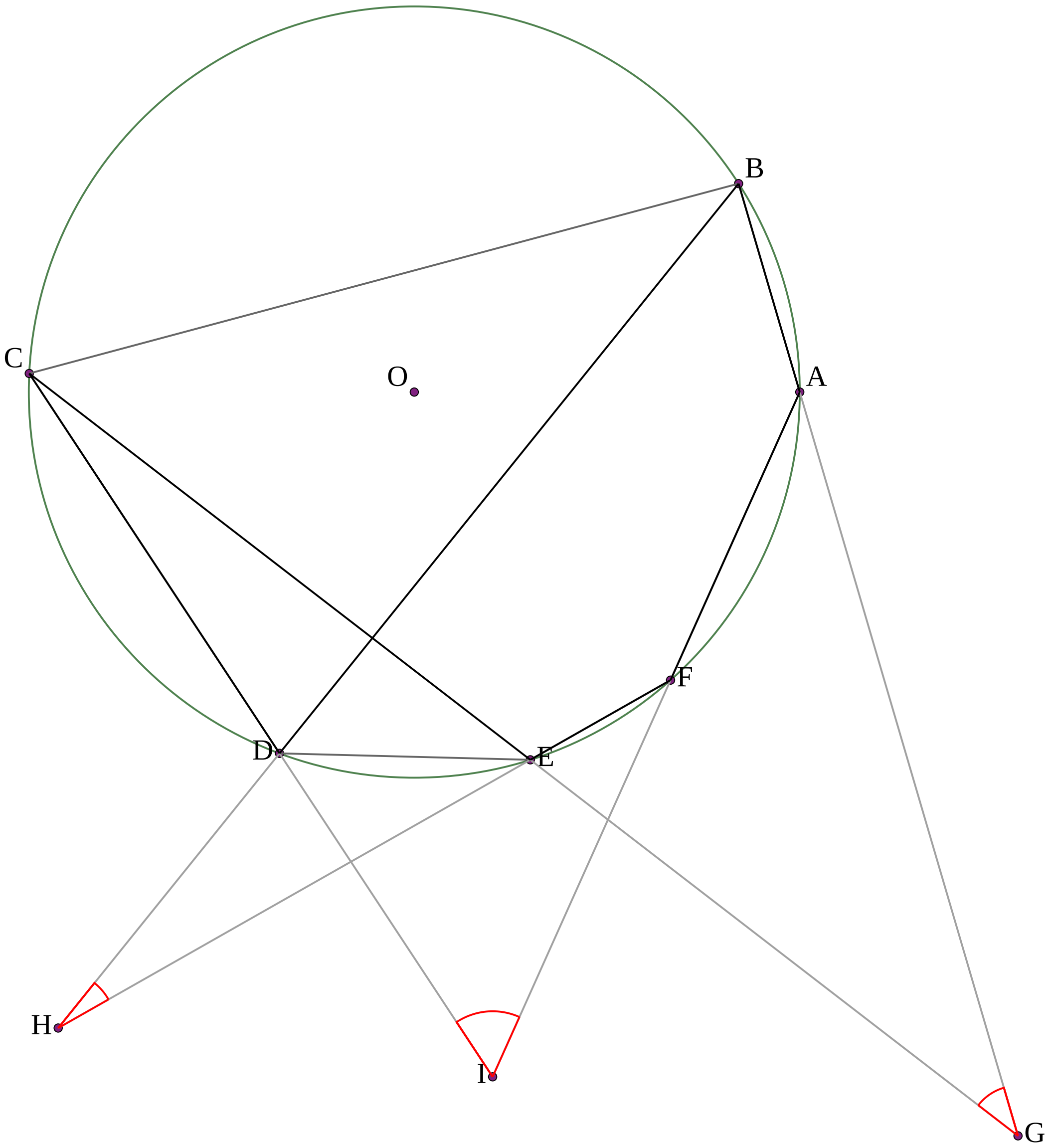
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CE. Let H be the intersection of BF and ED. Let I be the intersection of FC and DA.
Prove that BGC+EHF+AIC = 180

Example 41



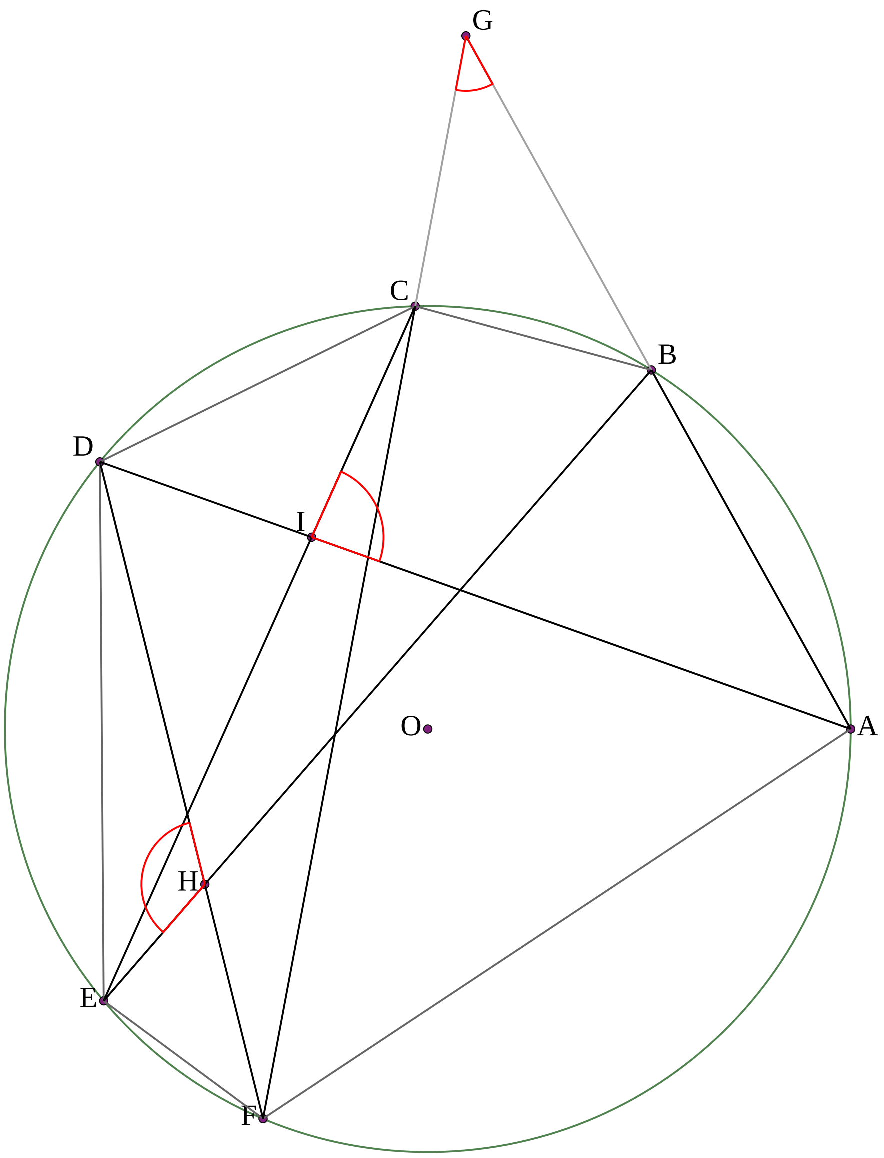
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and EC. Let H be the intersection of BF and CD. Let I be the intersection of FE and DA.
Prove that BGC = BHC+DIE

Example 42



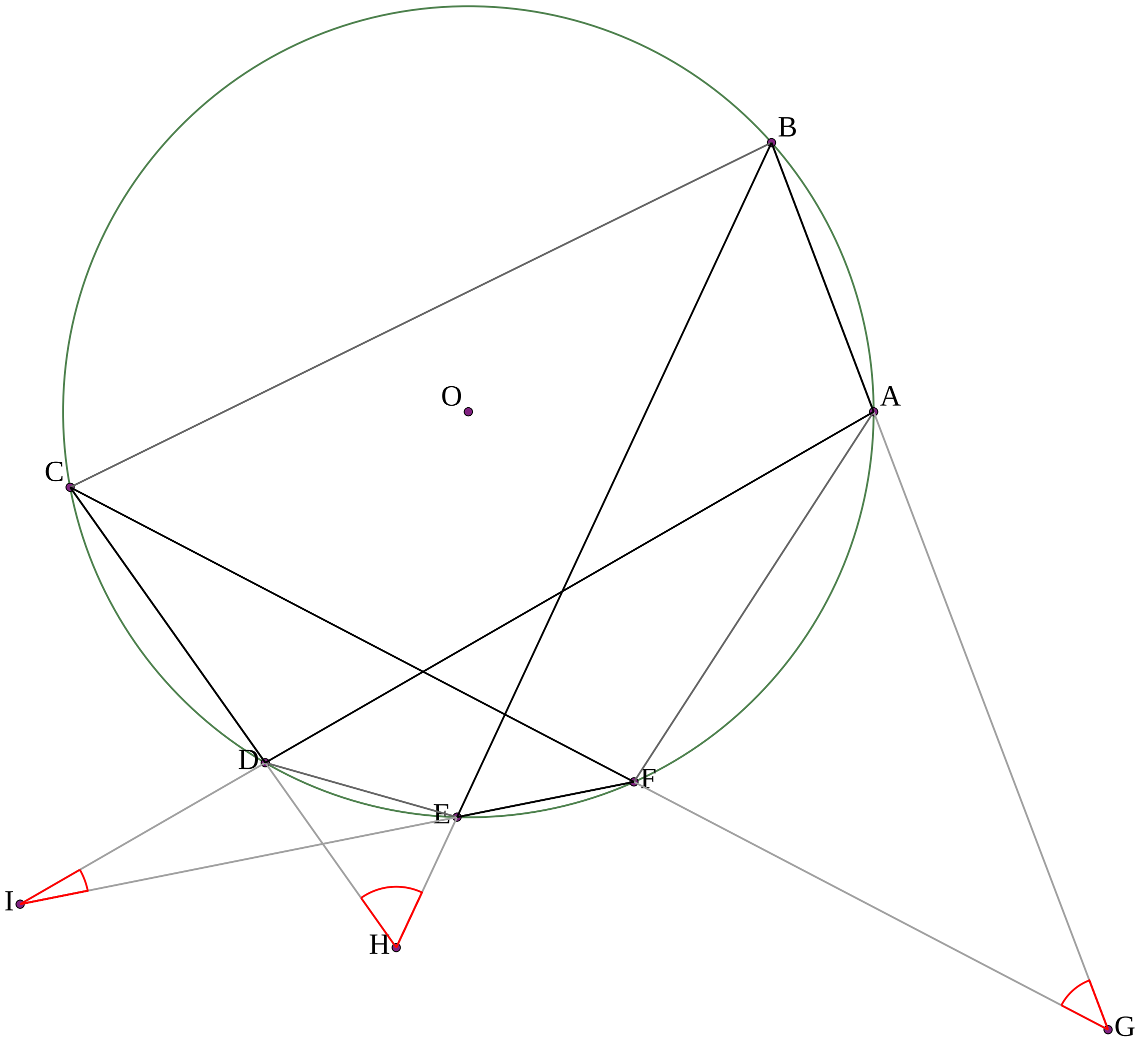
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CE. Let H be the intersection of BD and EF. Let I be the intersection of DC and FA.
Prove that DIF = AGE+DHE

Example 43



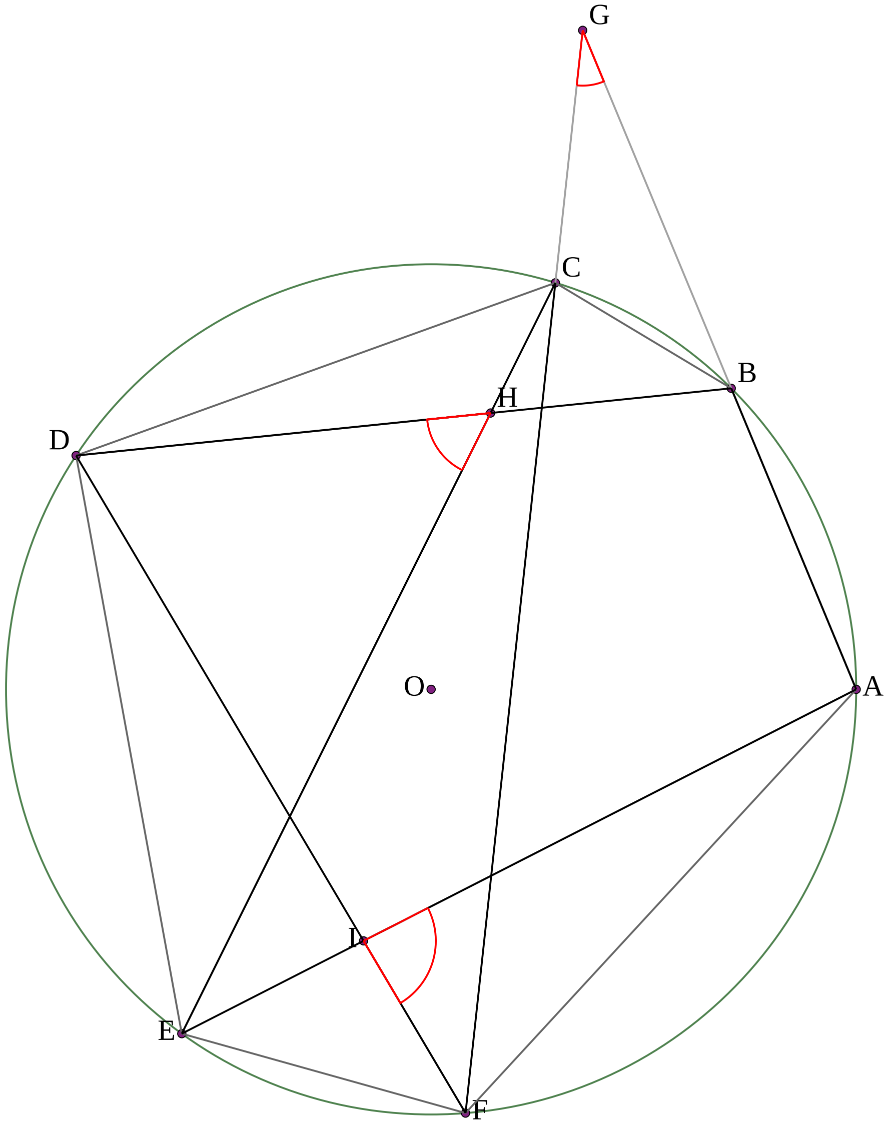
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CF. Let H be the intersection of BE and FD. Let I be the intersection of EC and DA.
Prove that BGC+AIC = DHE

Example 44



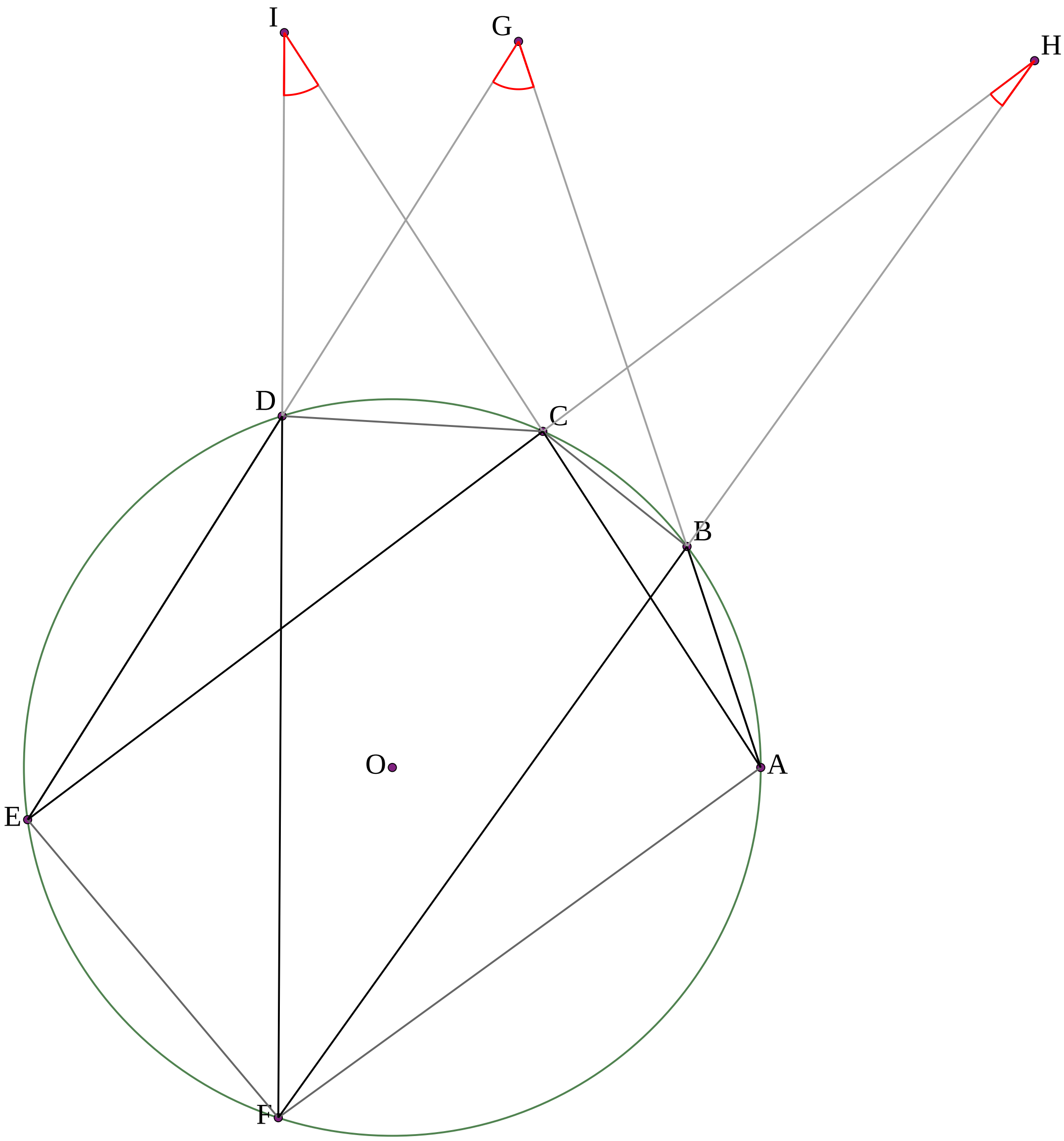
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and FC. Let H be the intersection of BE and CD. Let I be the intersection of EF and DA.
Prove that DHE = AGF+DIE

Example 45



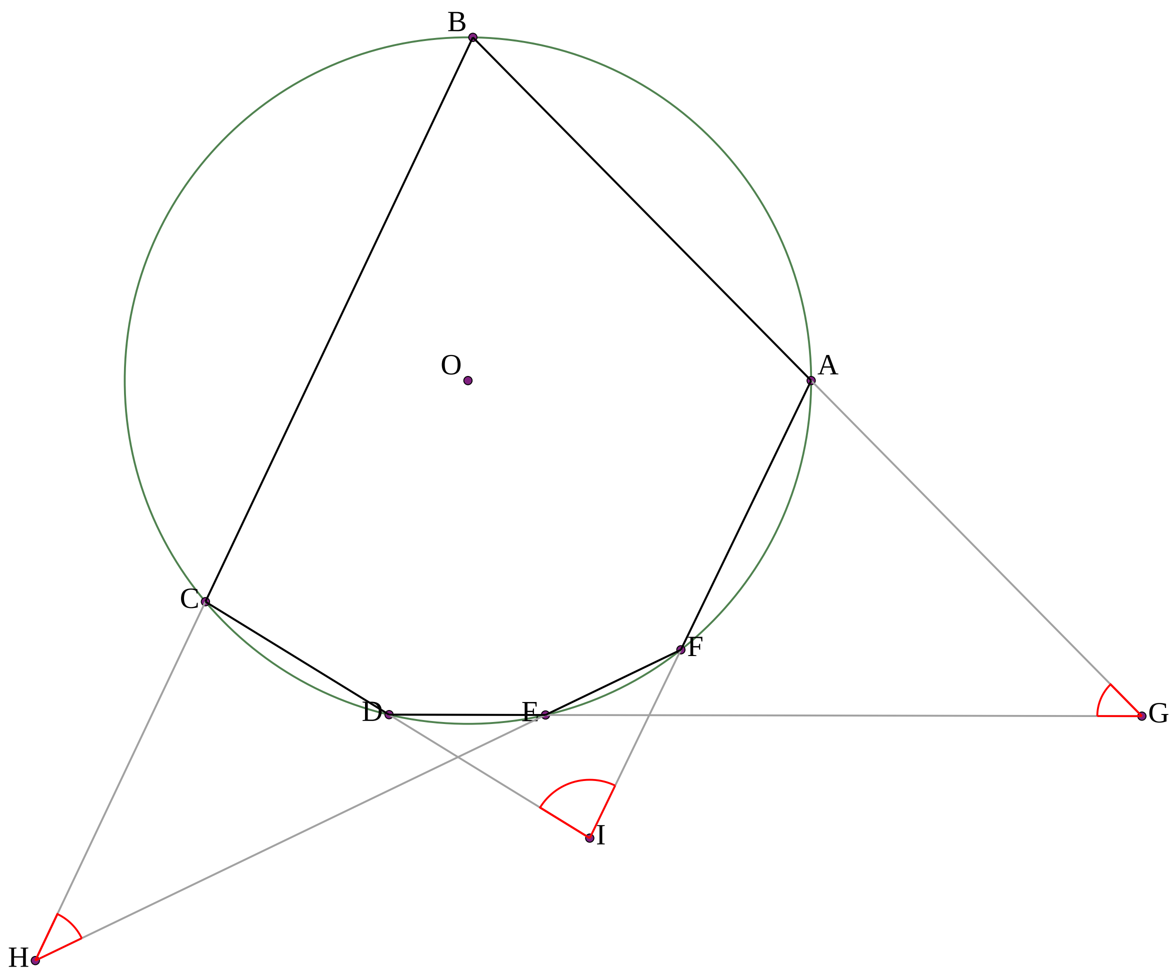
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and FC. Let H be the intersection of BD and CE. Let I be the intersection of DF and EA.
Prove that BGC+DHE = AIF

Example 46



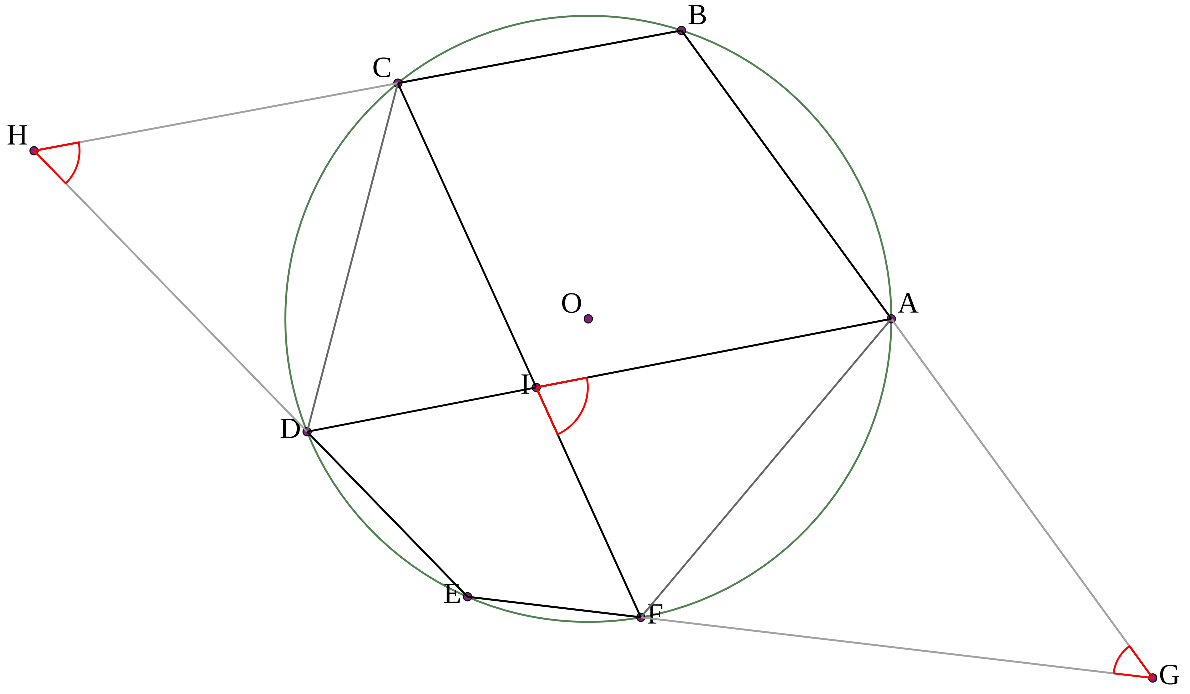
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and DE. Let H be the intersection of BF and EC. Let I be the intersection of FD and CA.
Prove that BGD = BHC+CID

Example 47



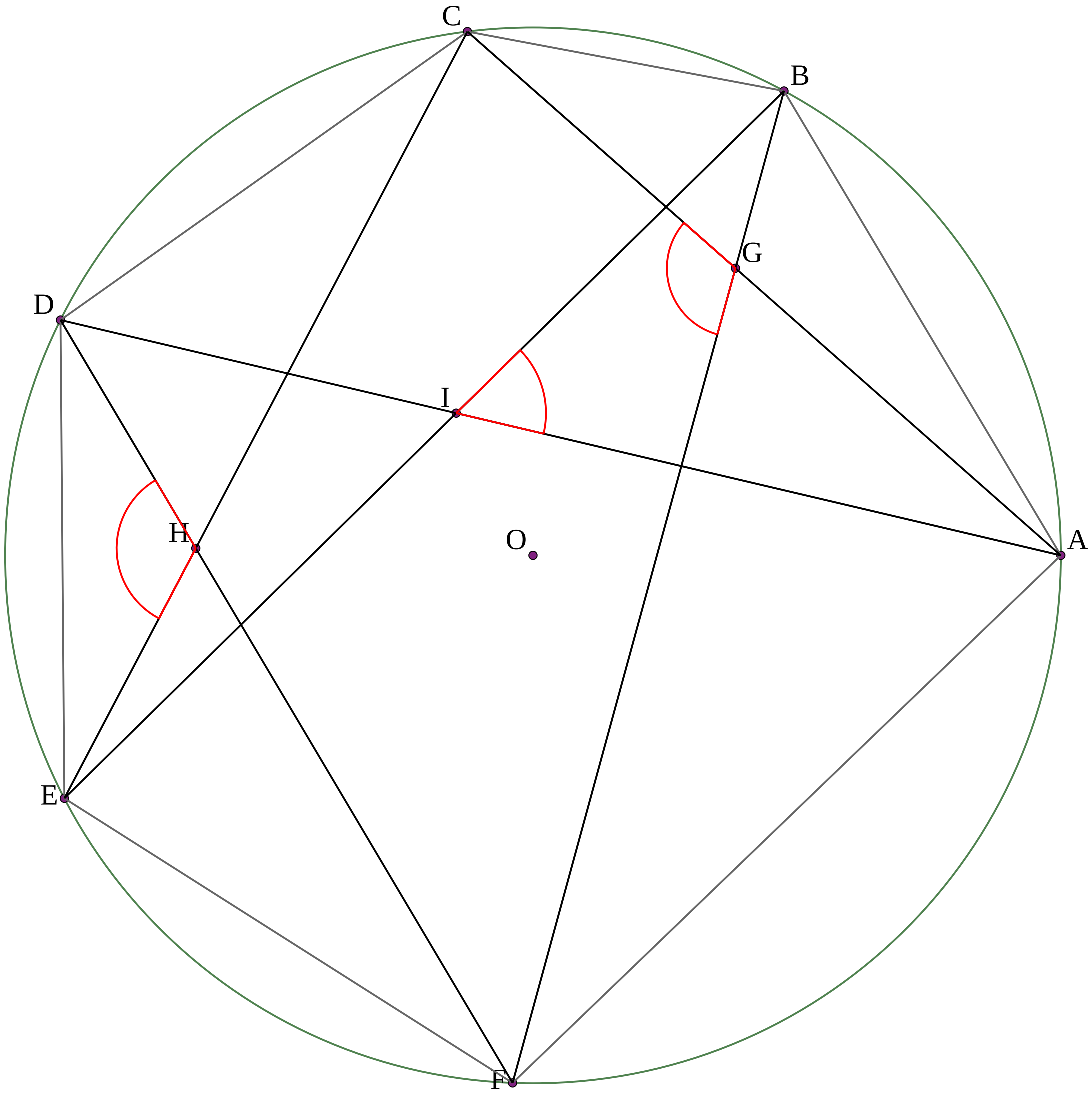
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and DE. Let H be the intersection of BC and EF. Let I be the intersection of CD and FA.
Prove that DIF = AGE+CHE

Example 48



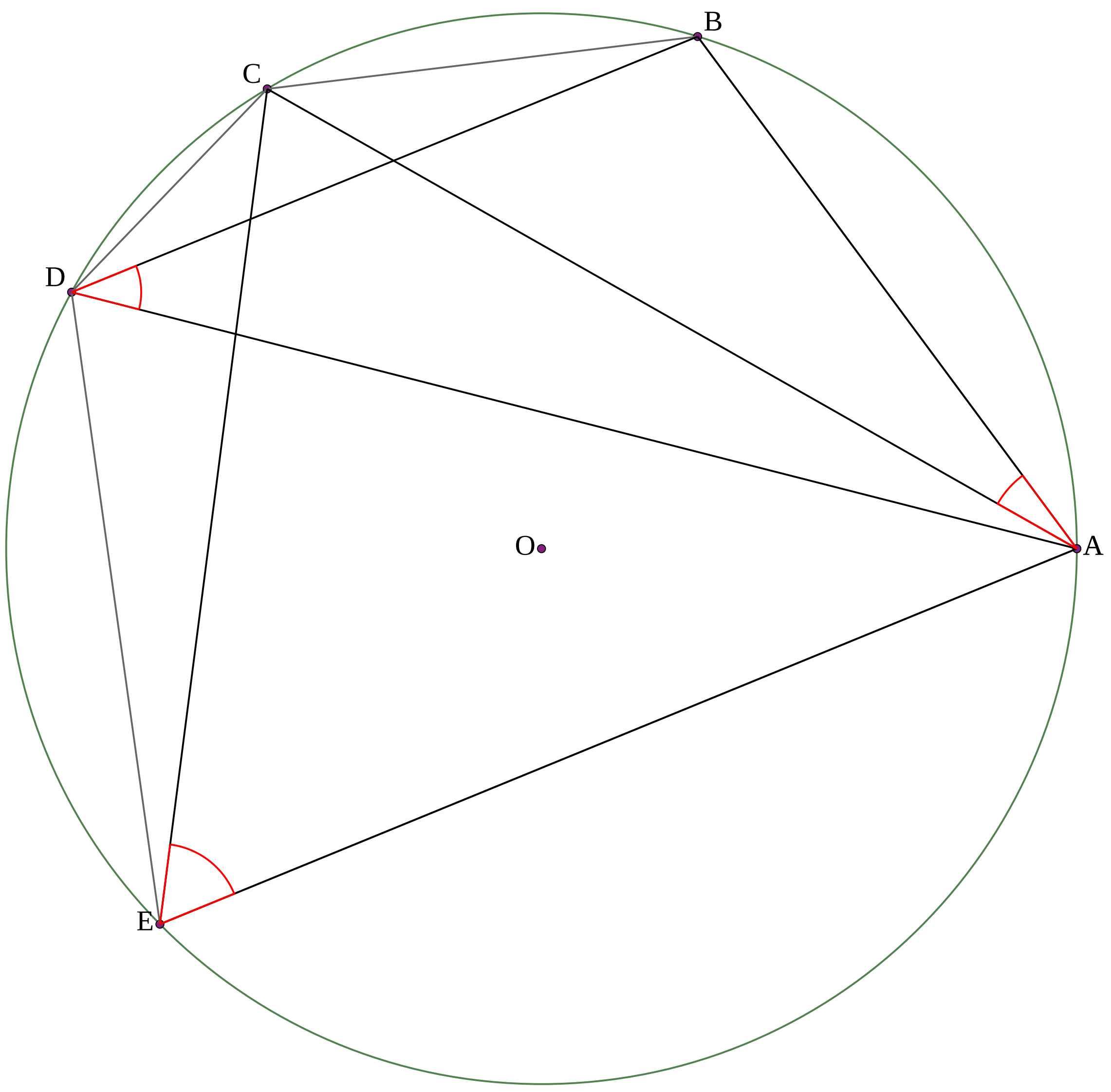
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and FE. Let H be the intersection of BC and ED. Let I be the intersection of CF and DA.
Prove that AGF+CHD+AIF = 180

Example 49



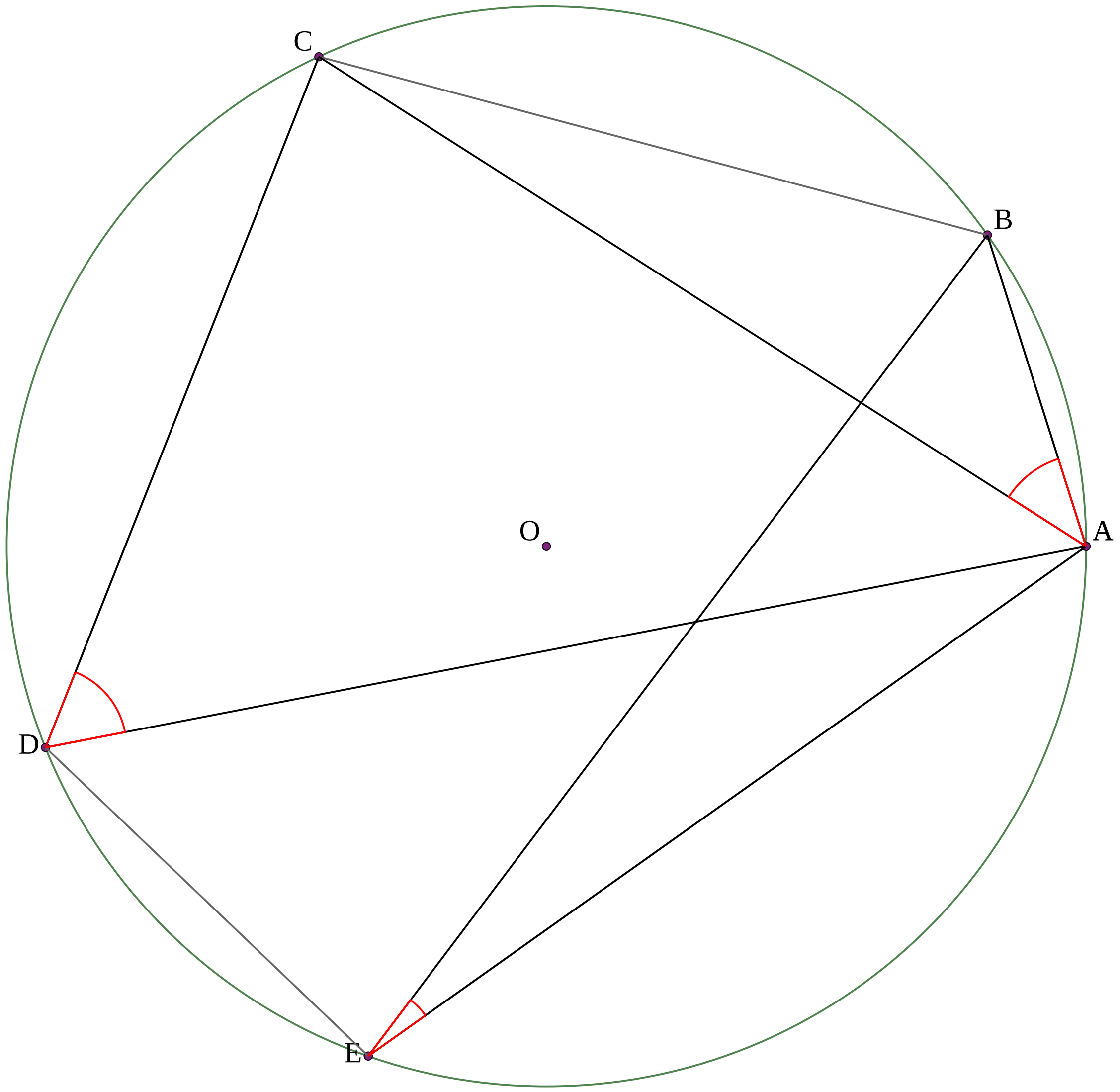
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AC and BF. Let H be the intersection of CE and FD. Let I be the intersection of EB and DA.
Prove that CGF+DHE = AIB+180

Example 50



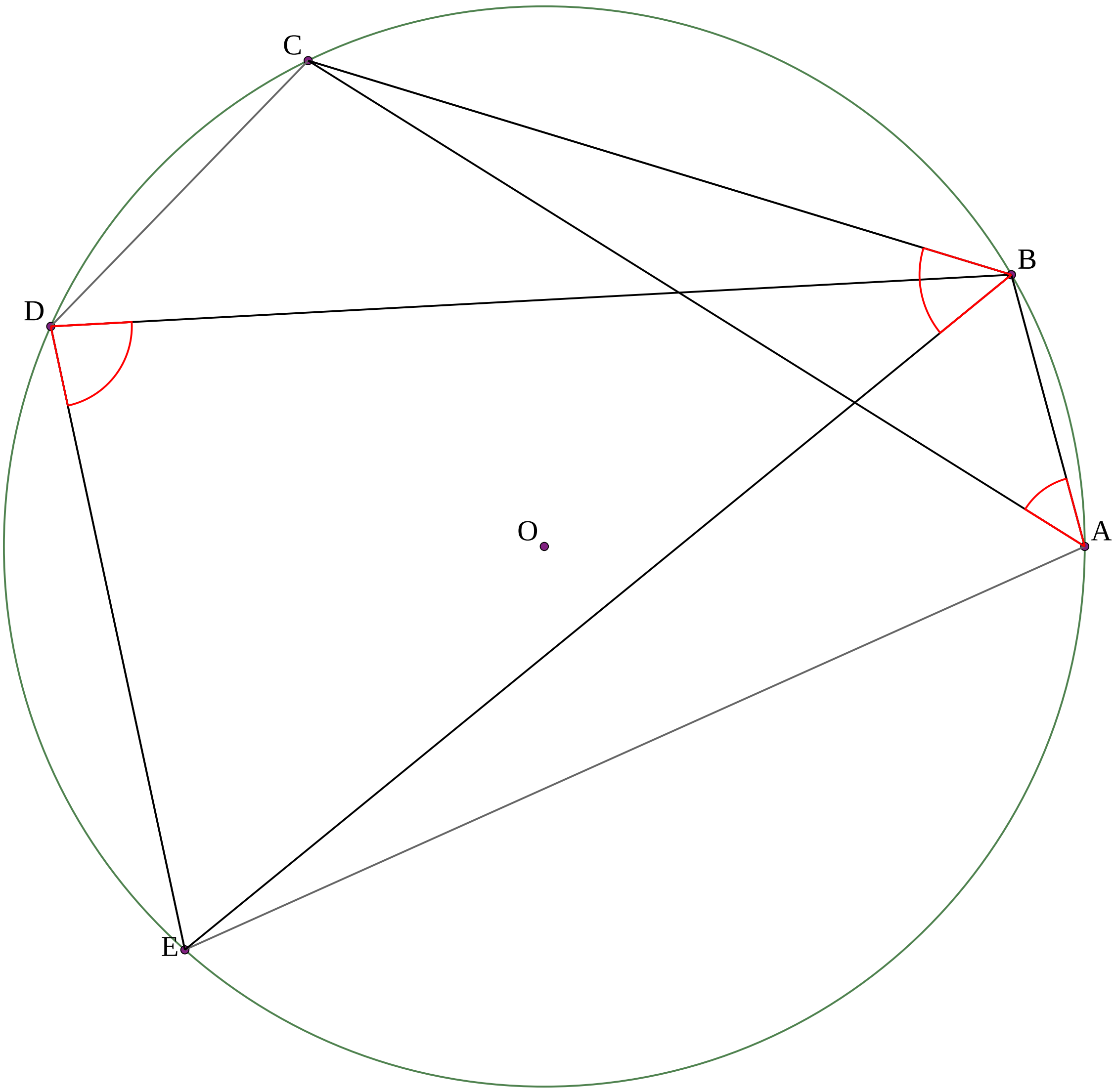
Let ABCDE be a cyclic pentagon with center O.
Prove that ADB+BAC = AEC

Example 51



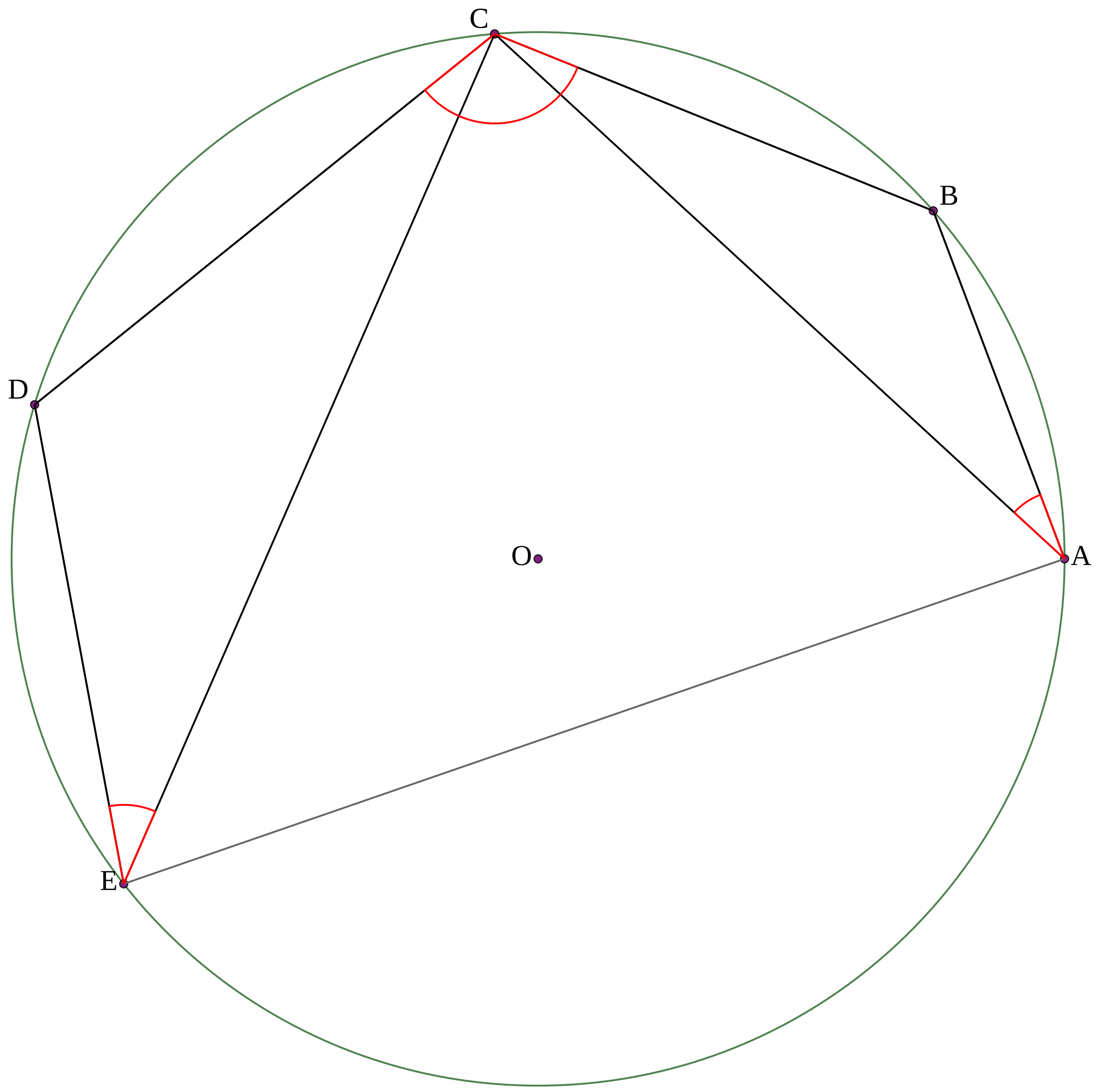
Let ABCDE be a cyclic pentagon with center O.
Prove that AEB+BAC = ADC

Example 52



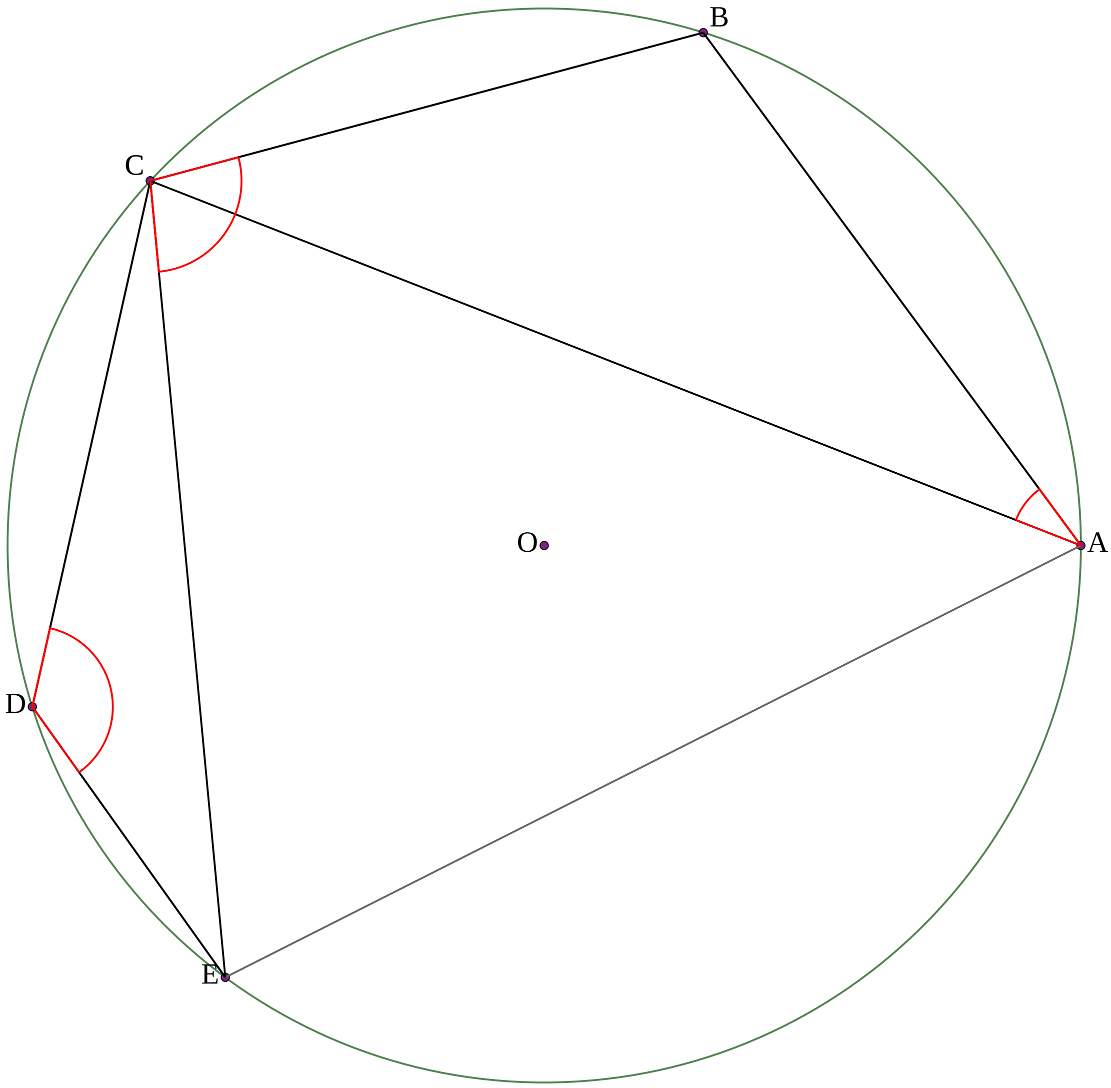
Let ABCDE be a cyclic pentagon with center O.
Prove that BAC+BDE+CBE = 180

Example 53



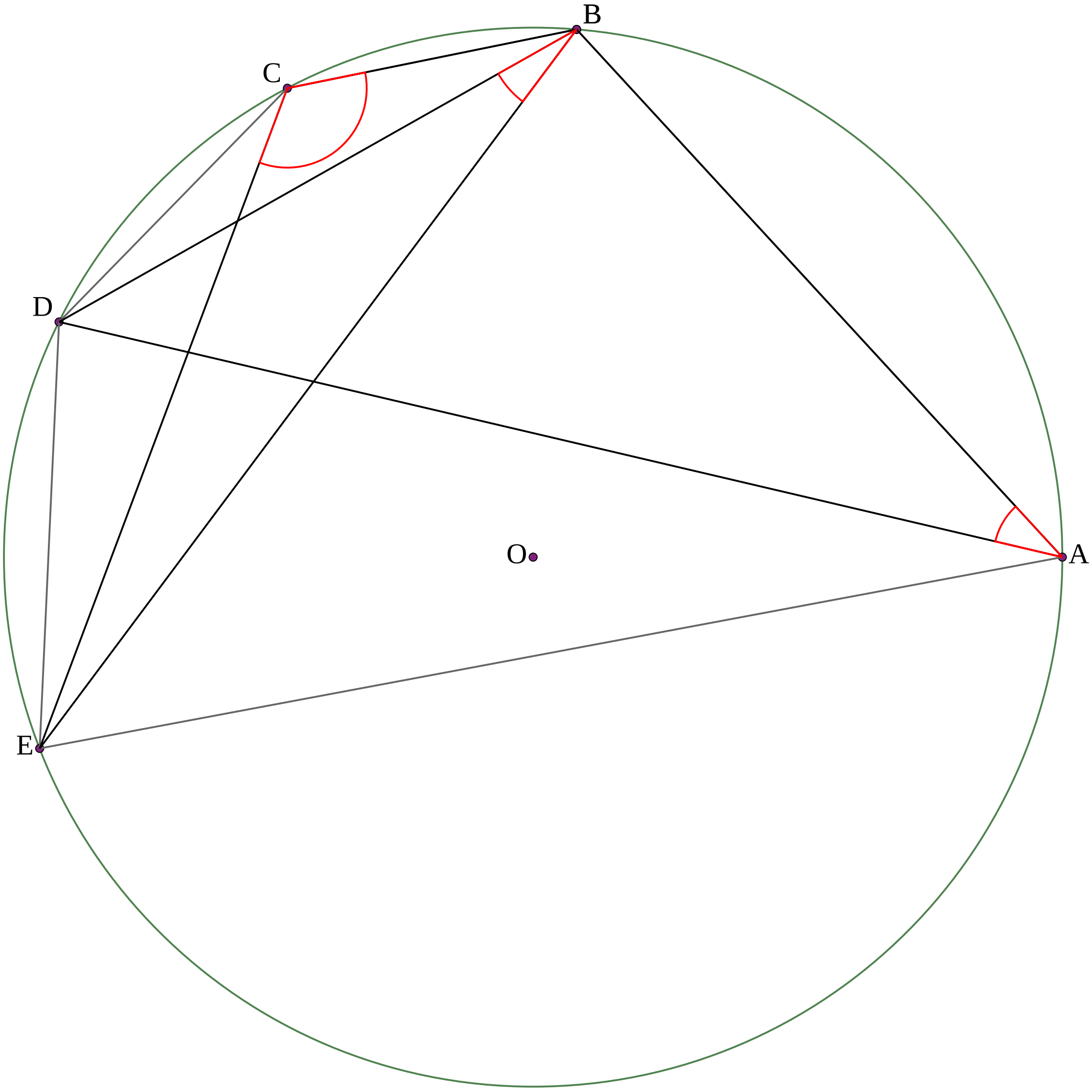
Let ABCDE be a cyclic pentagon with center O.
Prove that BAC+CED+BCD = 180

Example 54



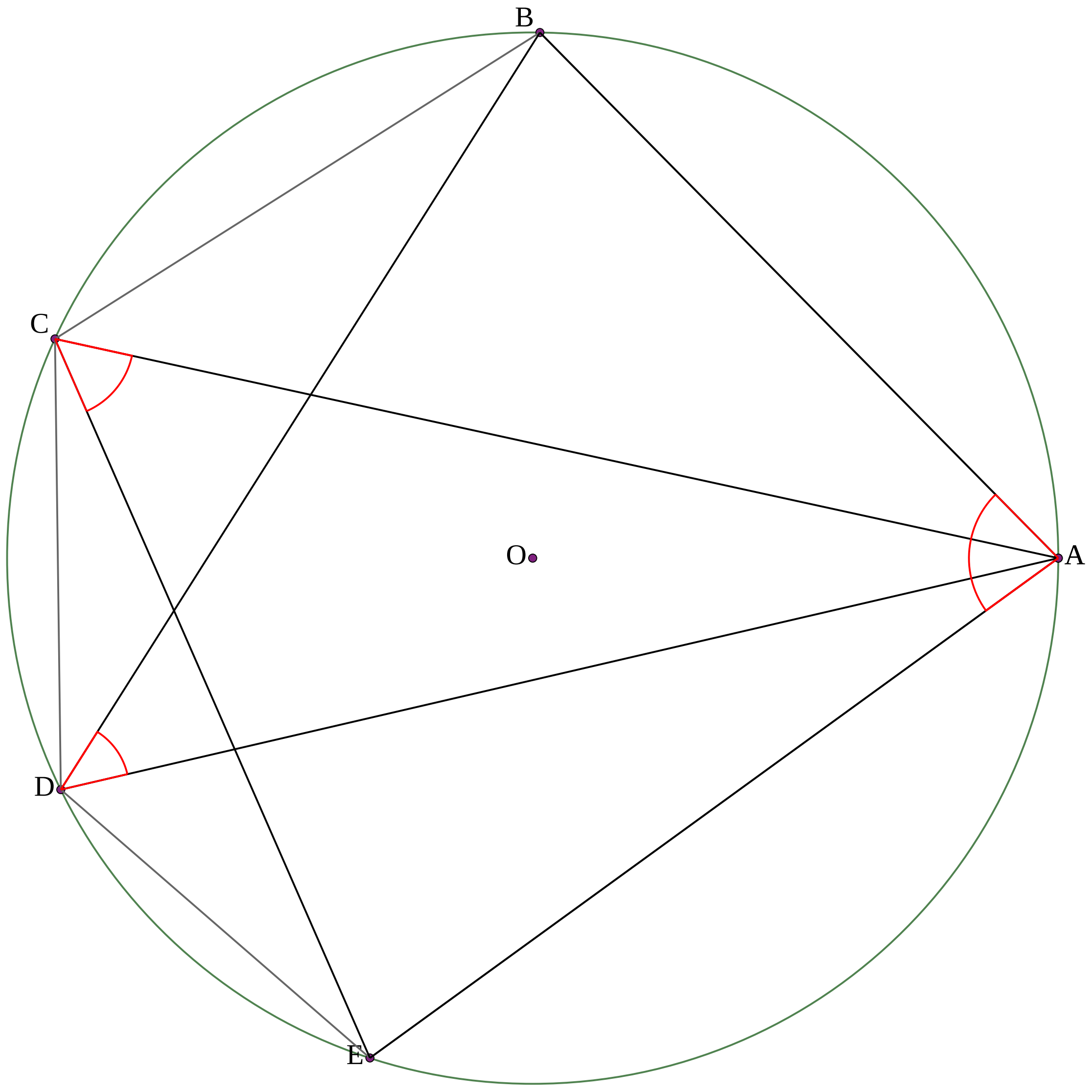
Let ABCDE be a cyclic pentagon with center O.
Prove that CDE = BAC+BCE

Example 55



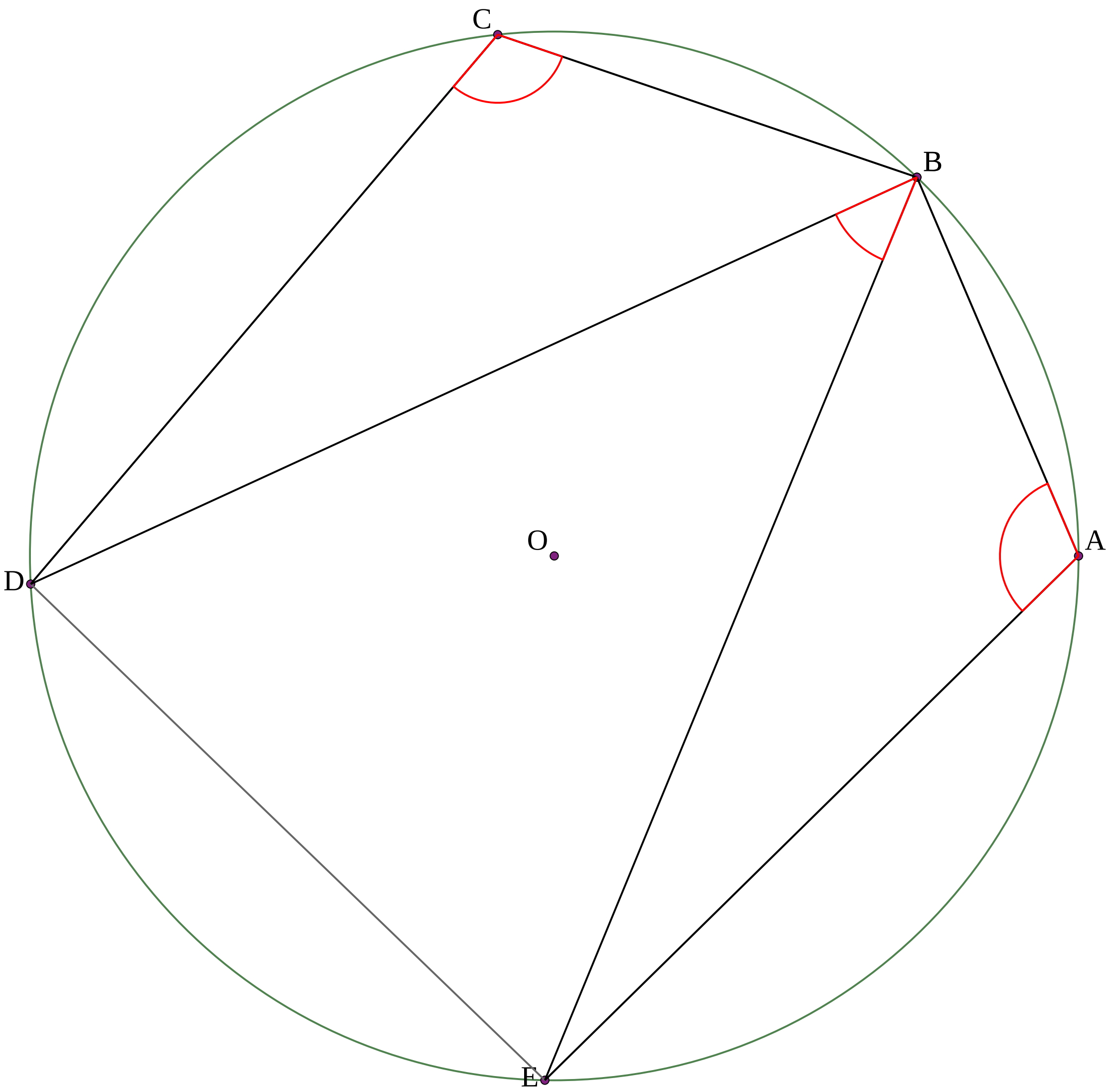
Let ABCDE be a cyclic pentagon with center O.
Prove that BAD+BCE+DBE = 180

Example 56



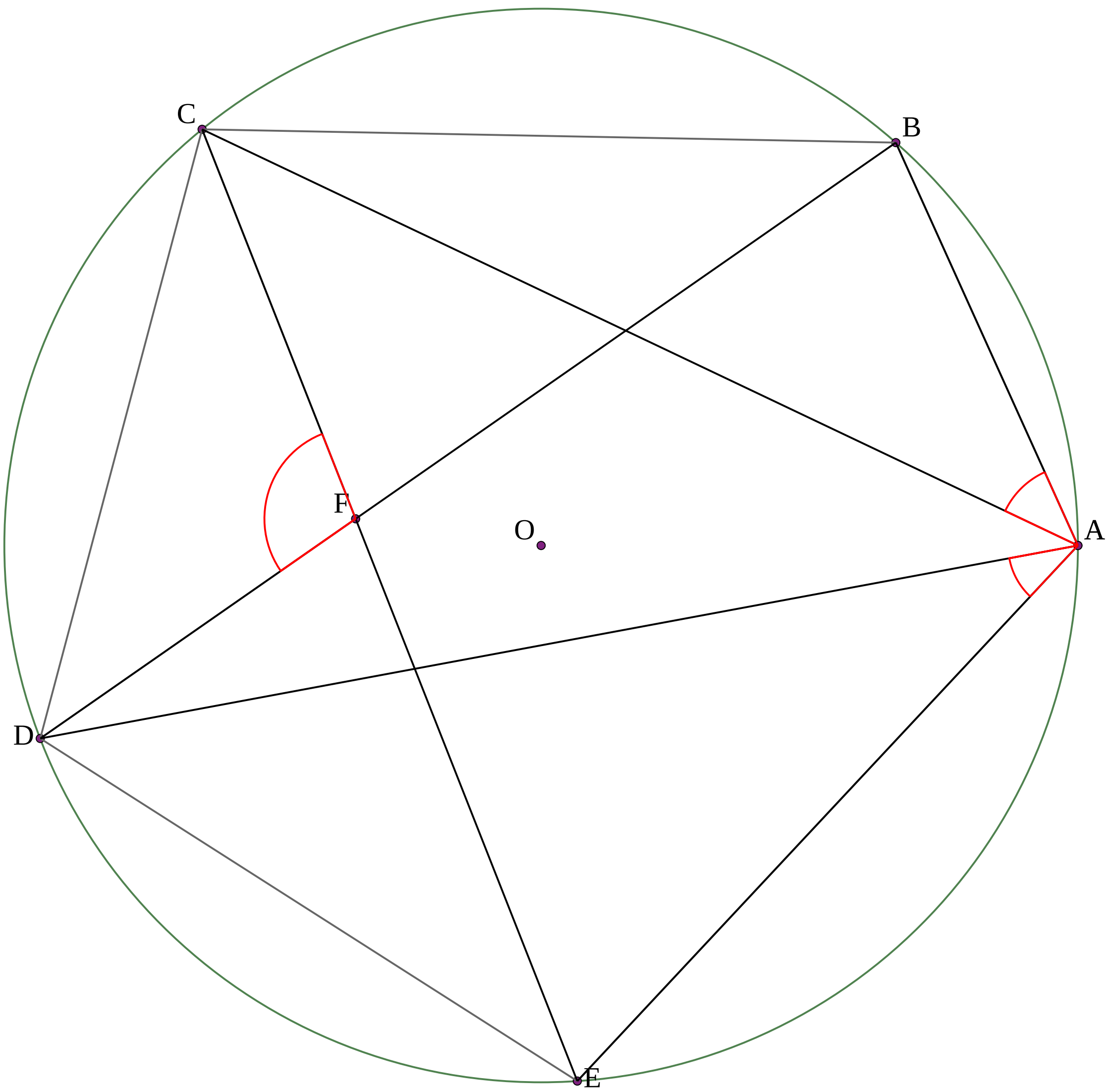
Let ABCDE be a cyclic pentagon with center O.
Prove that ADB+ACE+BAE = 180

Example 57



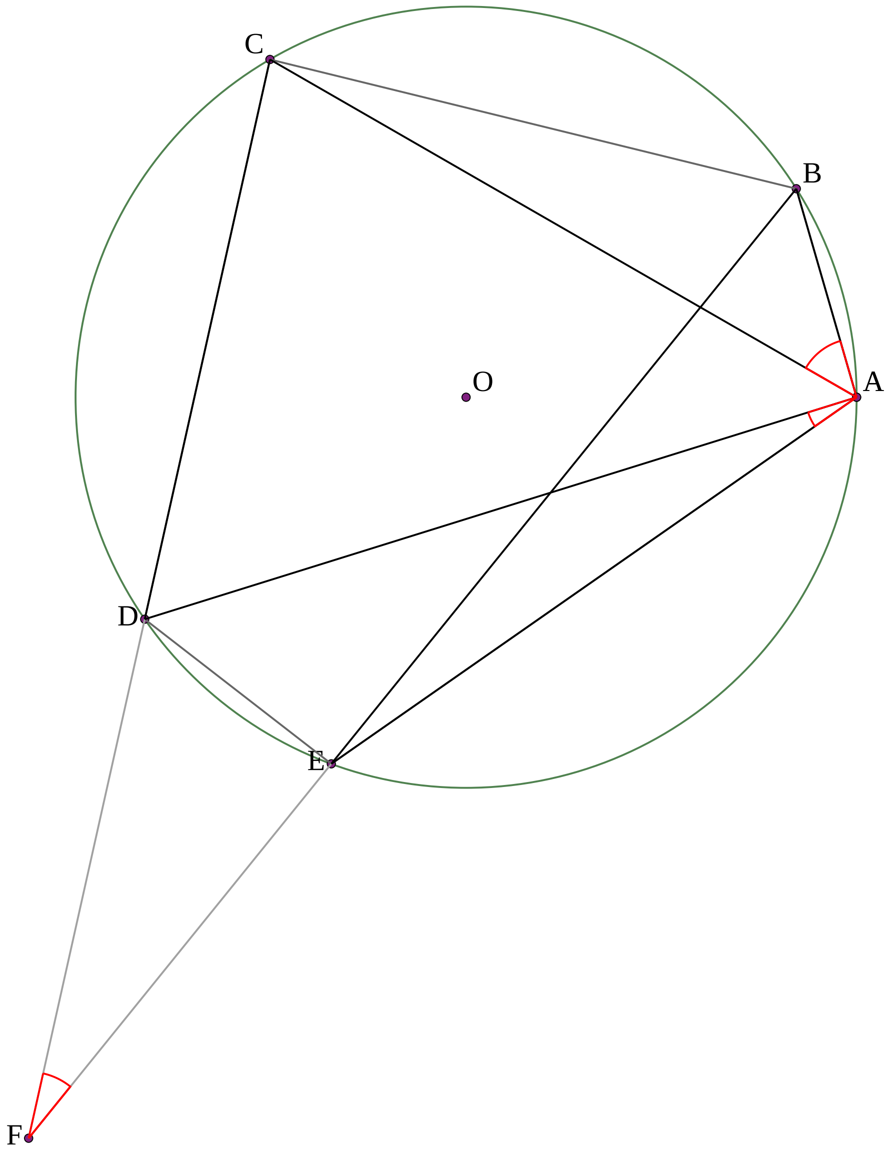
Let ABCDE be a cyclic pentagon with center O.
Prove that BCD+BAE = DBE+180

Example 58



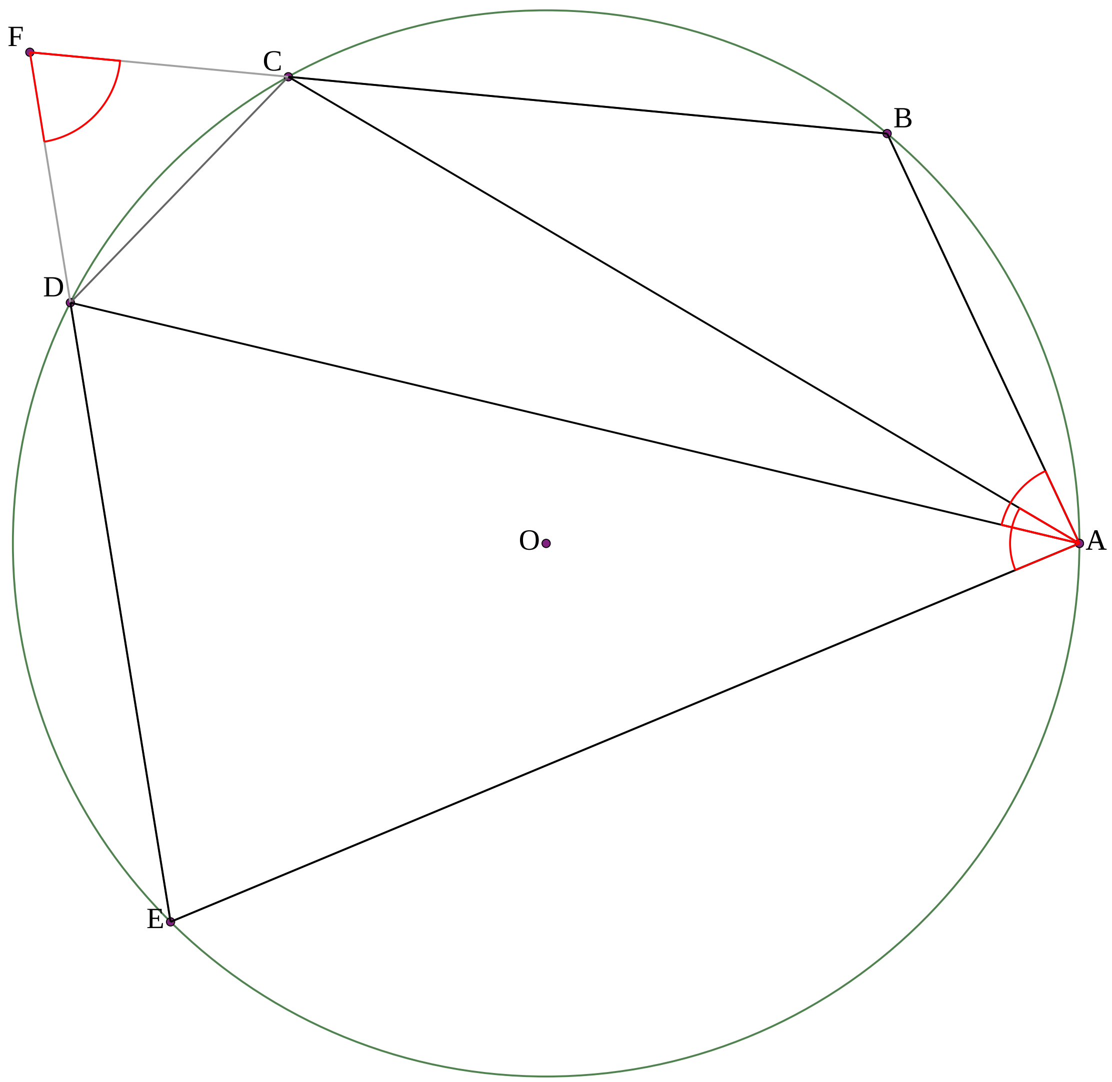
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EC.
Prove that DAE+BAC+CFD = 180

Example 59



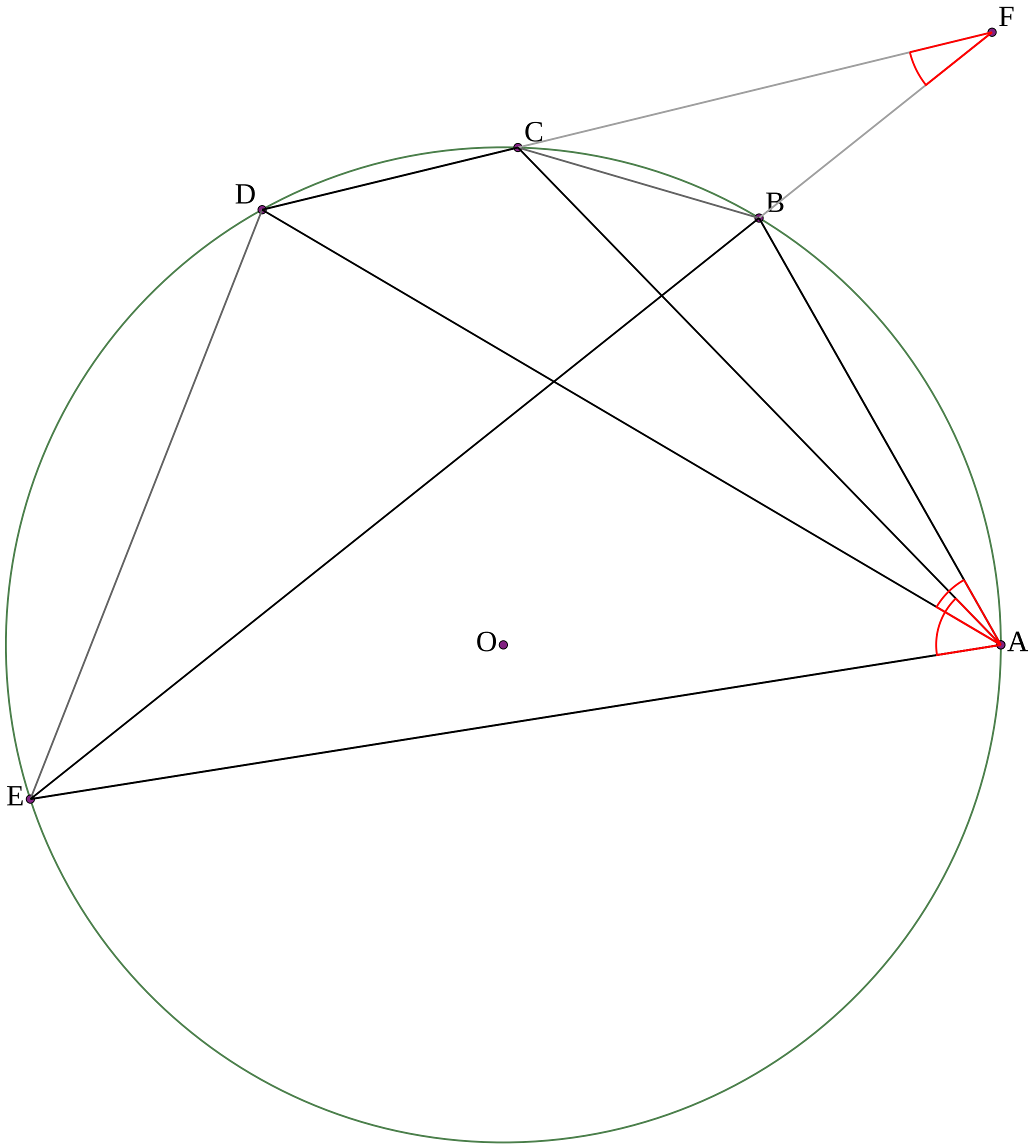
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BE and DC.
Prove that BAC = DAE+DFE

Example 60



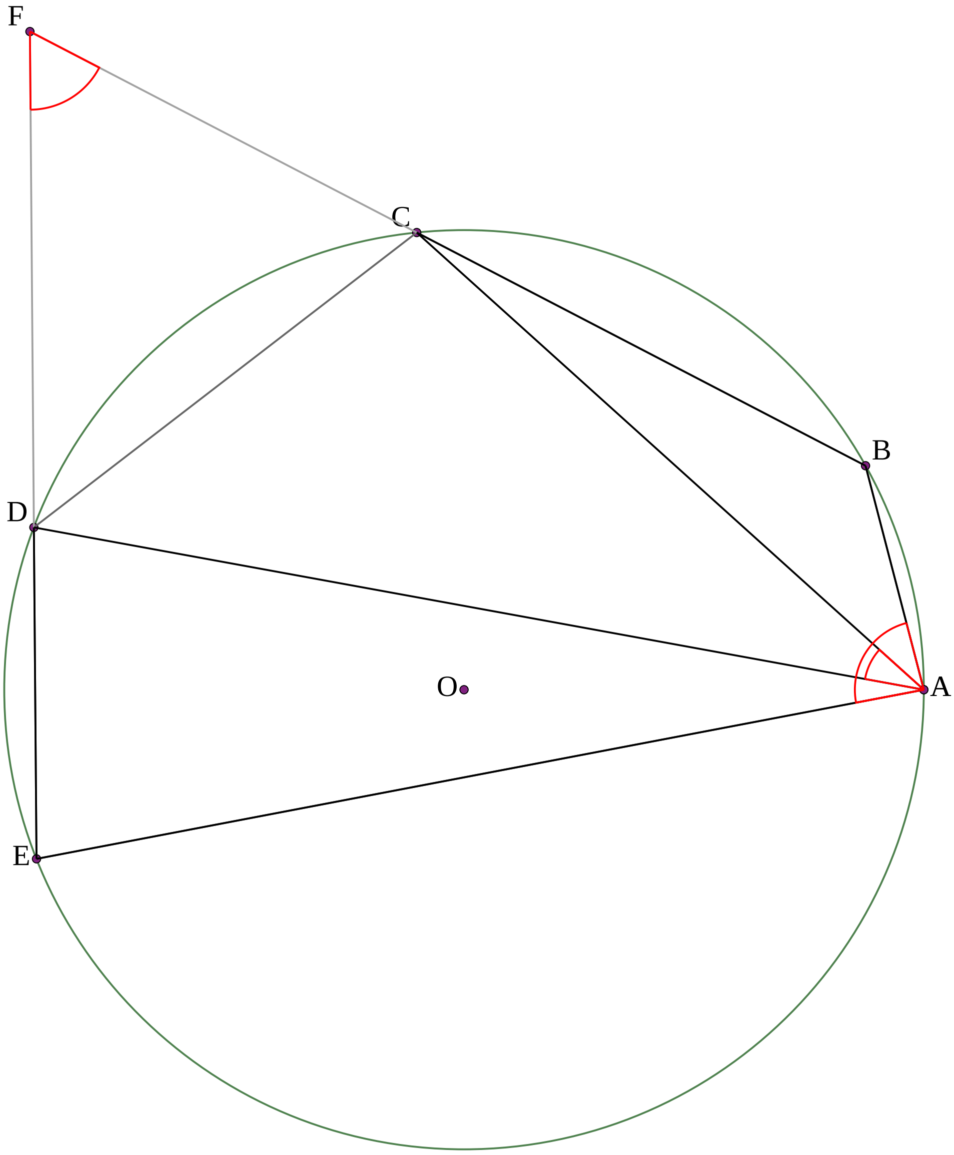
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BC and ED.
Prove that CAE+BAD+CFD = 180

Example 61



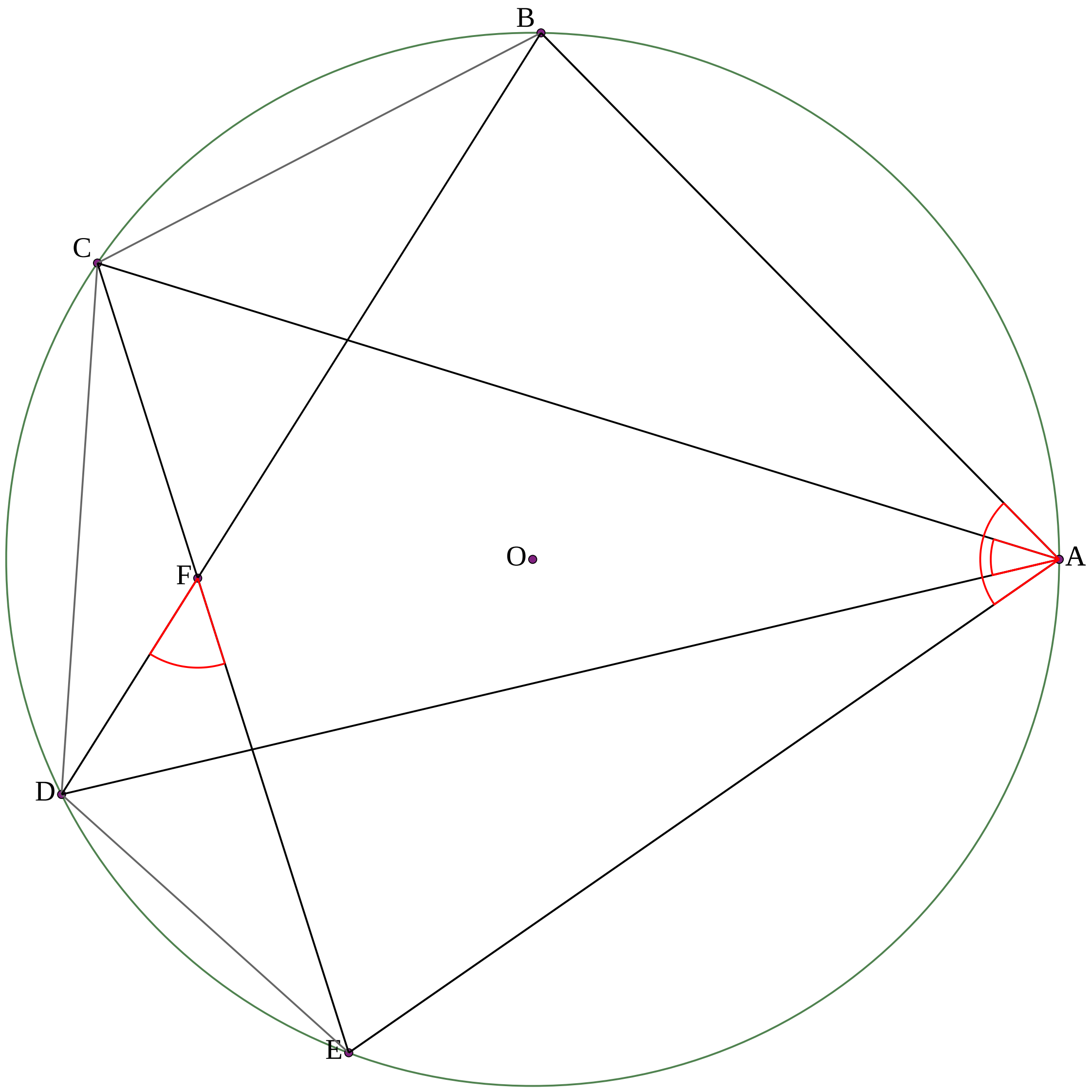
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BE and CD.
Prove that BAD+BFC = CAE

Example 62



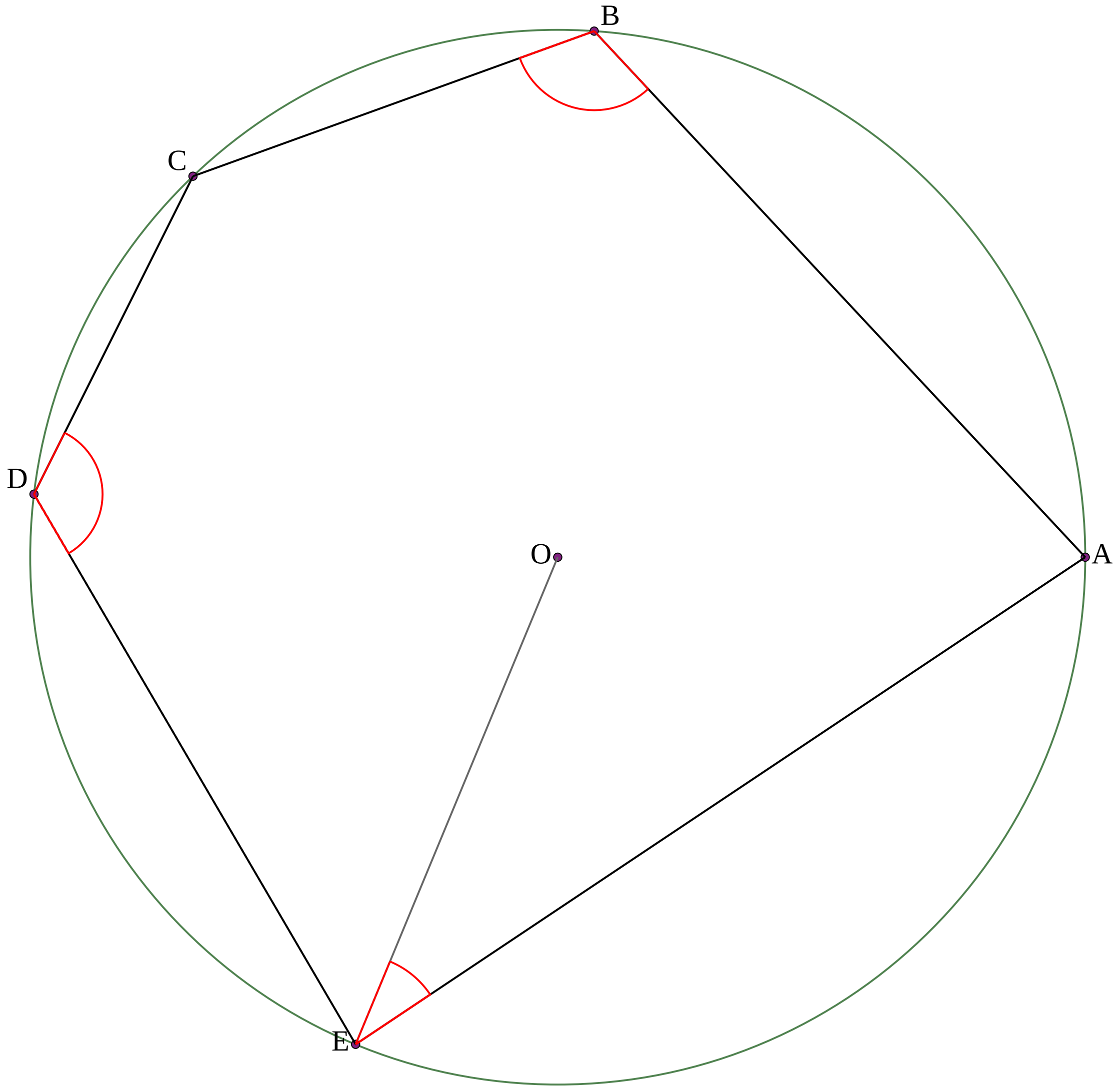
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BC and DE.
Prove that CAD+BAE+CFD = 180

Example 63



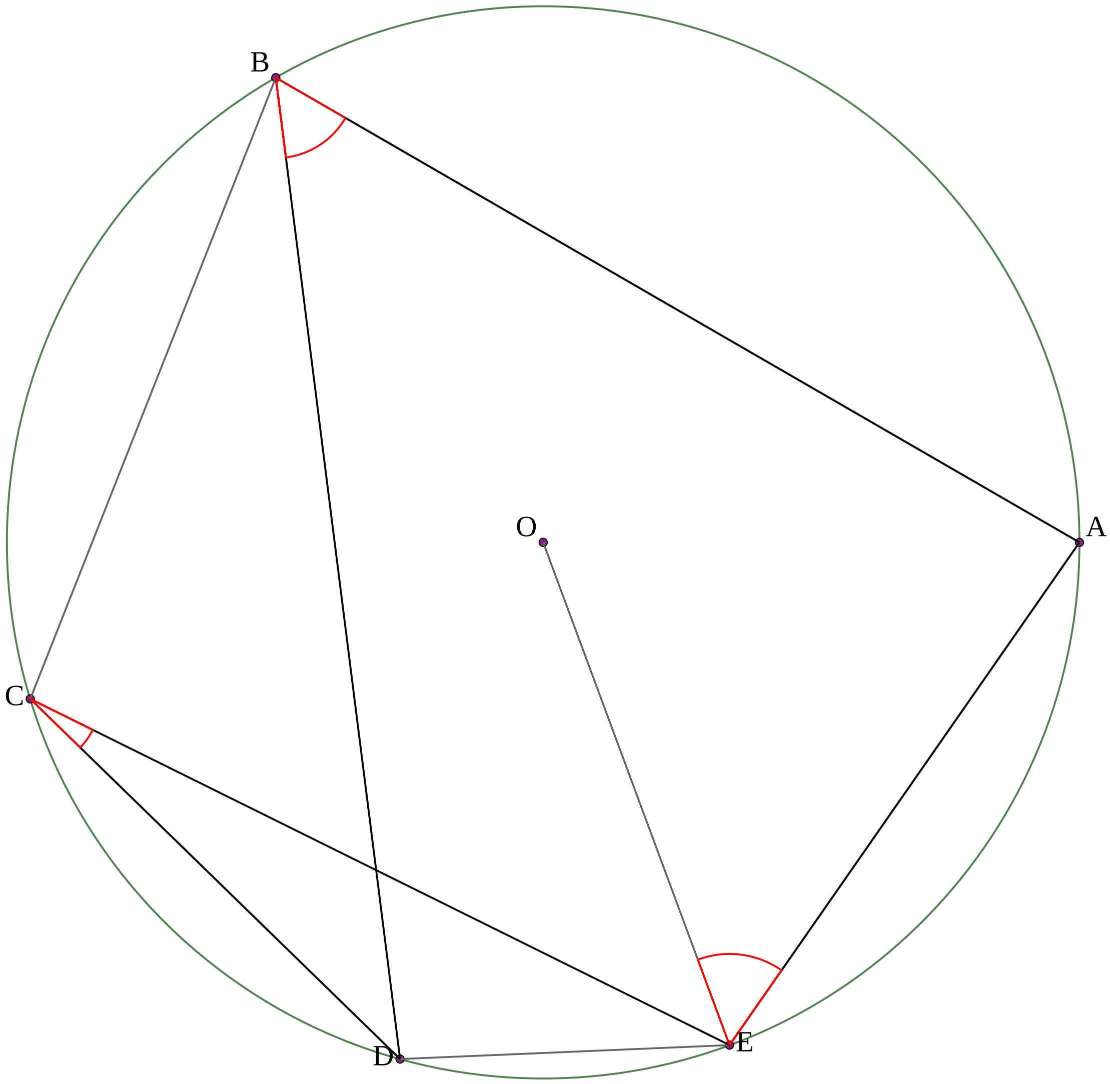
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and CE.
Prove that BAE = CAD+DFE

Example 64



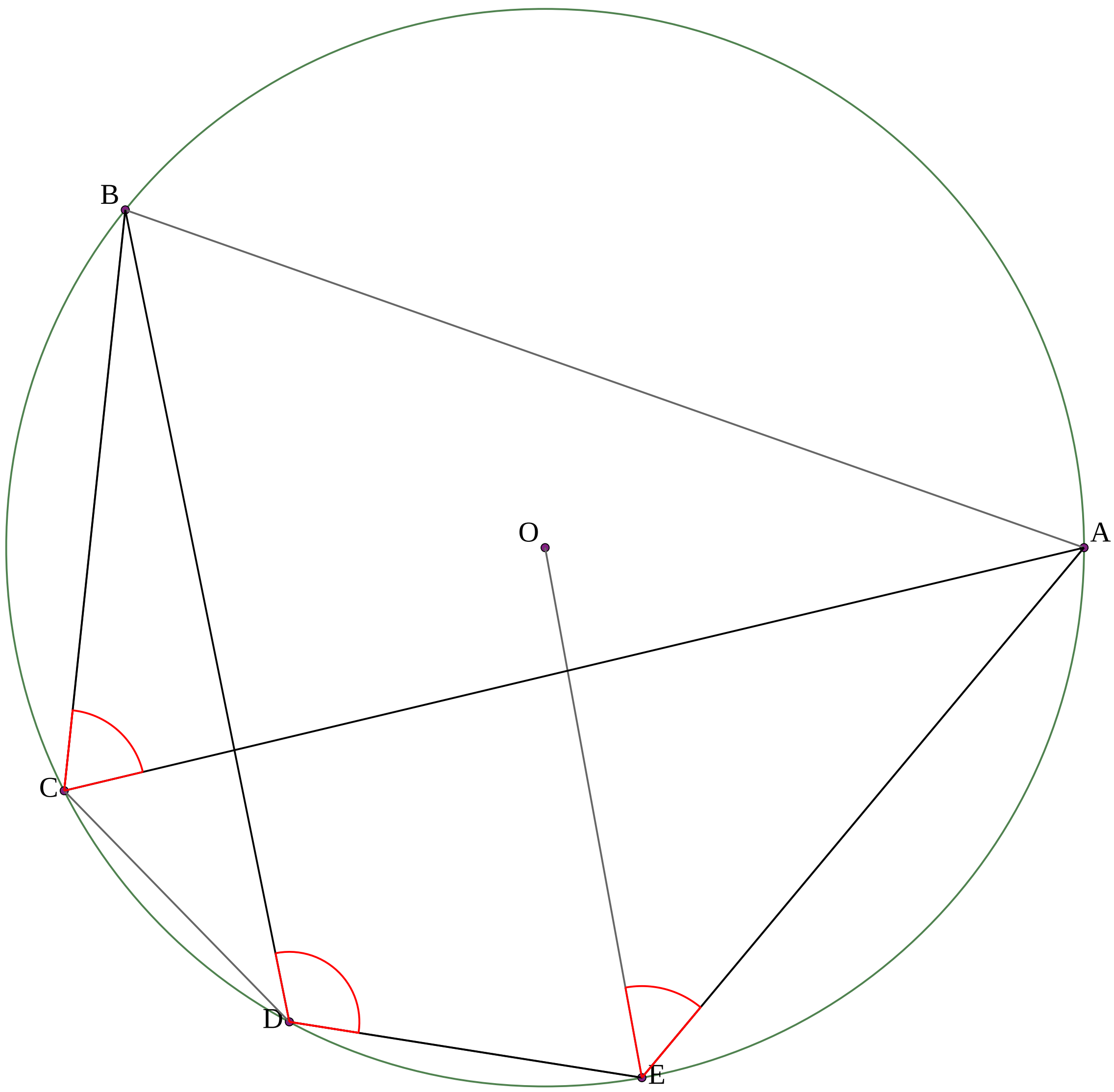
Let ABCDE be a cyclic pentagon with center O.
Prove that CDE+ABC+AEO = 270

Example 65



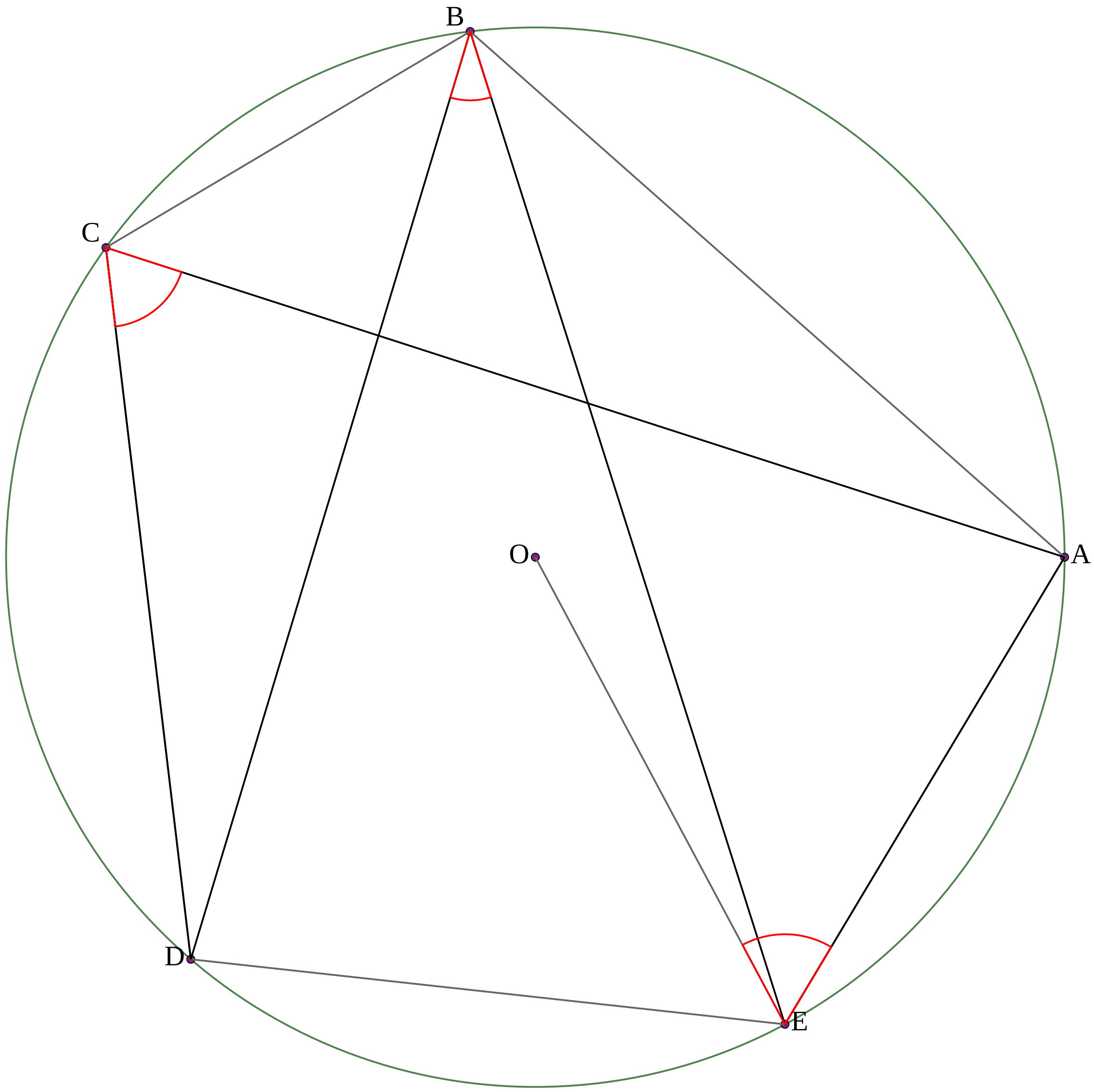
Let ABCDE be a cyclic pentagon with center O.
Prove that ABD+AEO = DCE+90

Example 66



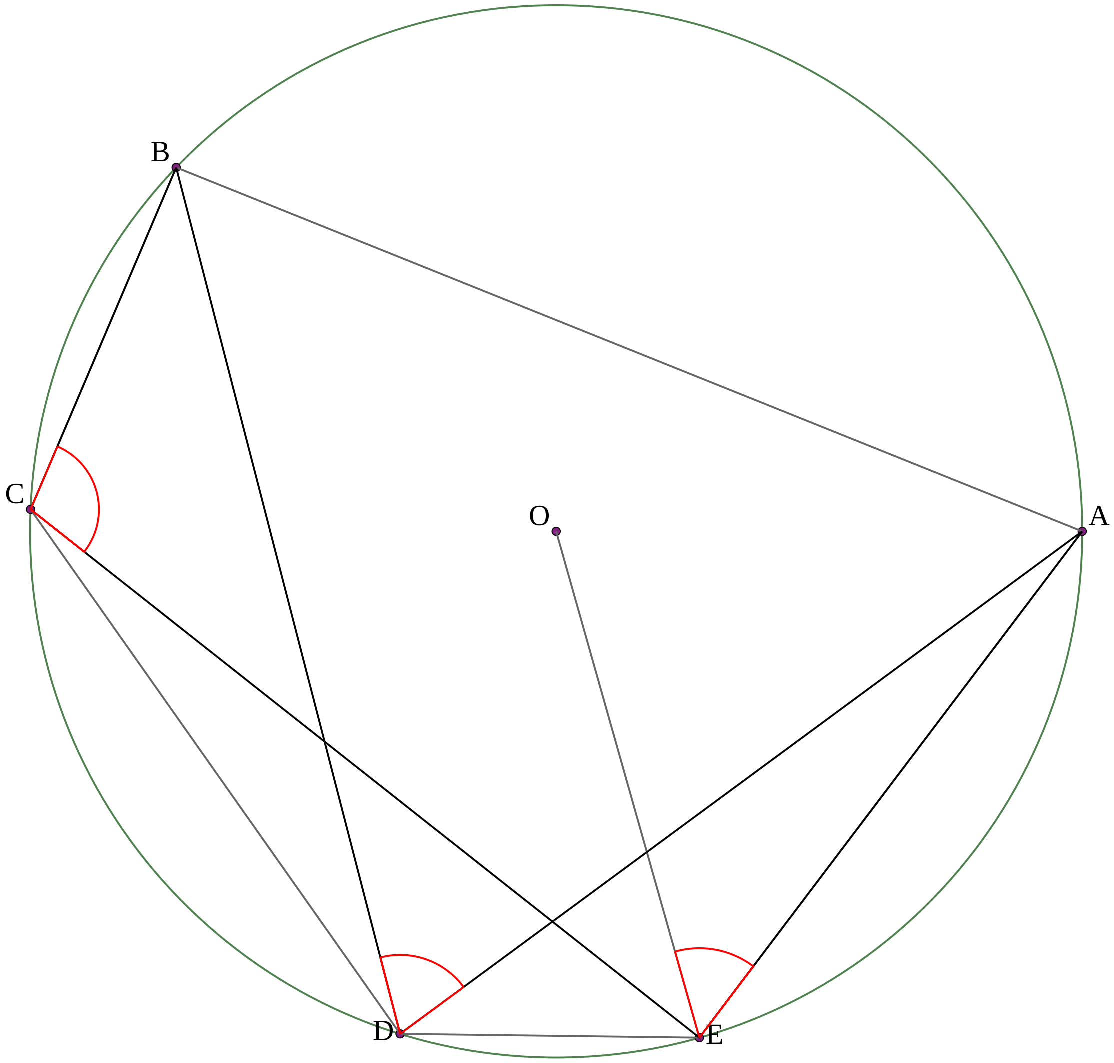
Let ABCDE be a cyclic pentagon with center O.
Prove that BDE+AEO = ACB+90

Example 67



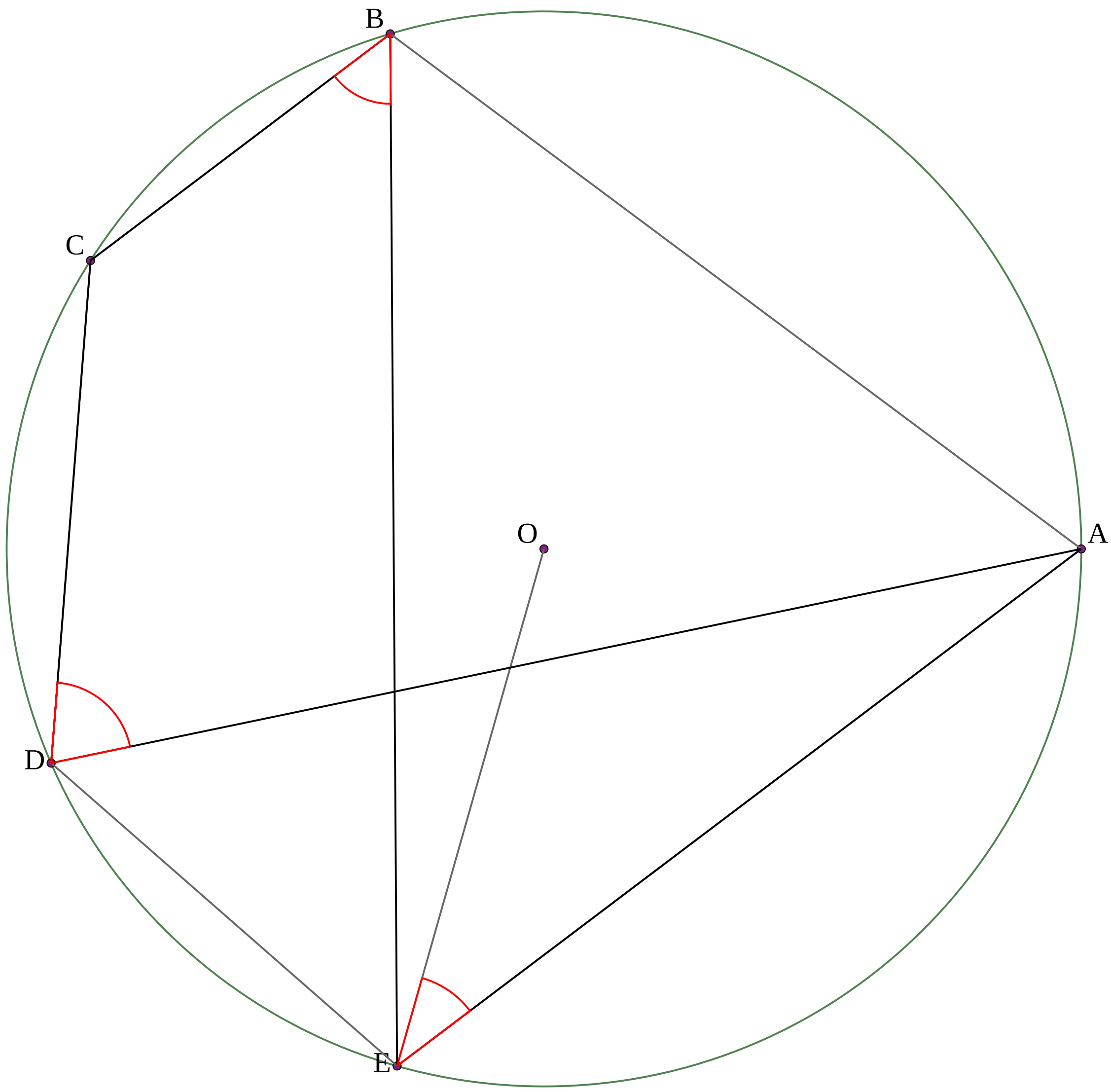
Let ABCDE be a cyclic pentagon with center O.
Prove that ACD+AEO = DBE+90

Example 68



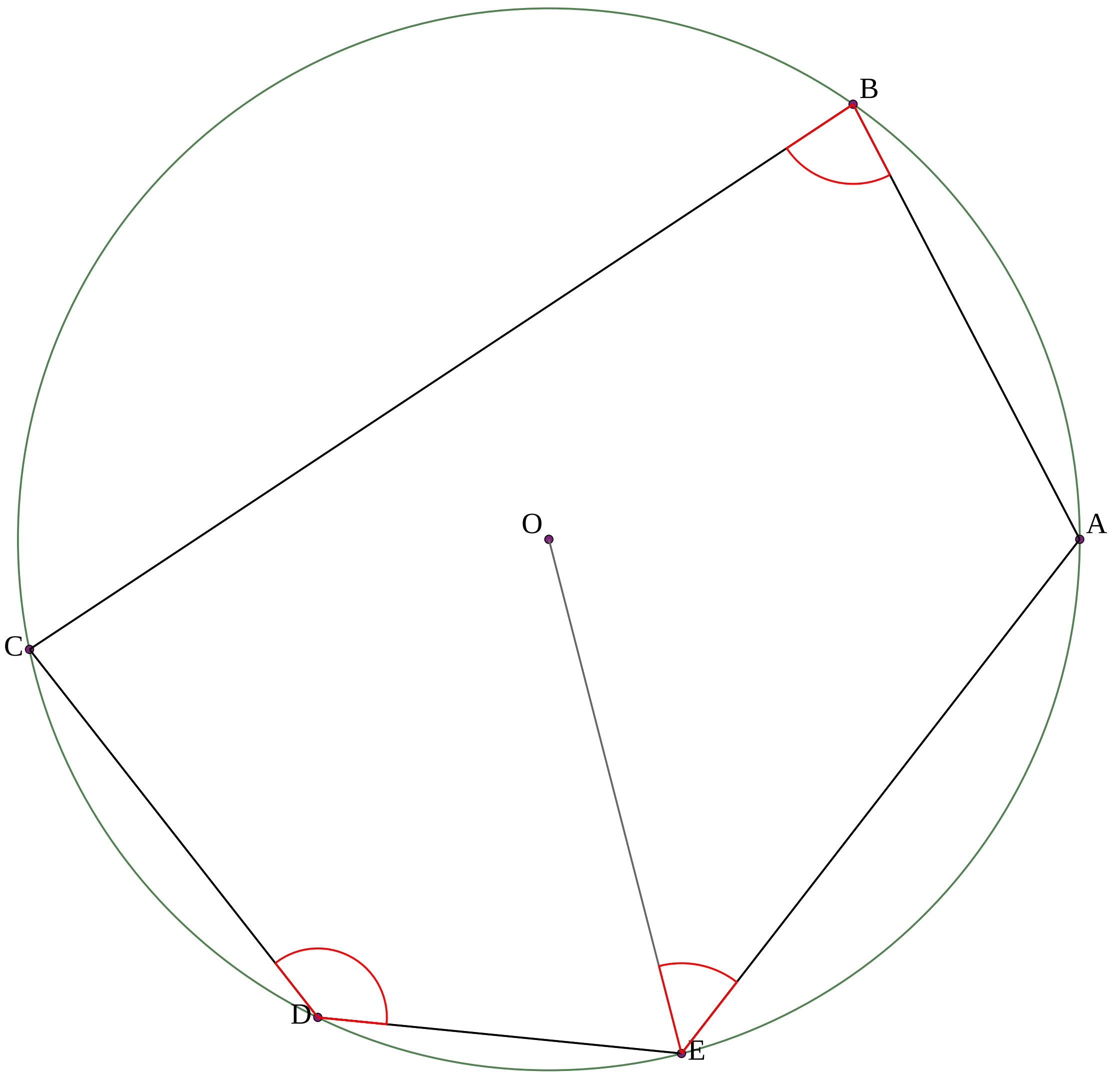
Let ABCDE be a cyclic pentagon with center O.
Prove that BCE+AEO = ADB+90

Example 69



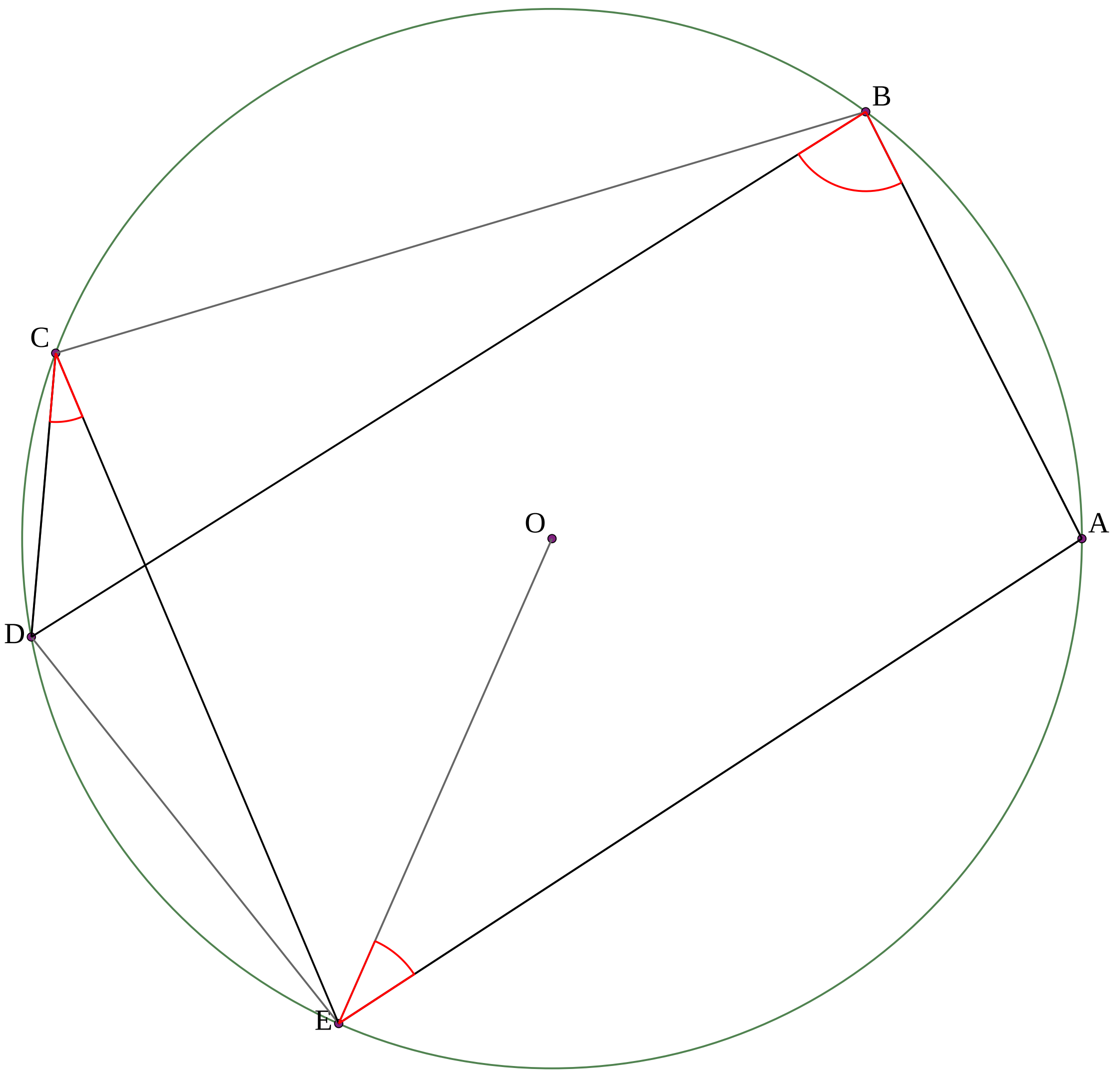
Let ABCDE be a cyclic pentagon with center O.
Prove that CBE+ADC = AEO+90

Example 70



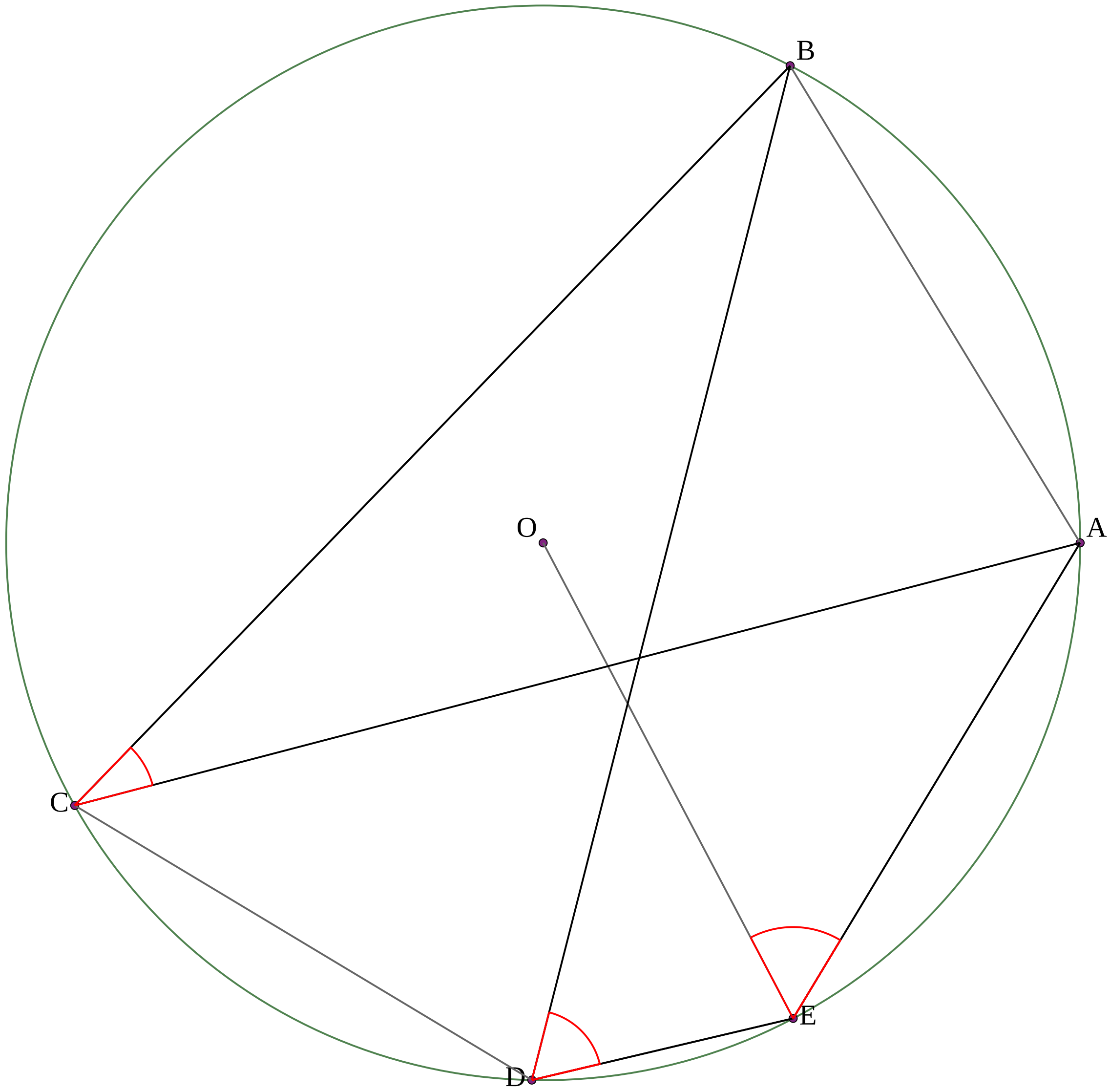
Let ABCDE be a cyclic pentagon with center O.
Prove that CDE+ABC+AEO = 270

Example 71



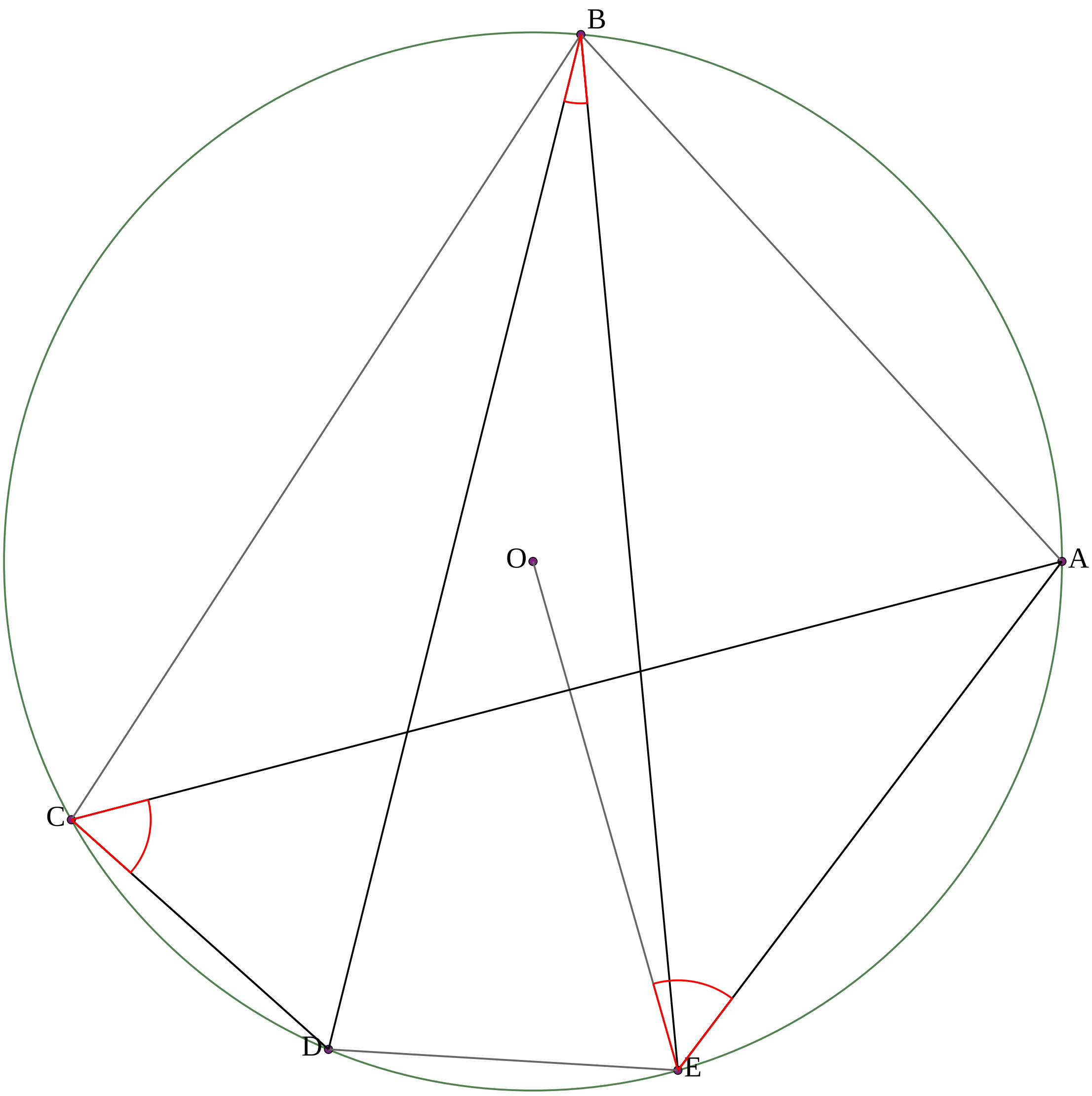
Let ABCDE be a cyclic pentagon with center O.
Prove that ABD+AEO = DCE+90

Example 72



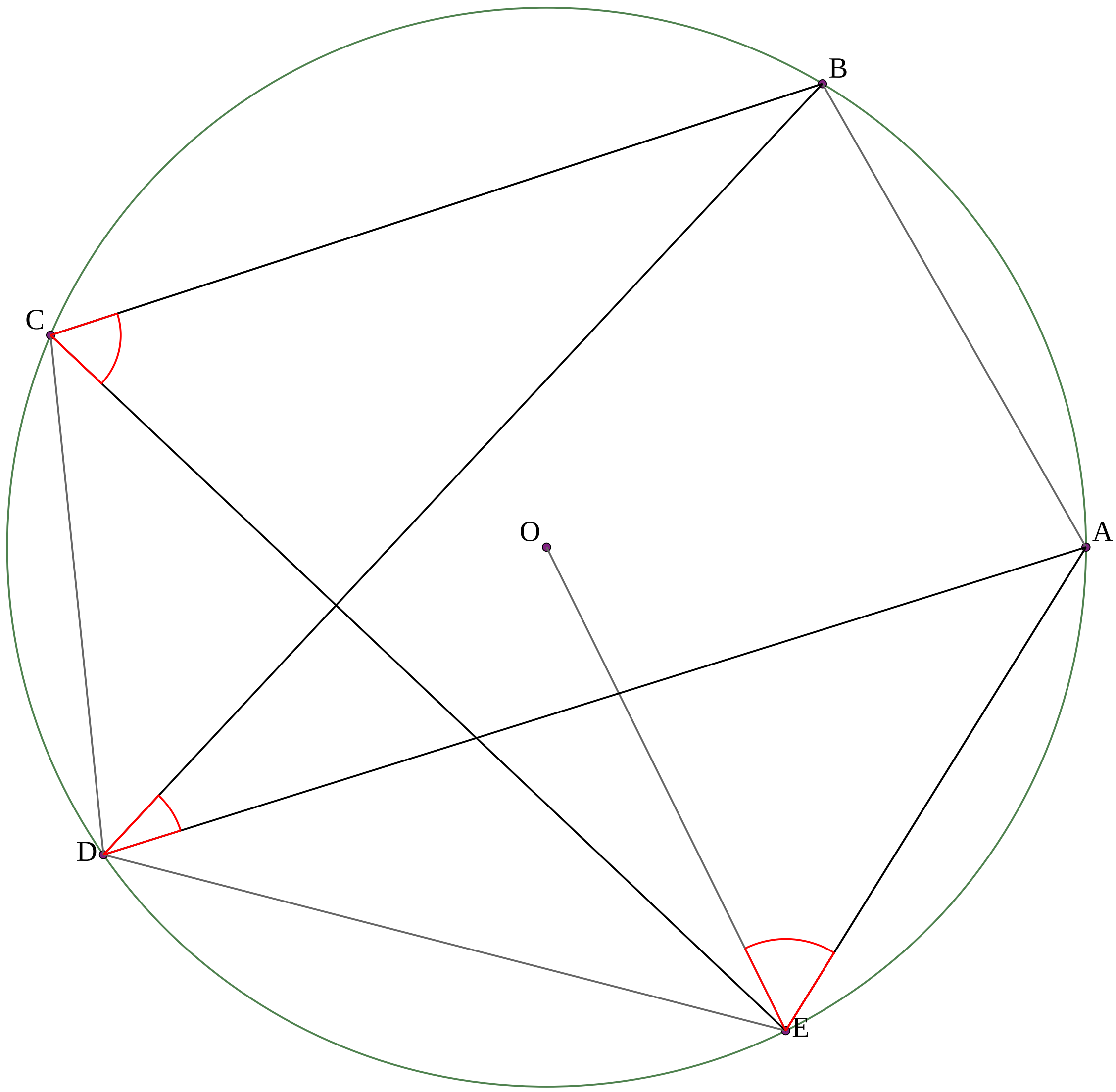
Let ABCDE be a cyclic pentagon with center O.
Prove that BDE+AEO = ACB+90

Example 73



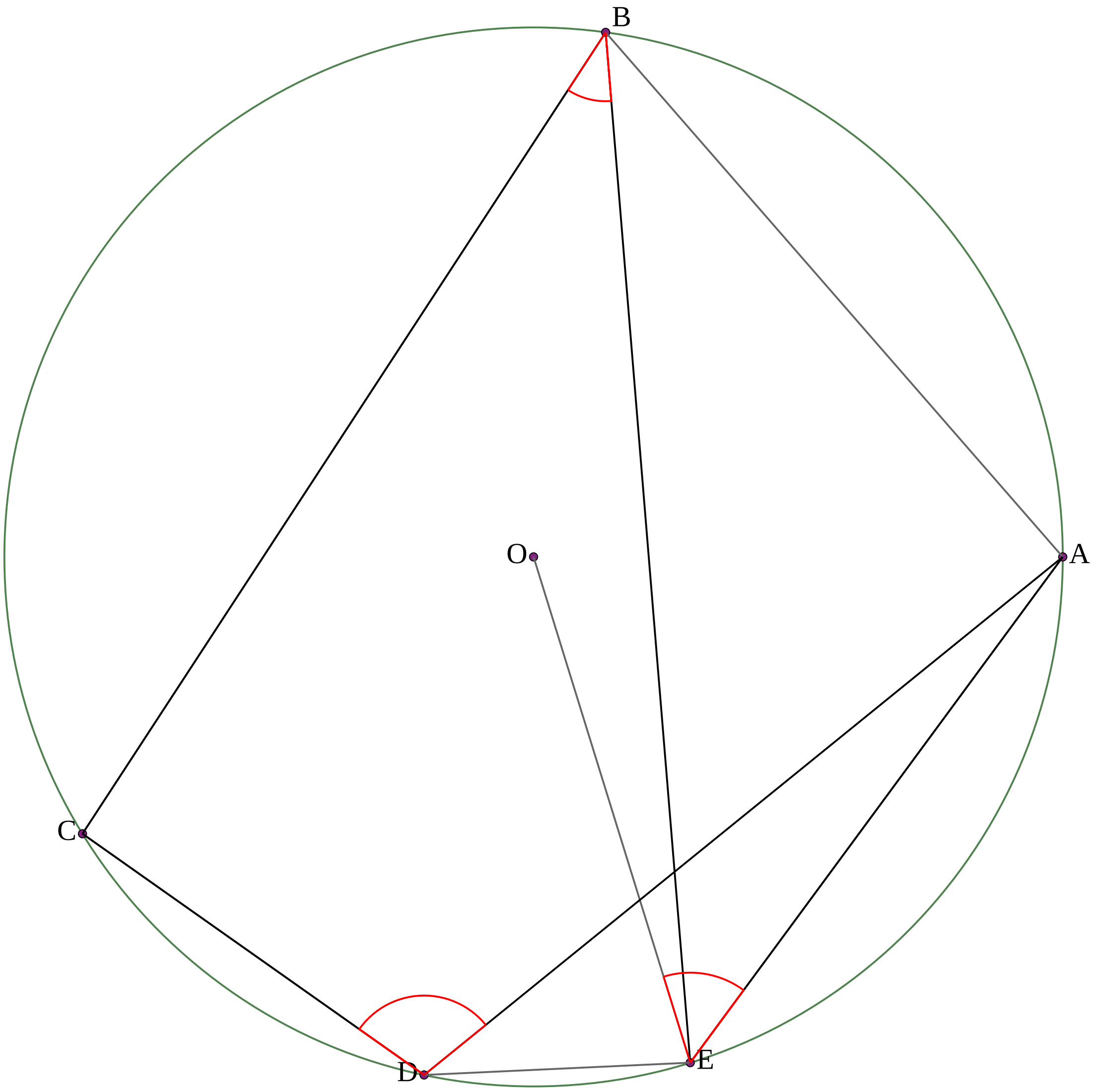
Let ABCDE be a cyclic pentagon with center O.
Prove that ACD+AEO = DBE+90

Example 74



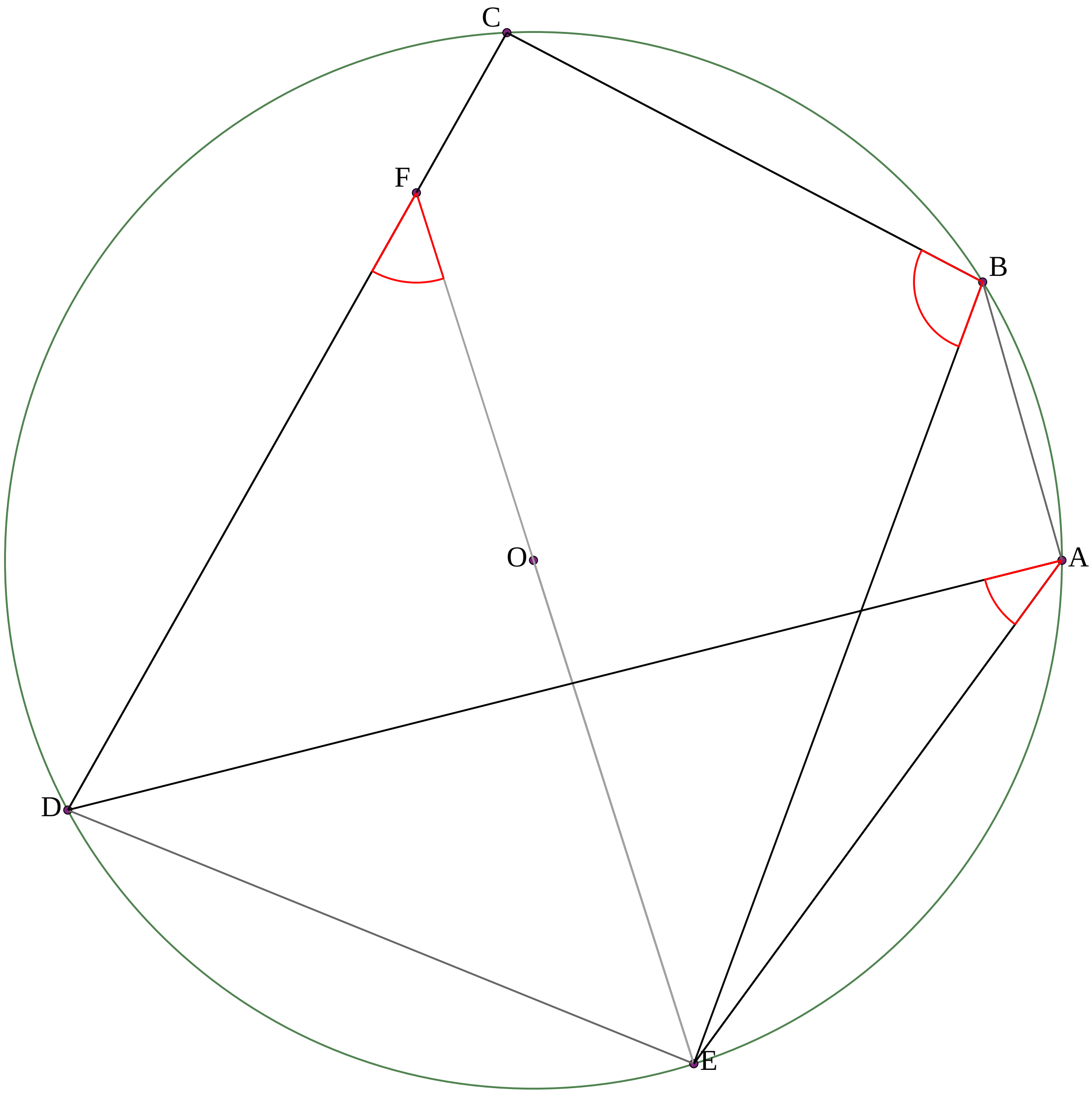
Let ABCDE be a cyclic pentagon with center O.
Prove that BCE+AEO = ADB+90

Example 75



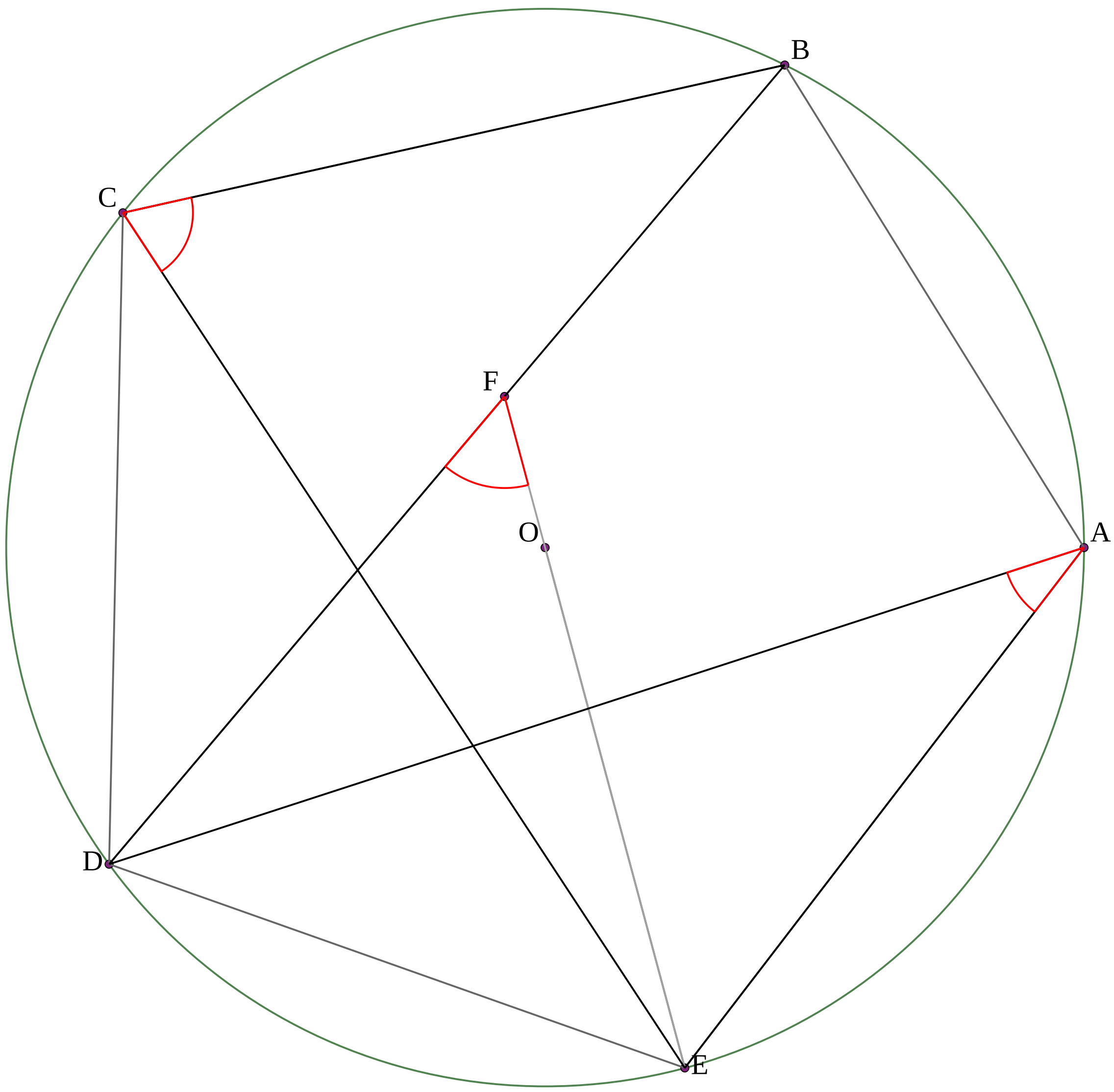
Let ABCDE be a cyclic pentagon with center O.
Prove that CBE+ADC = AEO+90

Example 76



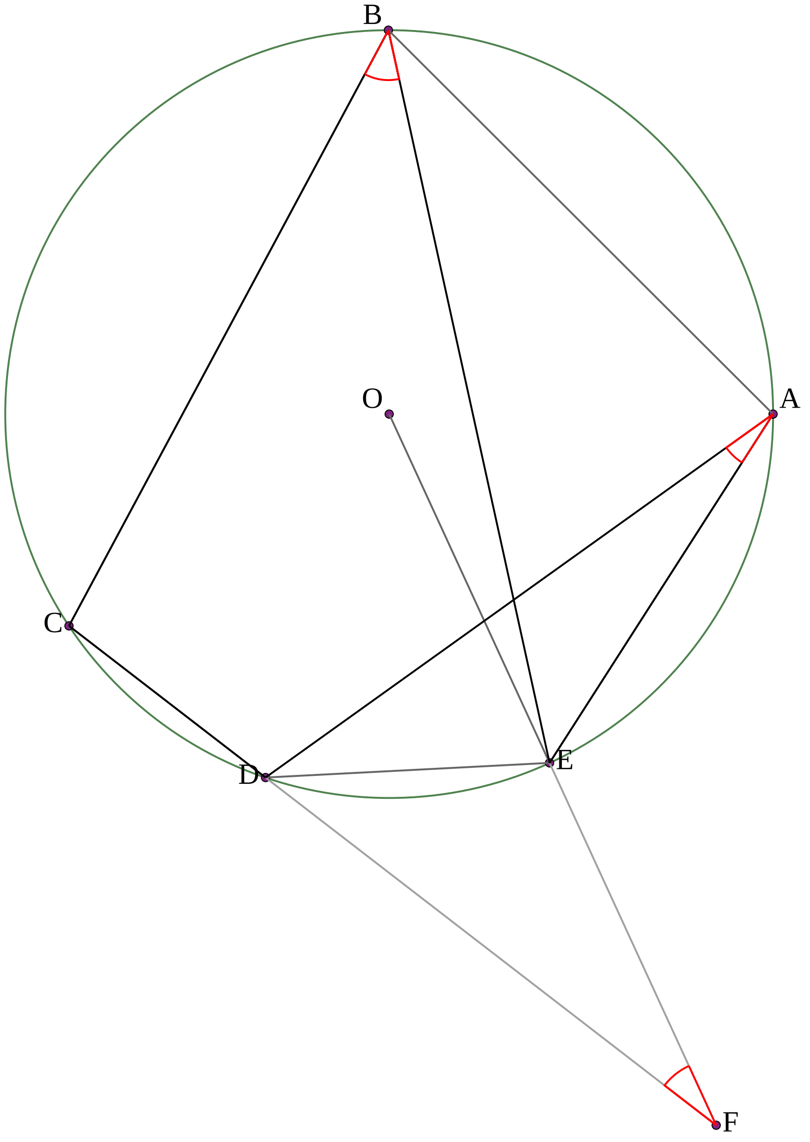
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of CD and EO.
Prove that CBE+DAE = DFE+90

Example 77



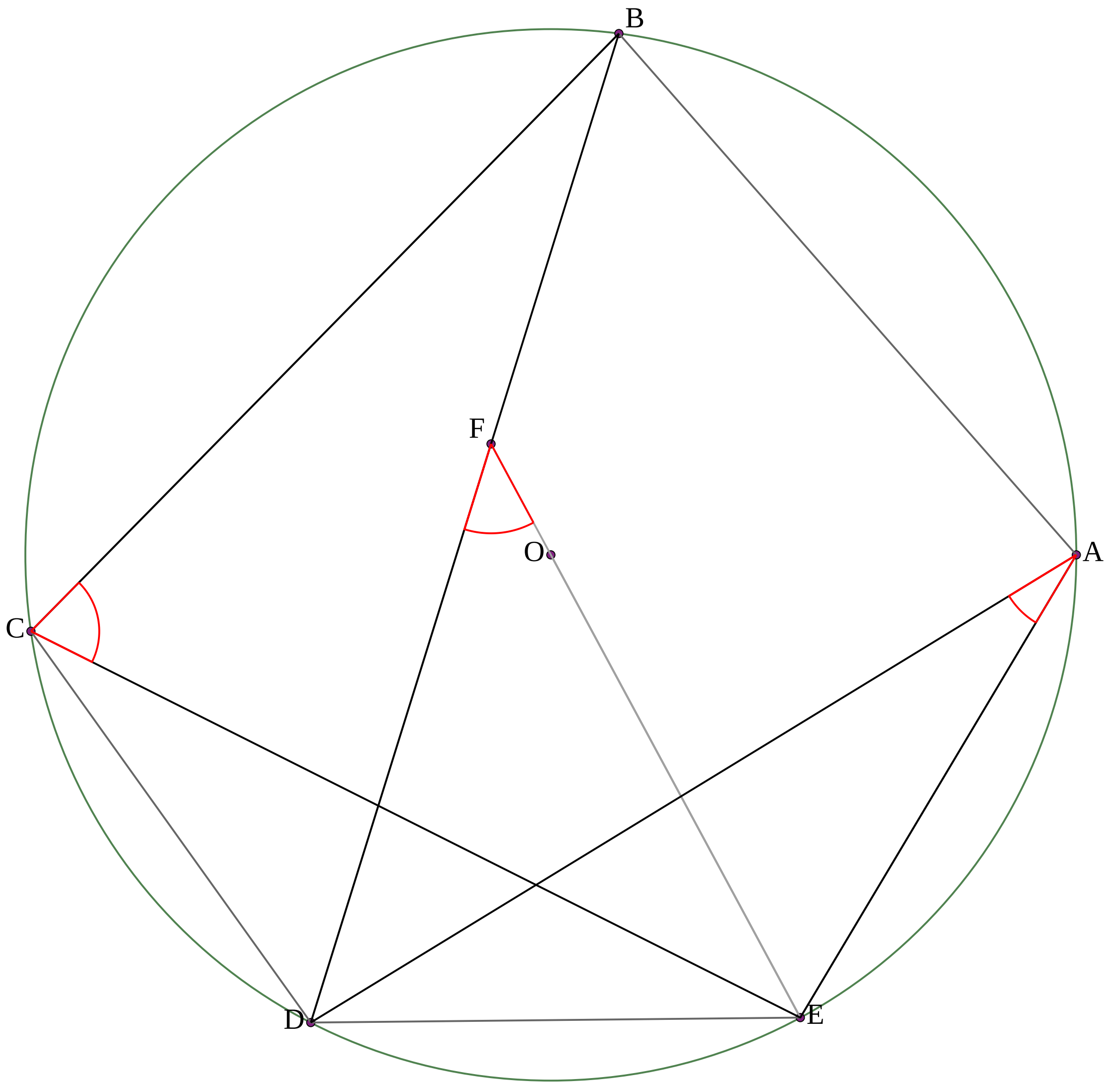
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EO.
Prove that BCE+DFE = DAE+90

Example 78



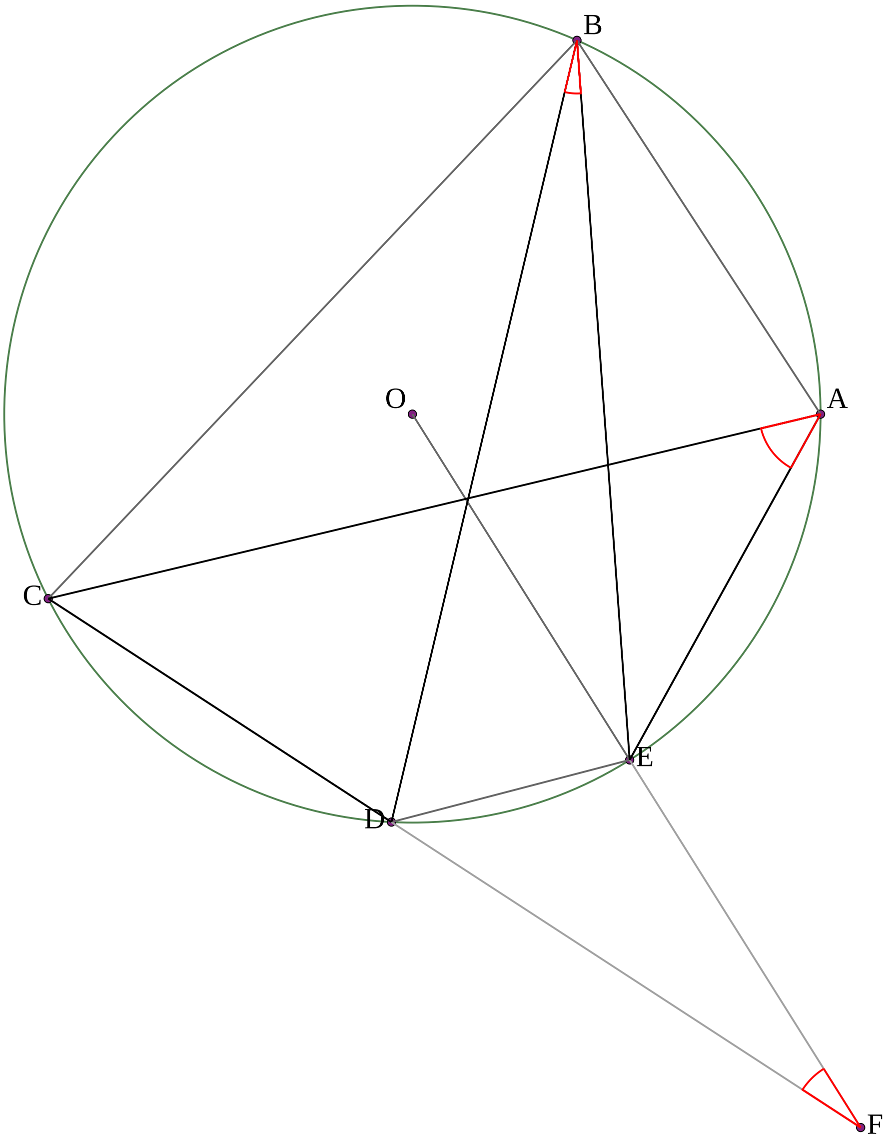
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of CD and EO.
Prove that CBE+DAE+DFE = 90

Example 79



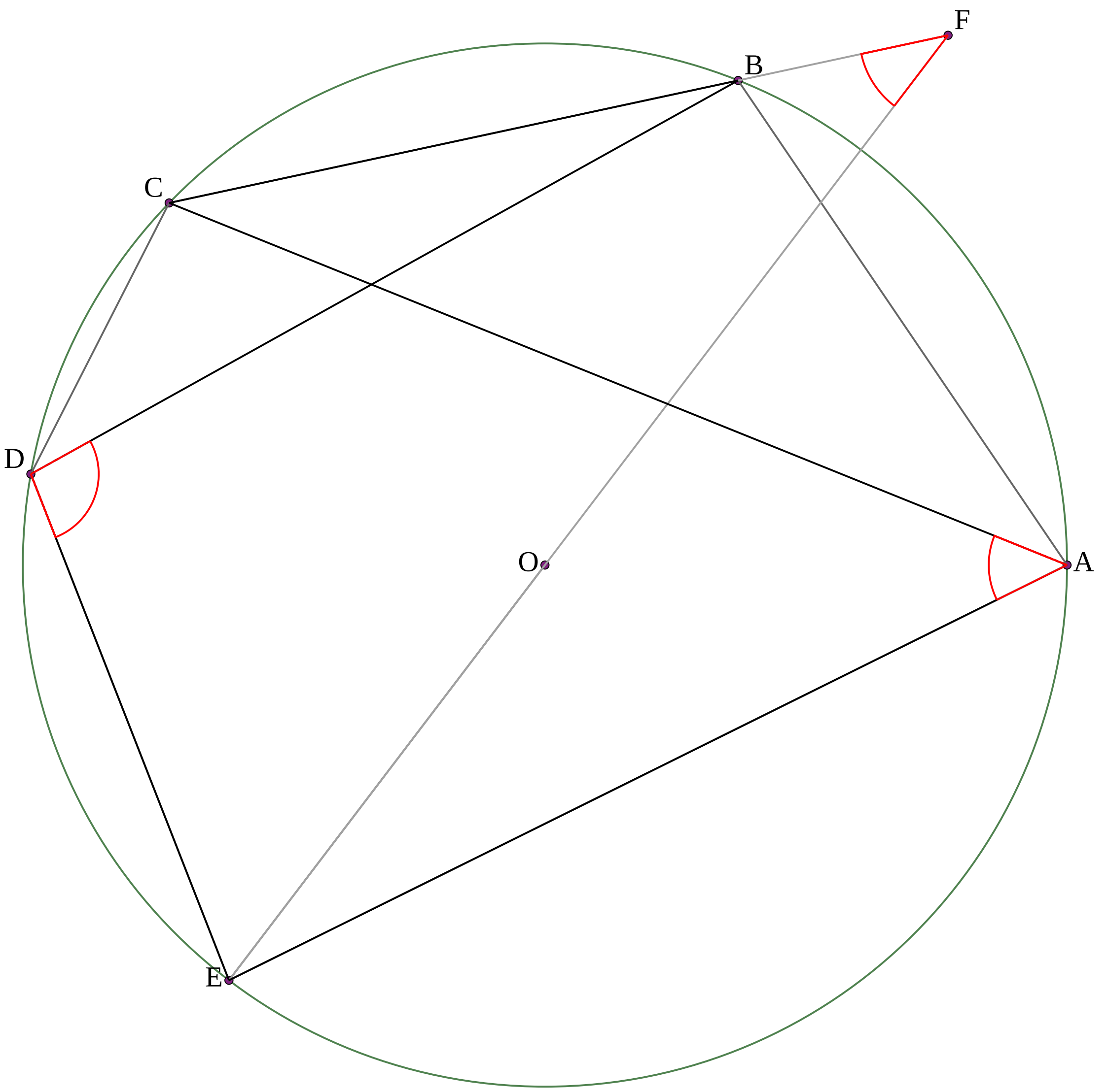
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EO.
Prove that BCE+DFE = DAE+90

Example 80



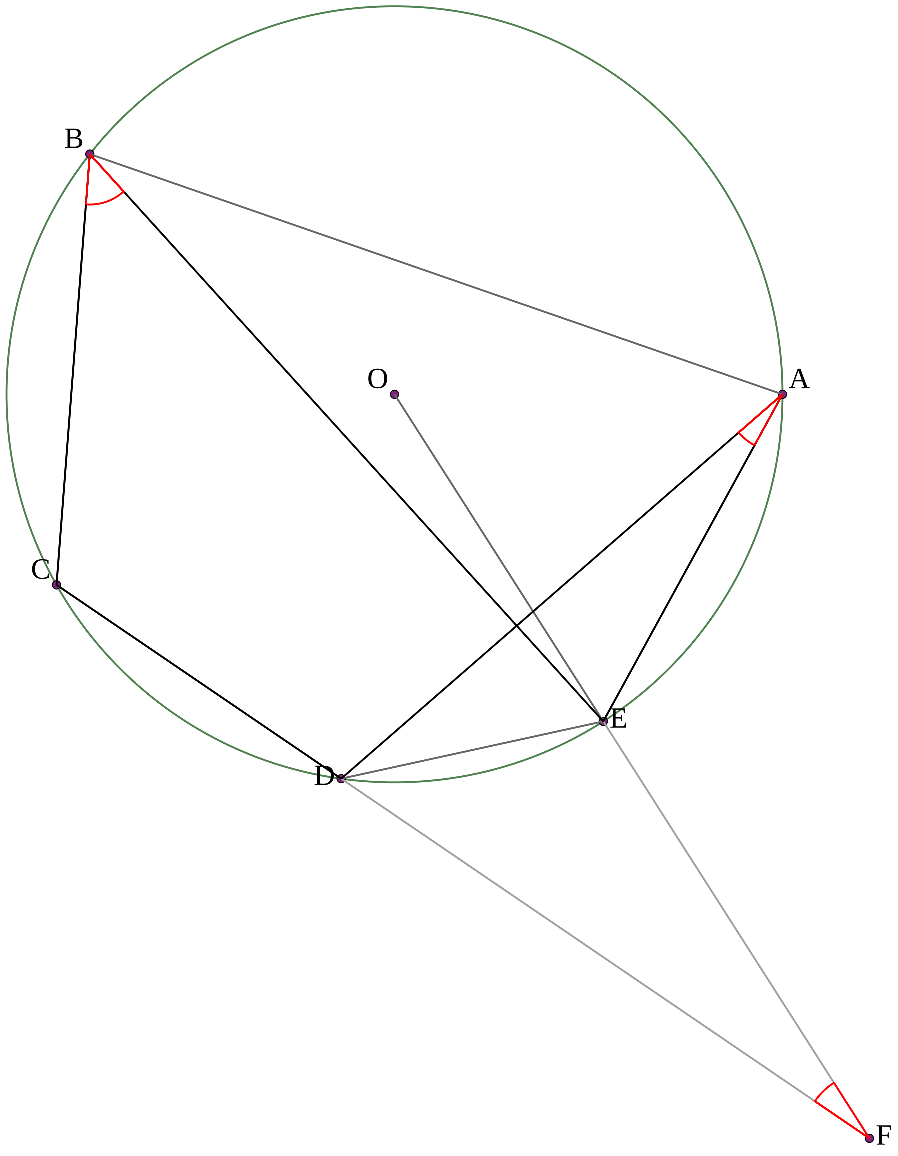
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of DC and EO.
Prove that DBE+CAE+DFE = 90

Example 81



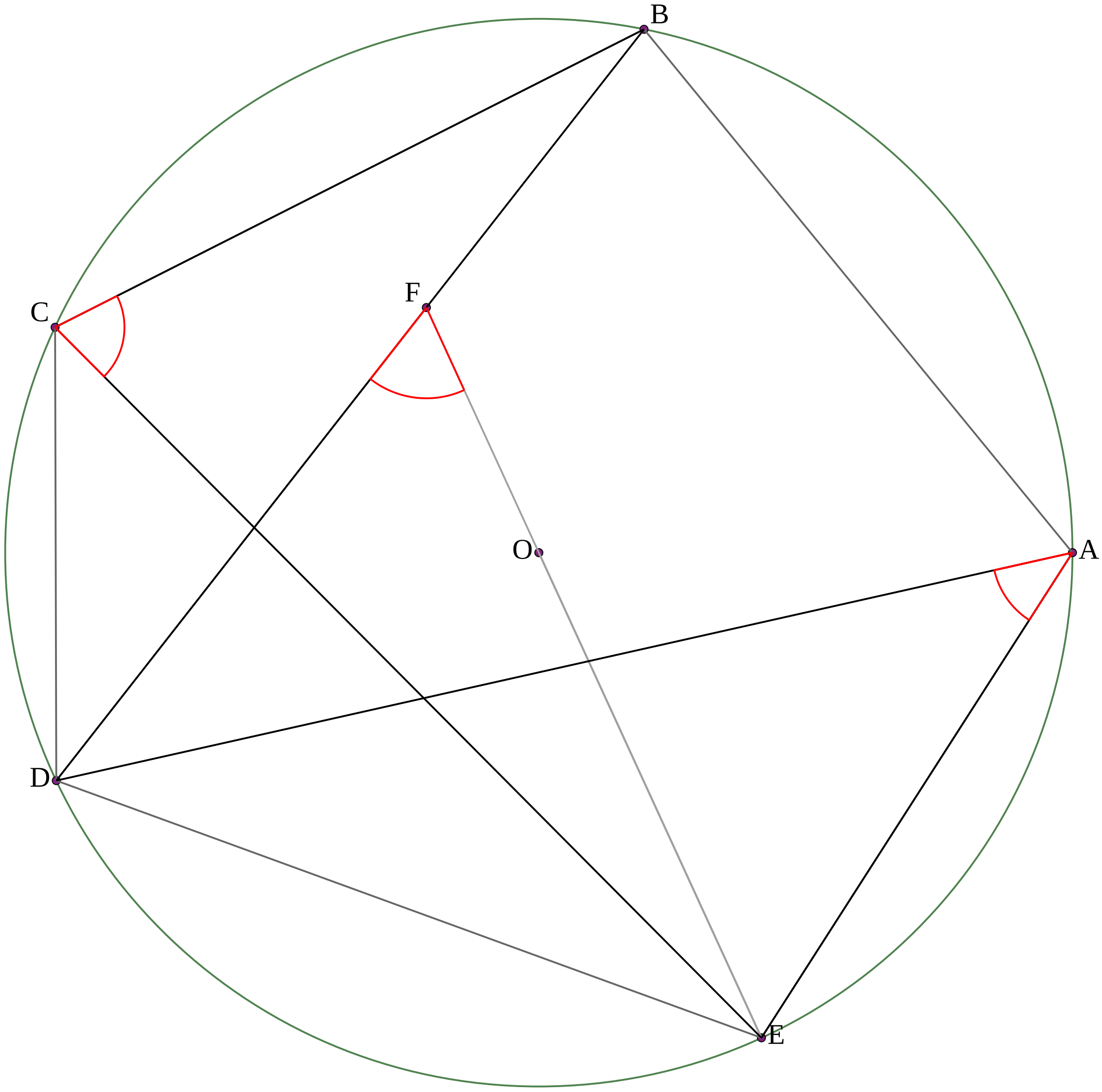
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BC and EO.
Prove that BDE+BFE = CAE+90

Example 82



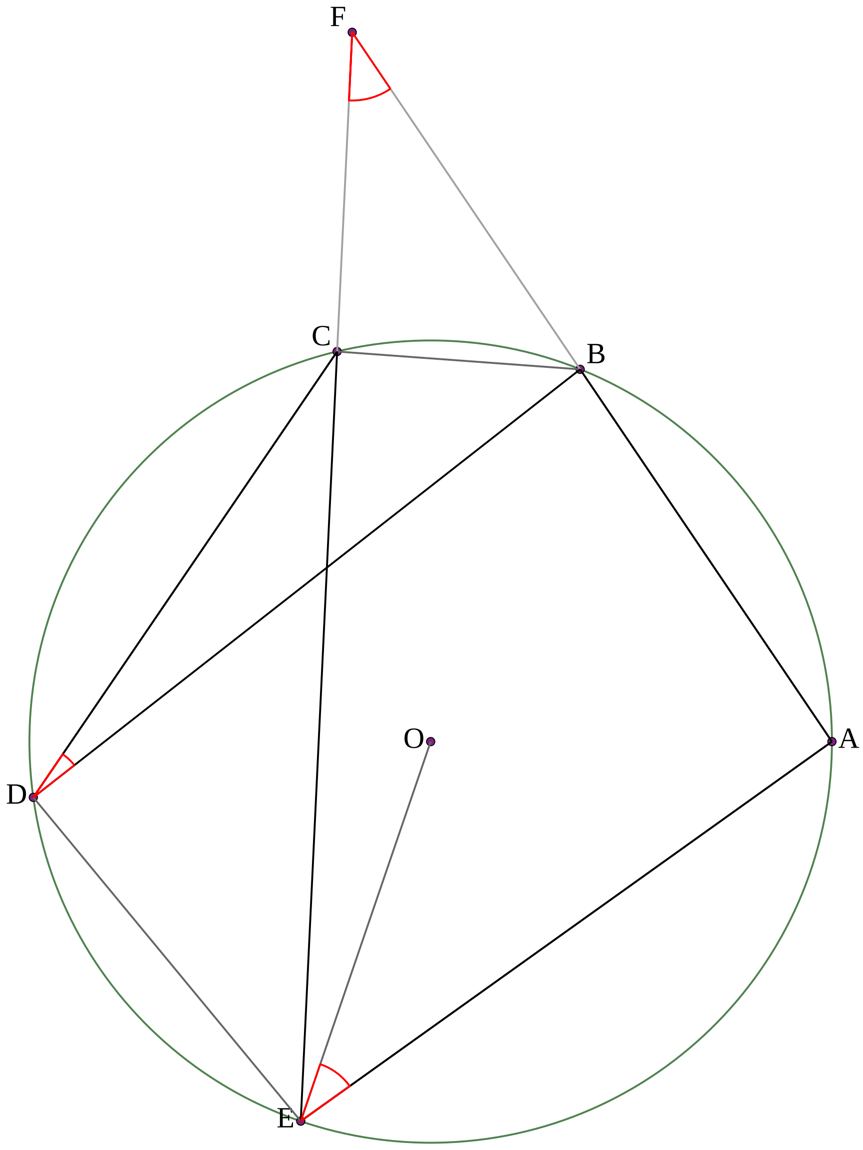
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of CD and EO.
Prove that CBE+DAE+DFE = 90

Example 83



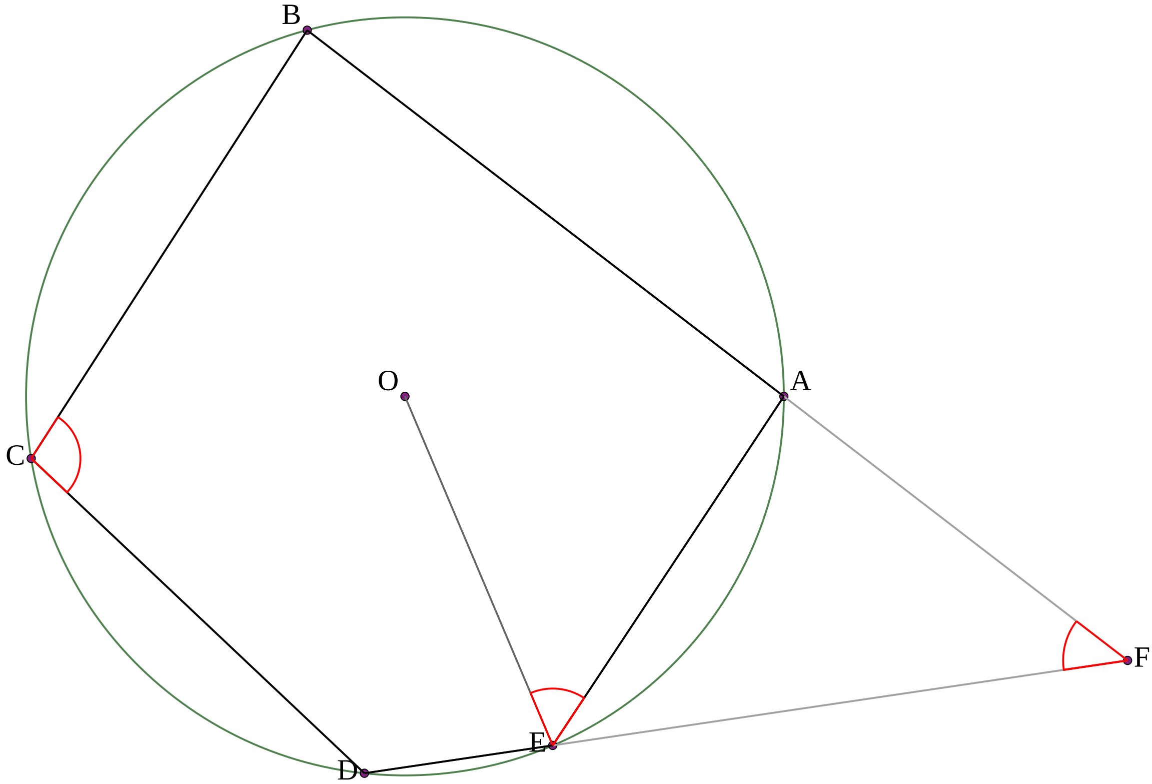
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EO.
Prove that BCE+DFE = DAE+90

Example 84



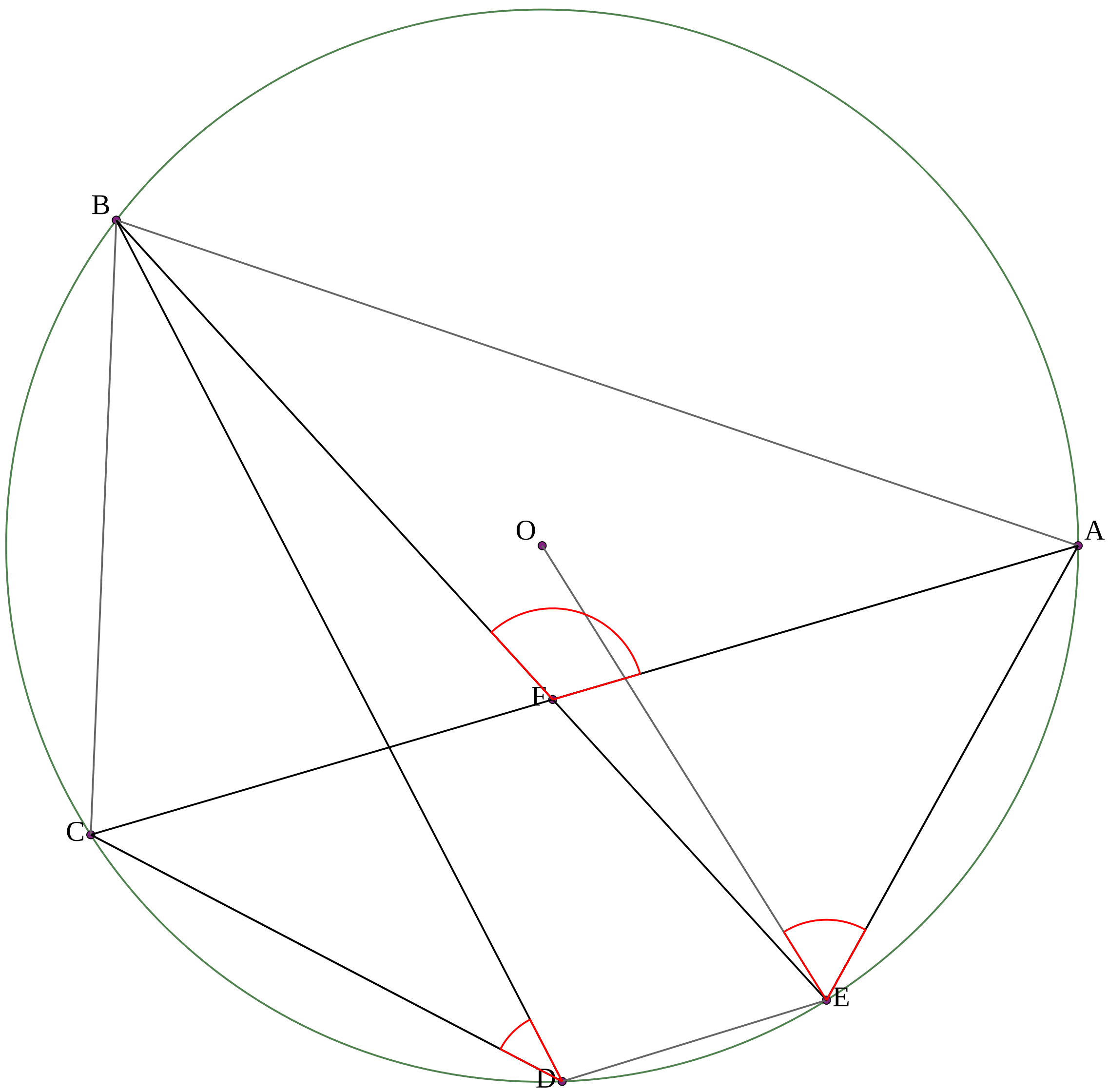
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and BA.
Prove that BDC+AEO+BFC = 90

Example 85



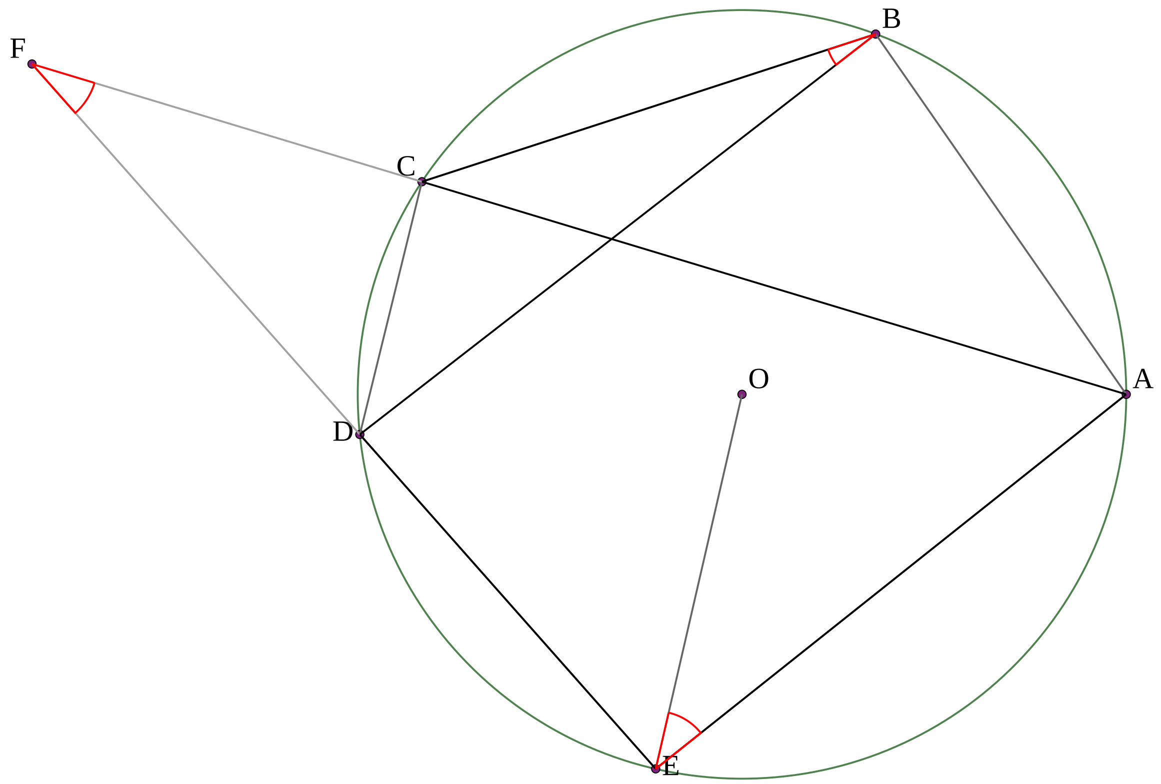
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and BA.
Prove that BCD+AFE = AEO+90

Example 86



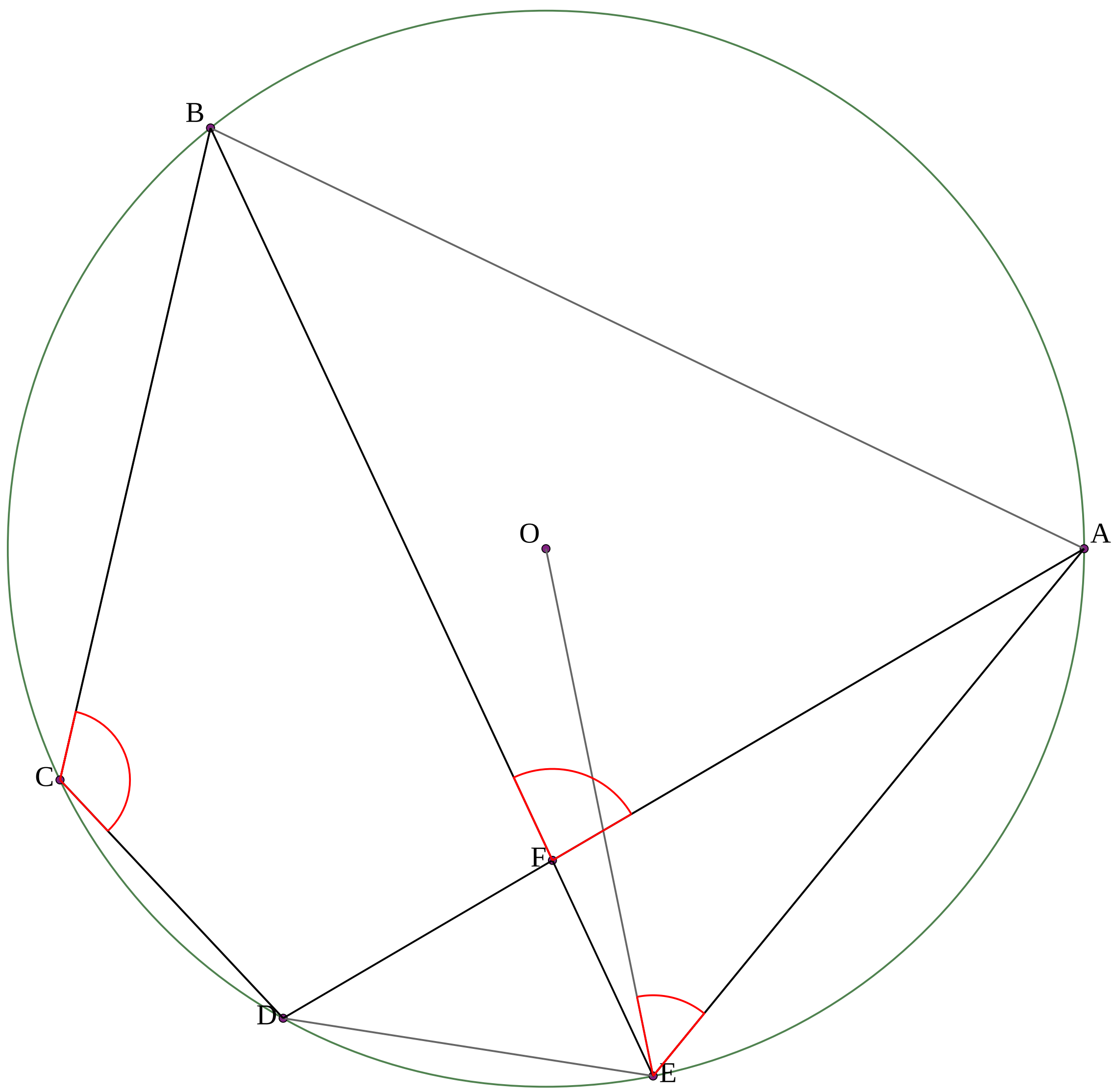
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and CA.
Prove that BDC+AFB = AEO+90

Example 87



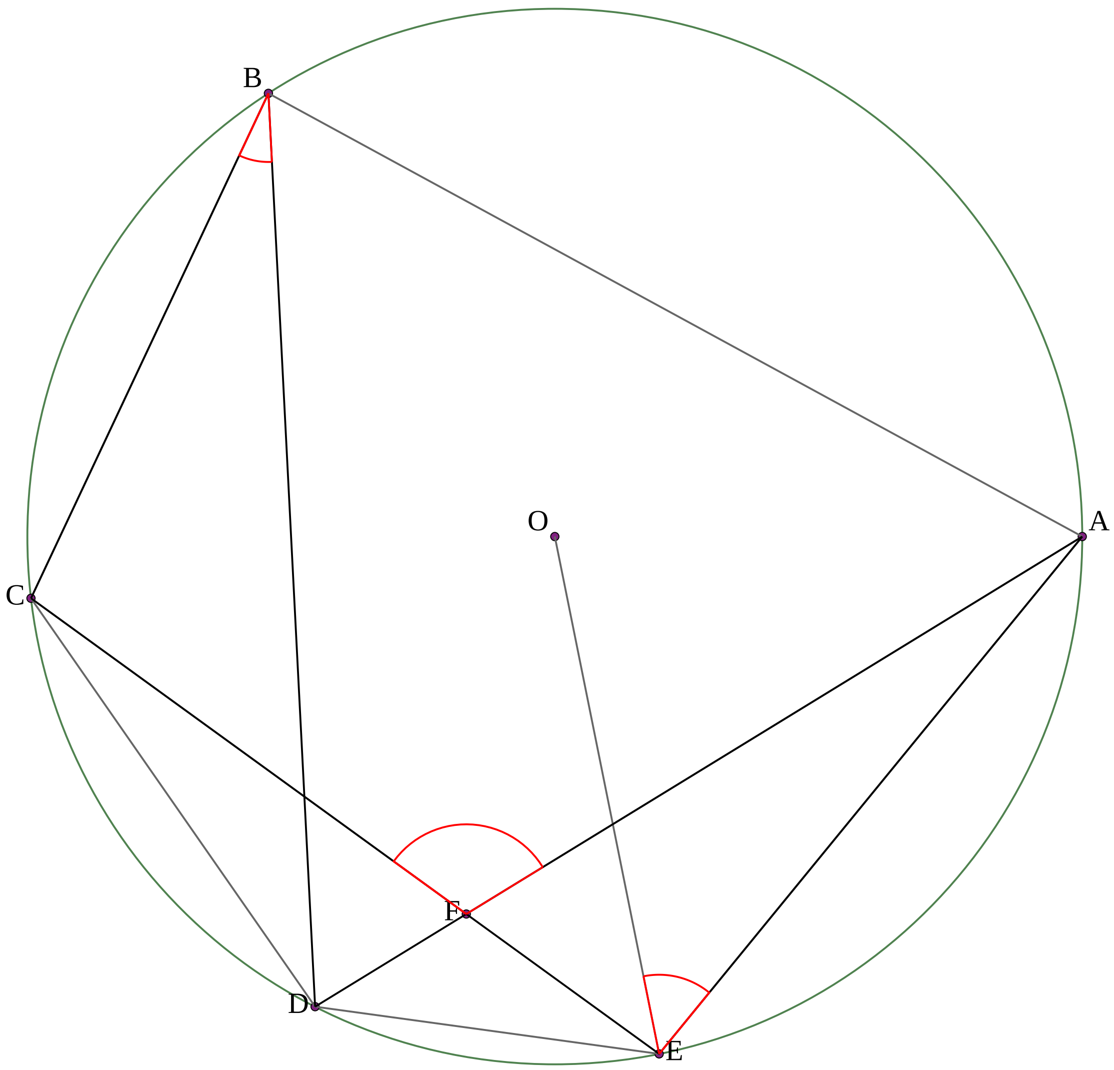
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and CA.
Prove that CBD+AEO+CFD = 90

Example 88



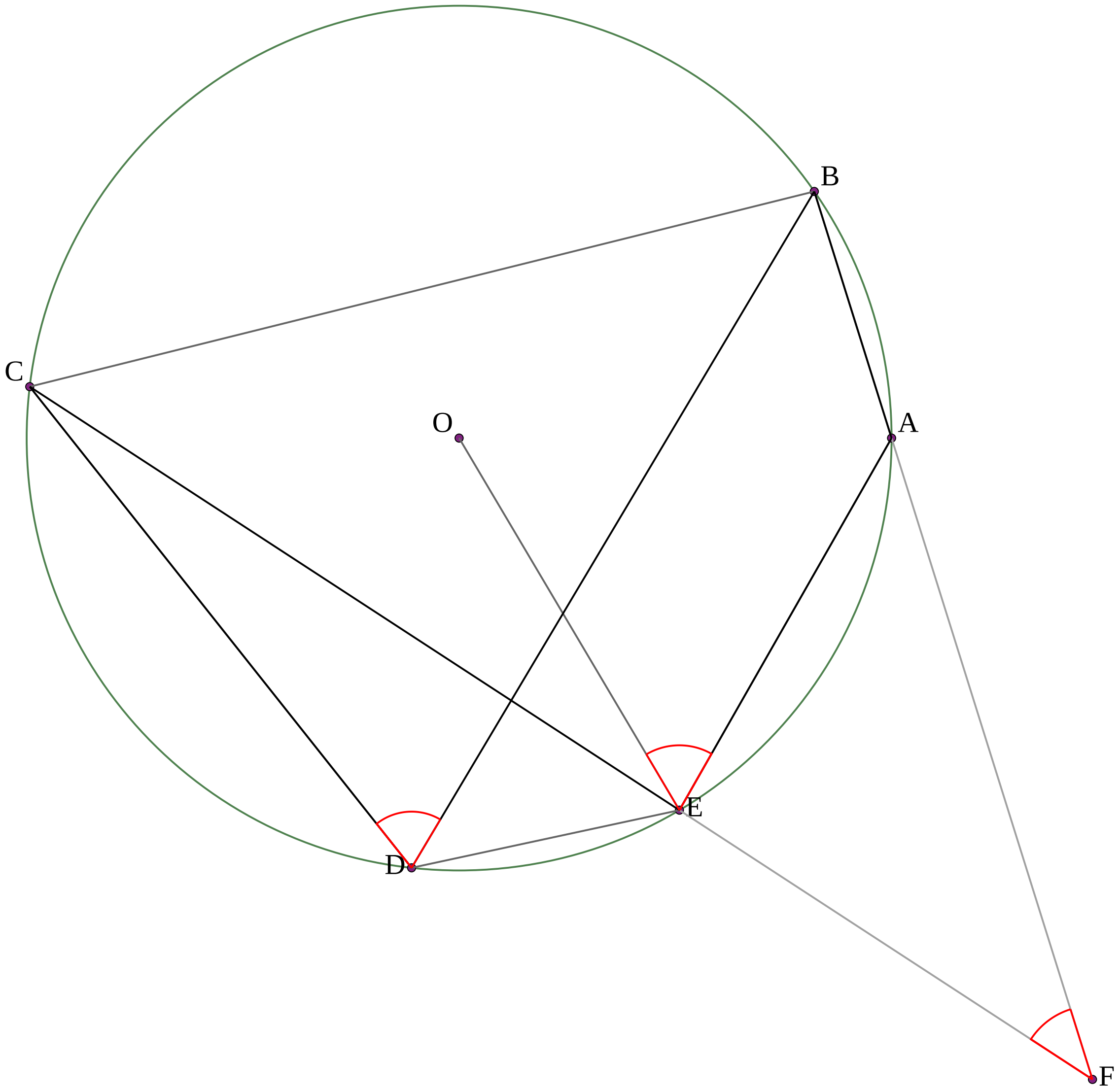
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA.
Prove that BCD+AEO = AFB+90

Example 89



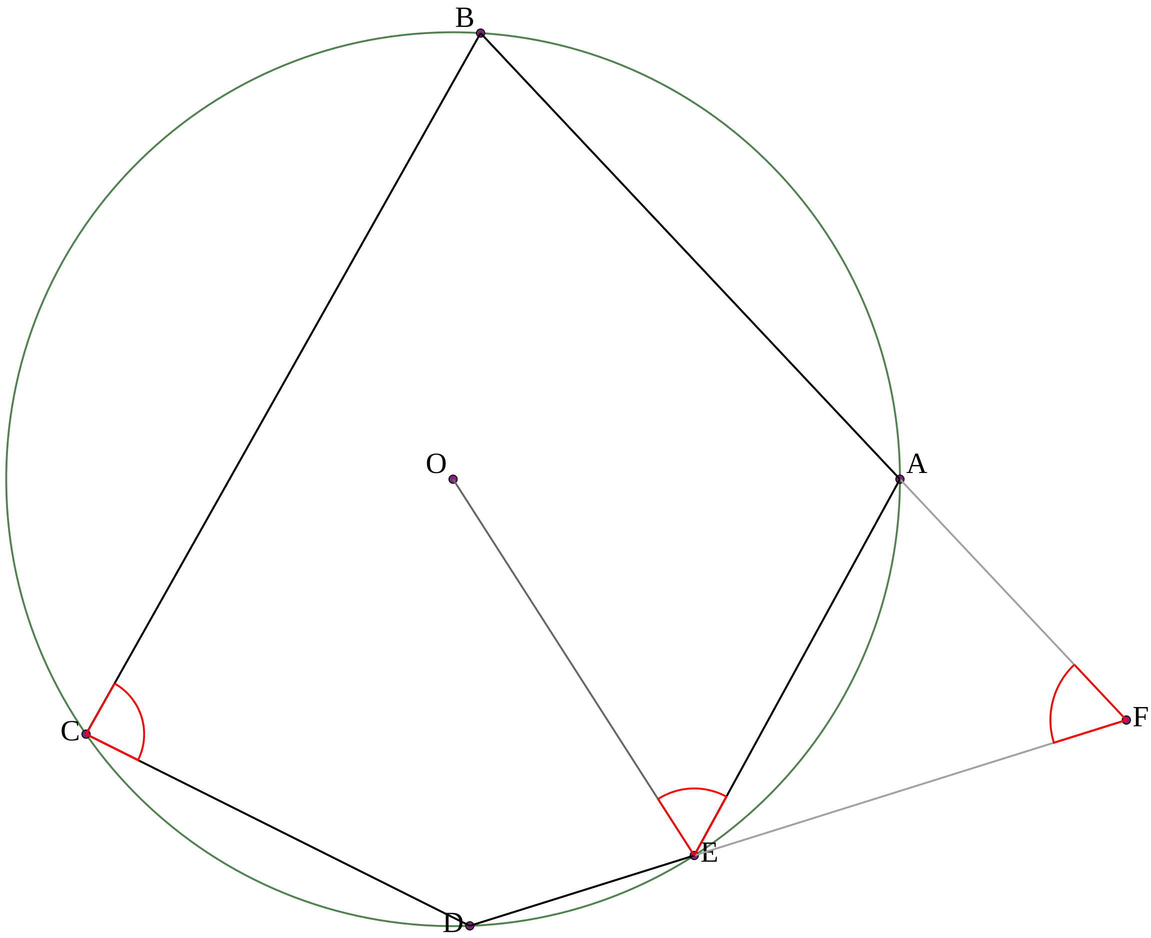
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA.
Prove that CBD+AFC = AEO+90

Example 90



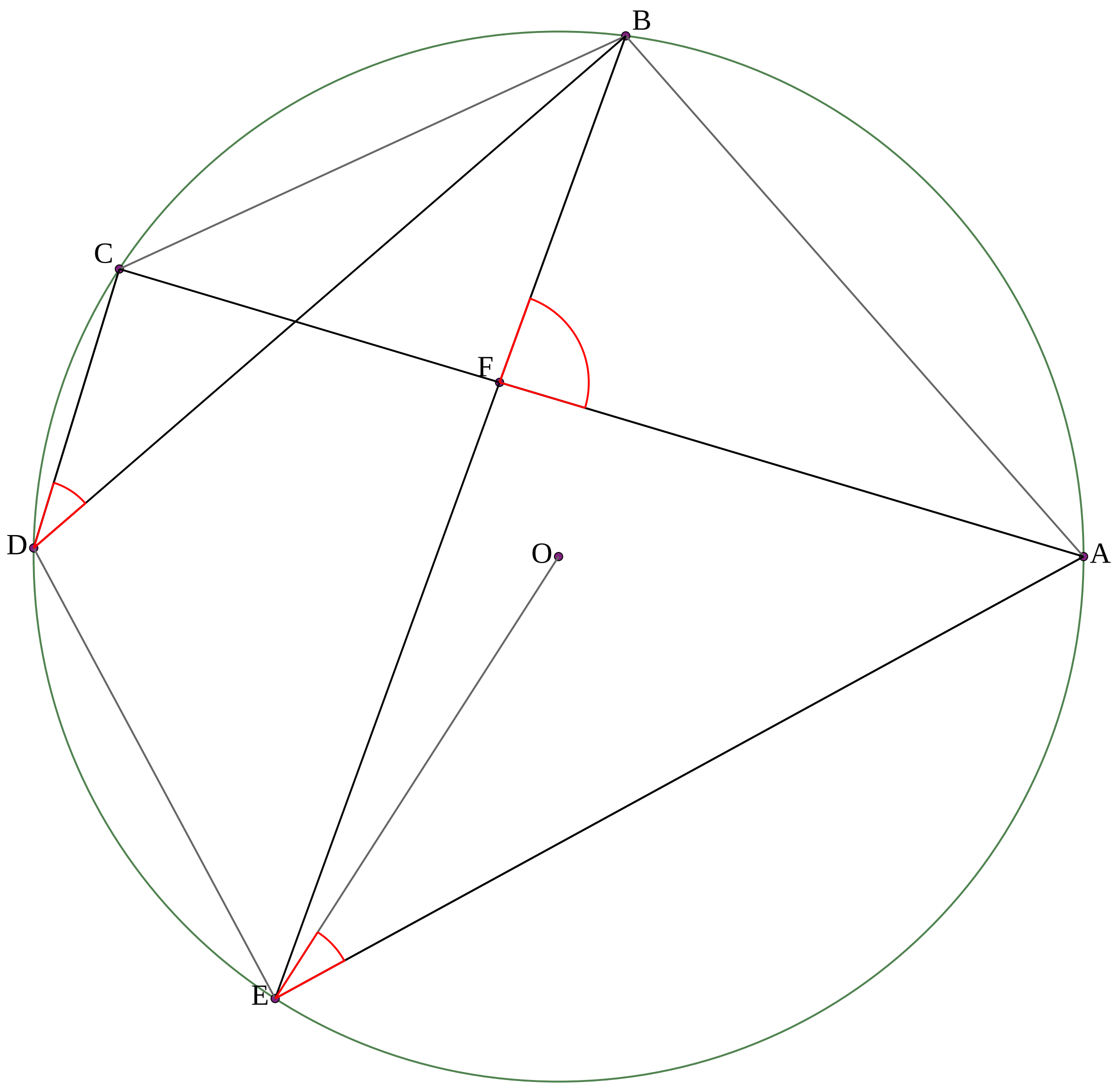
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and BA.
Prove that BDC+AEO = AFE+90

Example 91



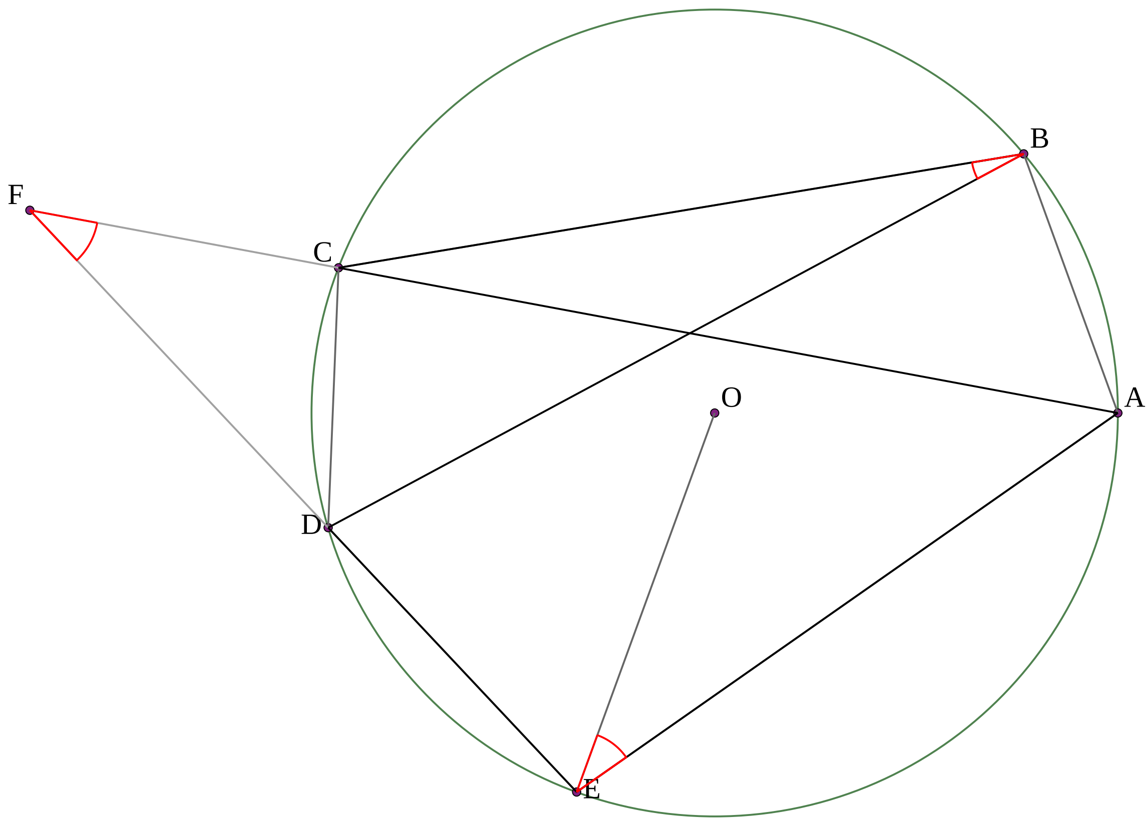
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and BA.
Prove that BCD+AFE = AEO+90

Example 92



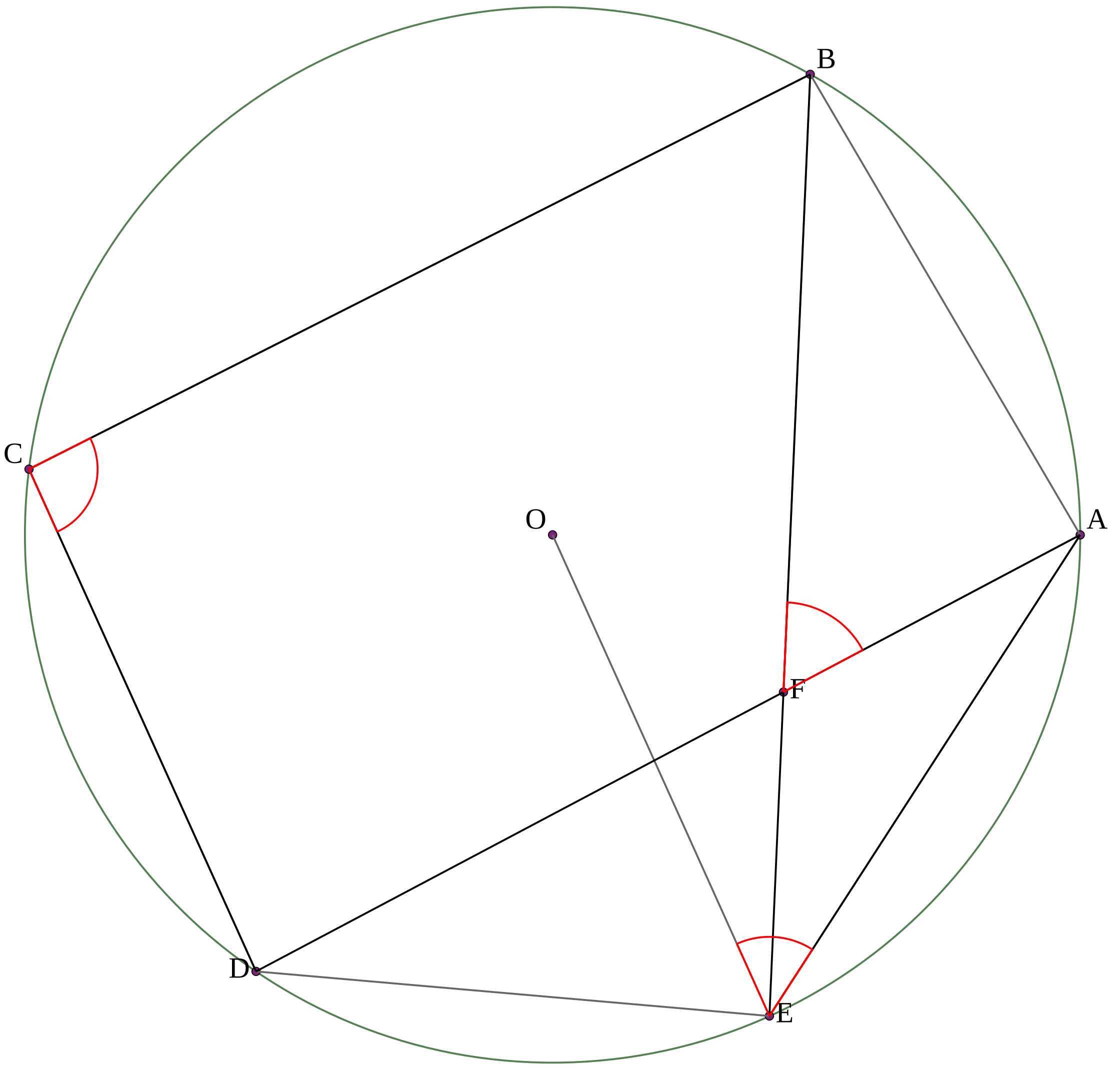
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and CA.
Prove that BDC+AFB = AEO+90

Example 93



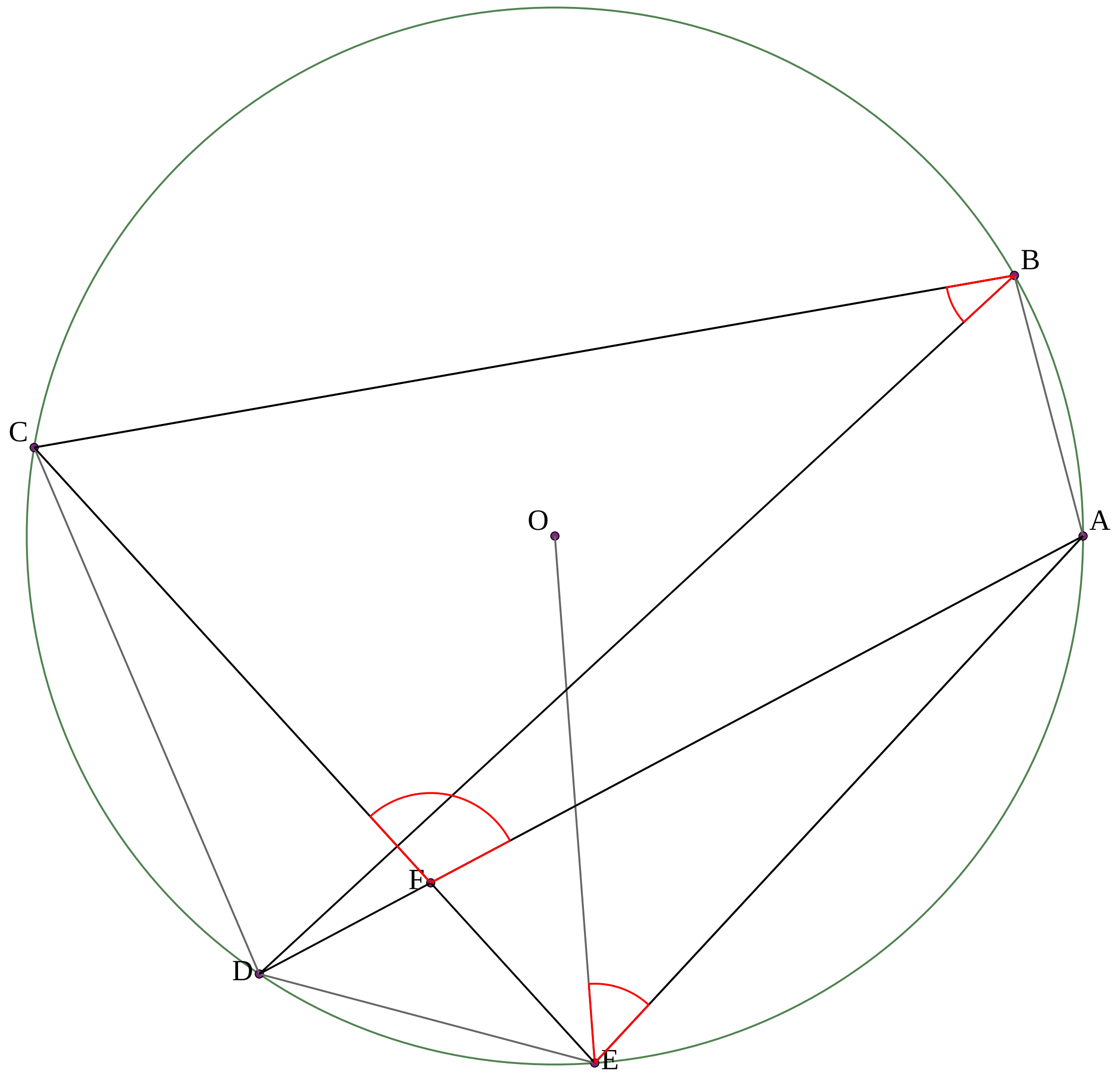
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and CA.
Prove that CBD+AEO+CFD = 90

Example 94



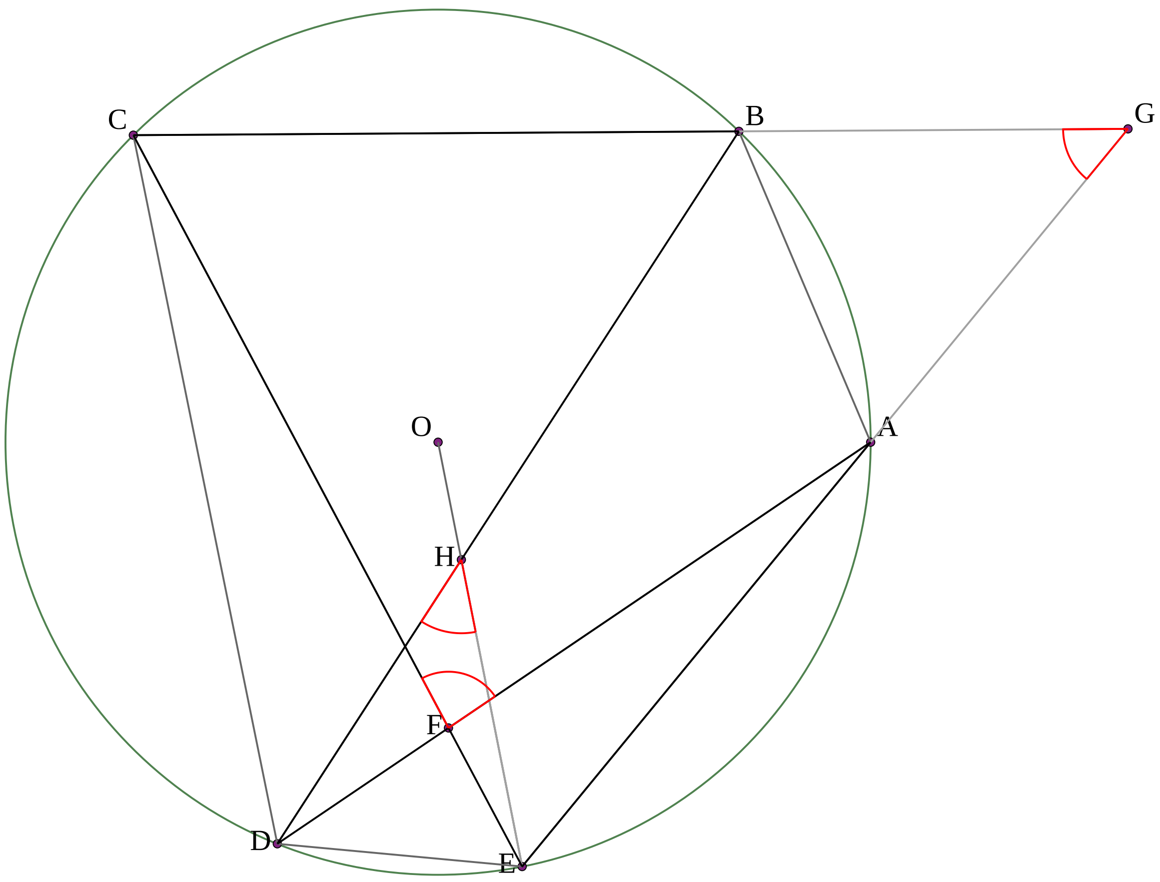
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA.
Prove that BCD+AEO = AFB+90

Example 95



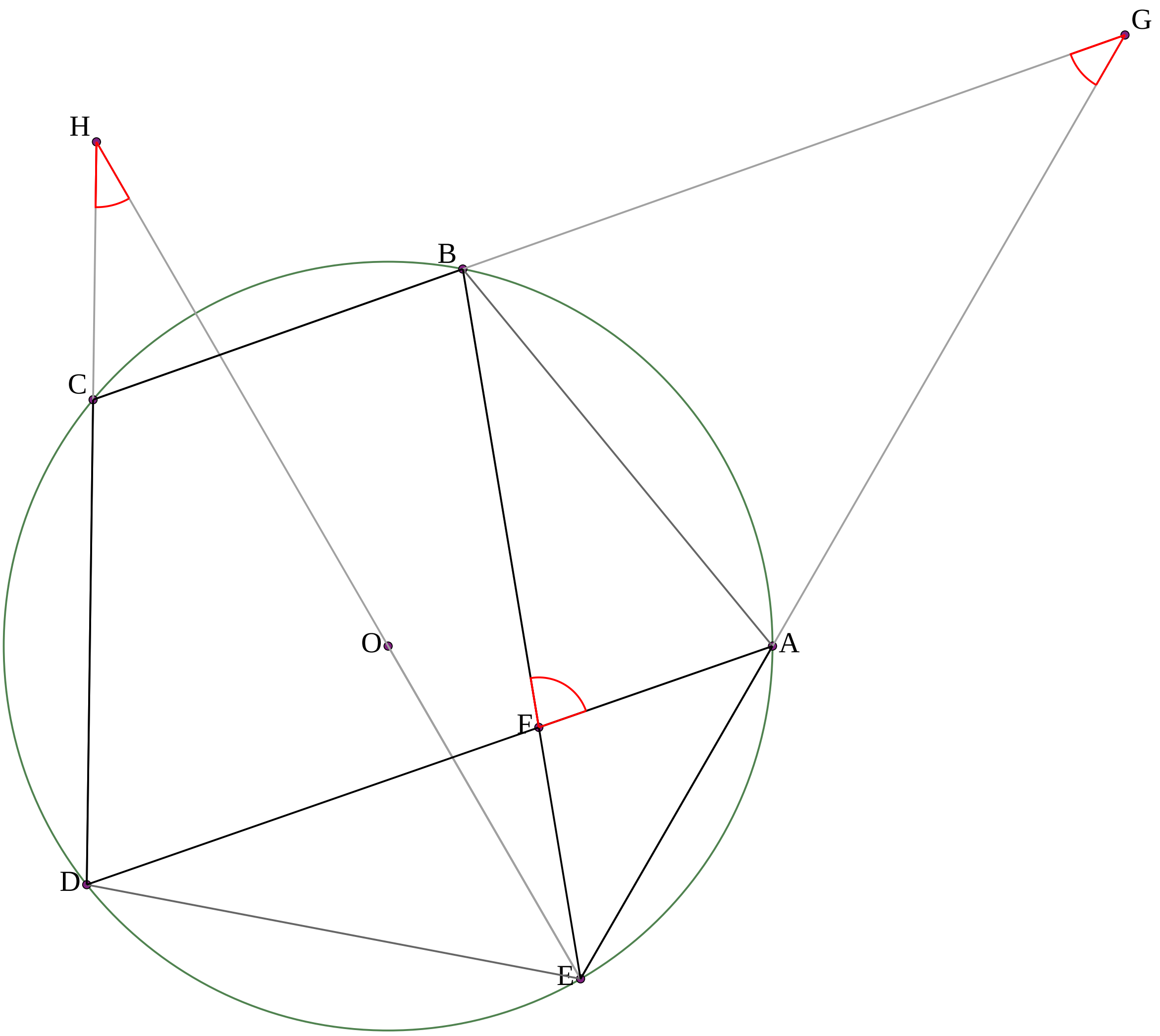
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA.
Prove that CBD+AFC = AEO+90

Example 96



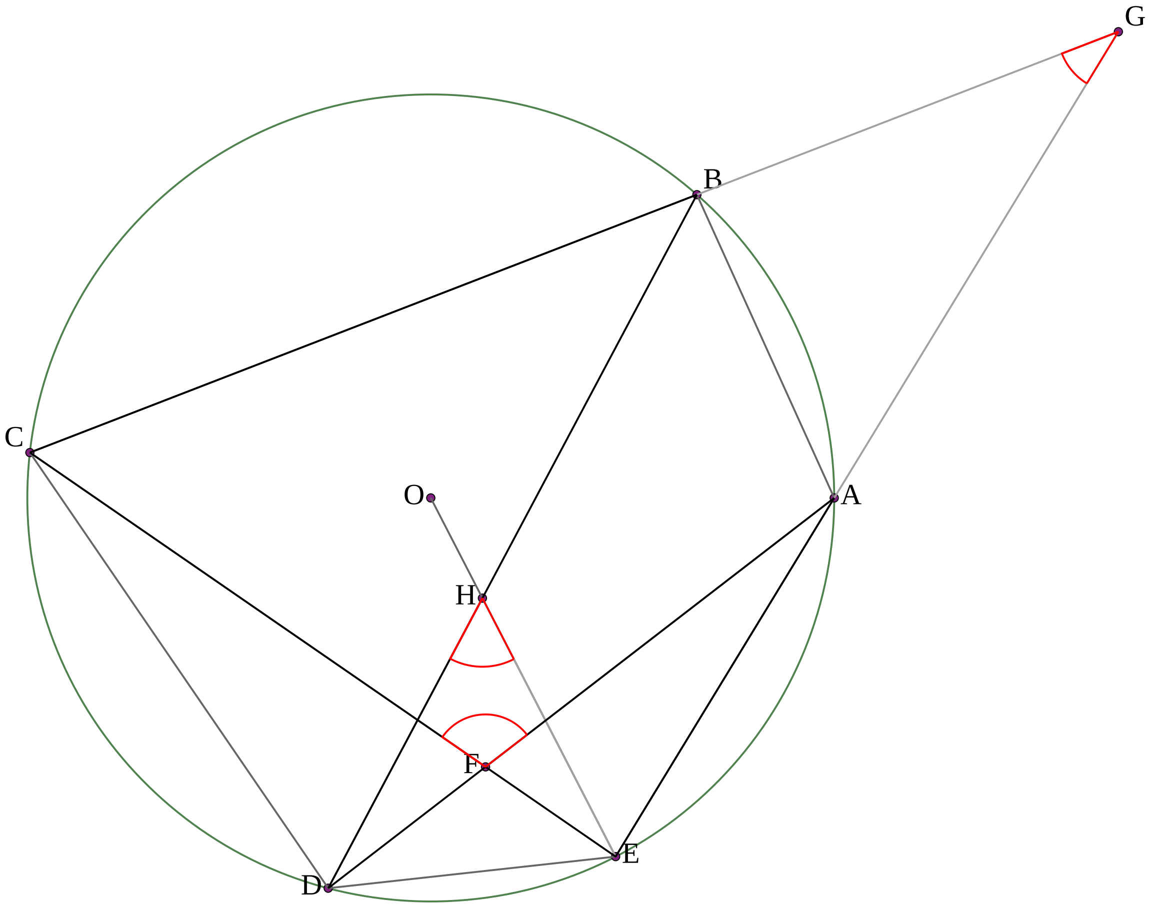
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA. Let G be the intersection of CB and AE. Let H be the intersection of BD and EO.
Prove that AFC+AGB = DHE+90

Example 97



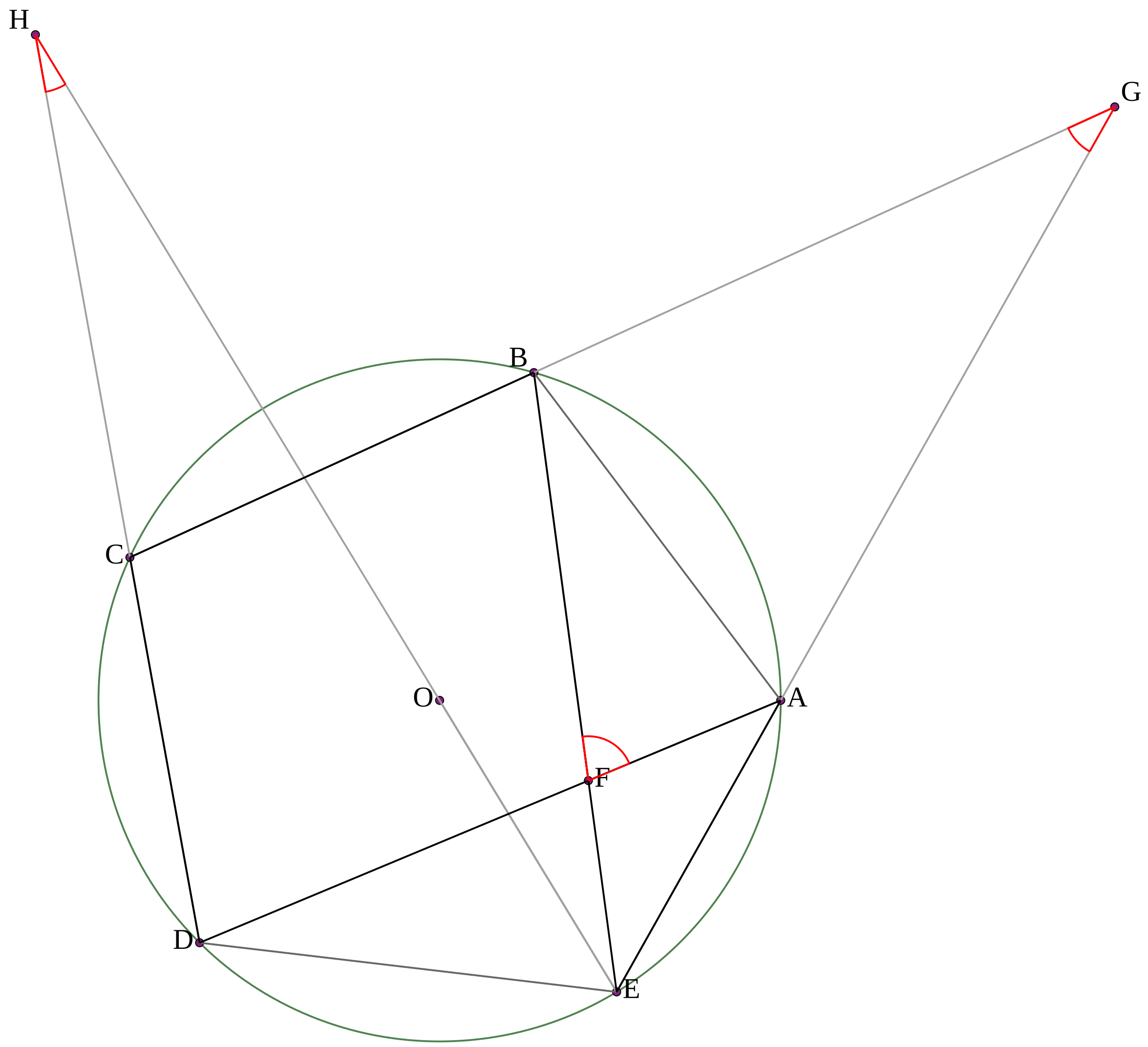
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA. Let G be the intersection of BC and AE. Let H be the intersection of CD and EO.
Prove that AFB+AGB = CHE+90

Example 98



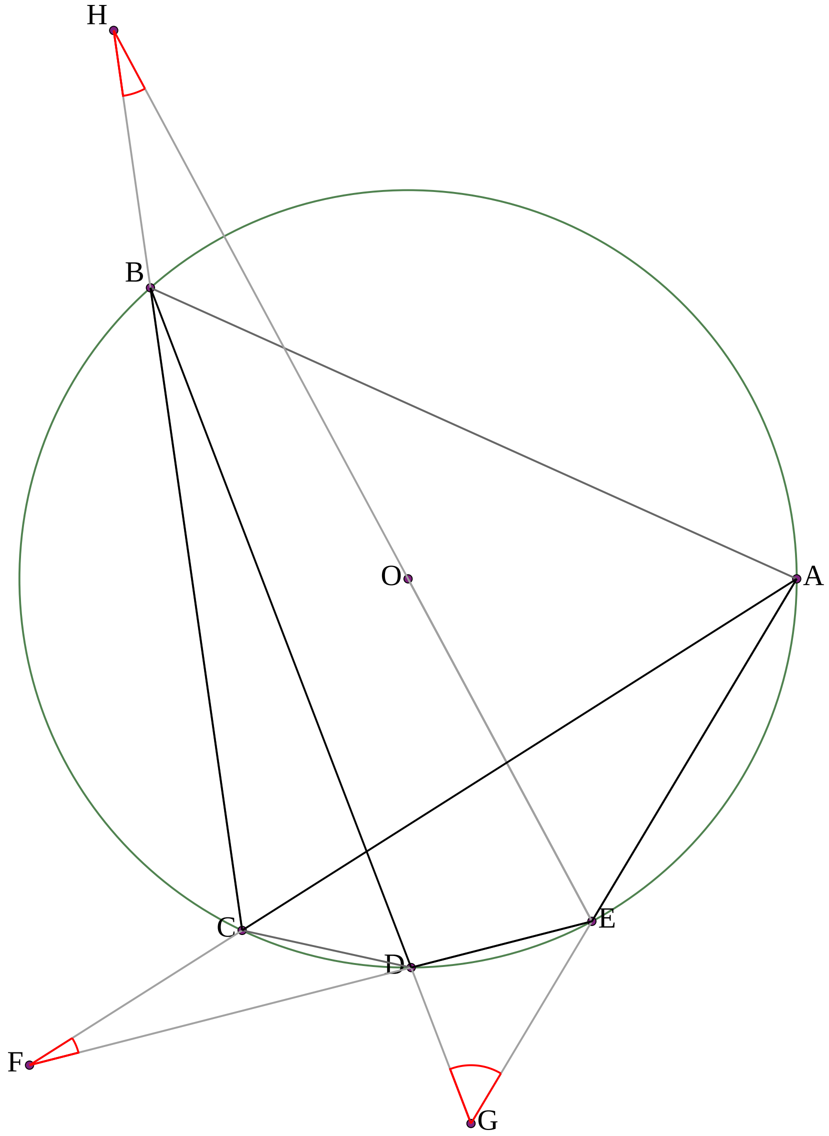
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA. Let G be the intersection of CB and AE. Let H be the intersection of BD and EO.
Prove that AFC+AGB = DHE+90

Example 99



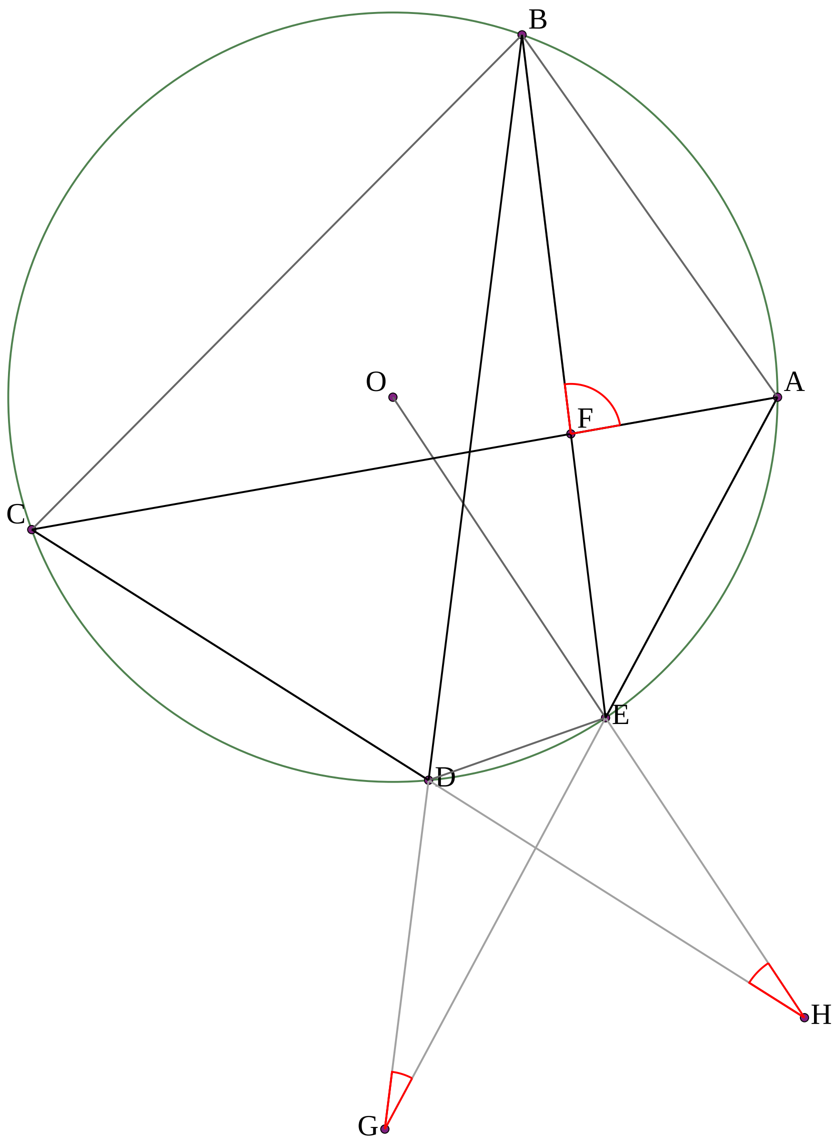
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA. Let G be the intersection of BC and AE. Let H be the intersection of CD and EO.
Prove that AFB+AGB = CHE+90

Example 100



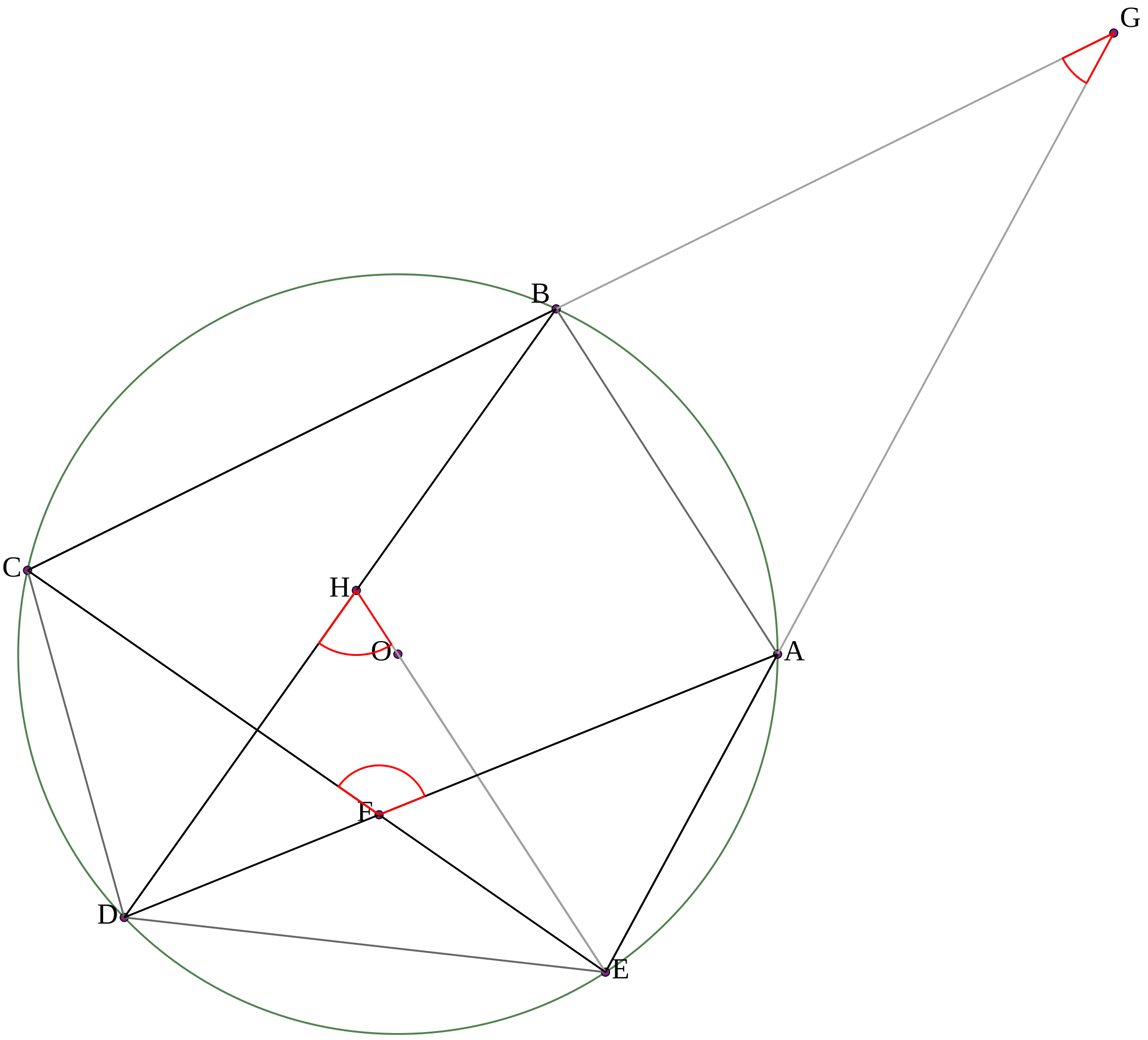
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and CA. Let G be the intersection of DB and AE. Let H be the intersection of BC and EO.
Prove that CFD+DGE+BHE = 90

Example 101



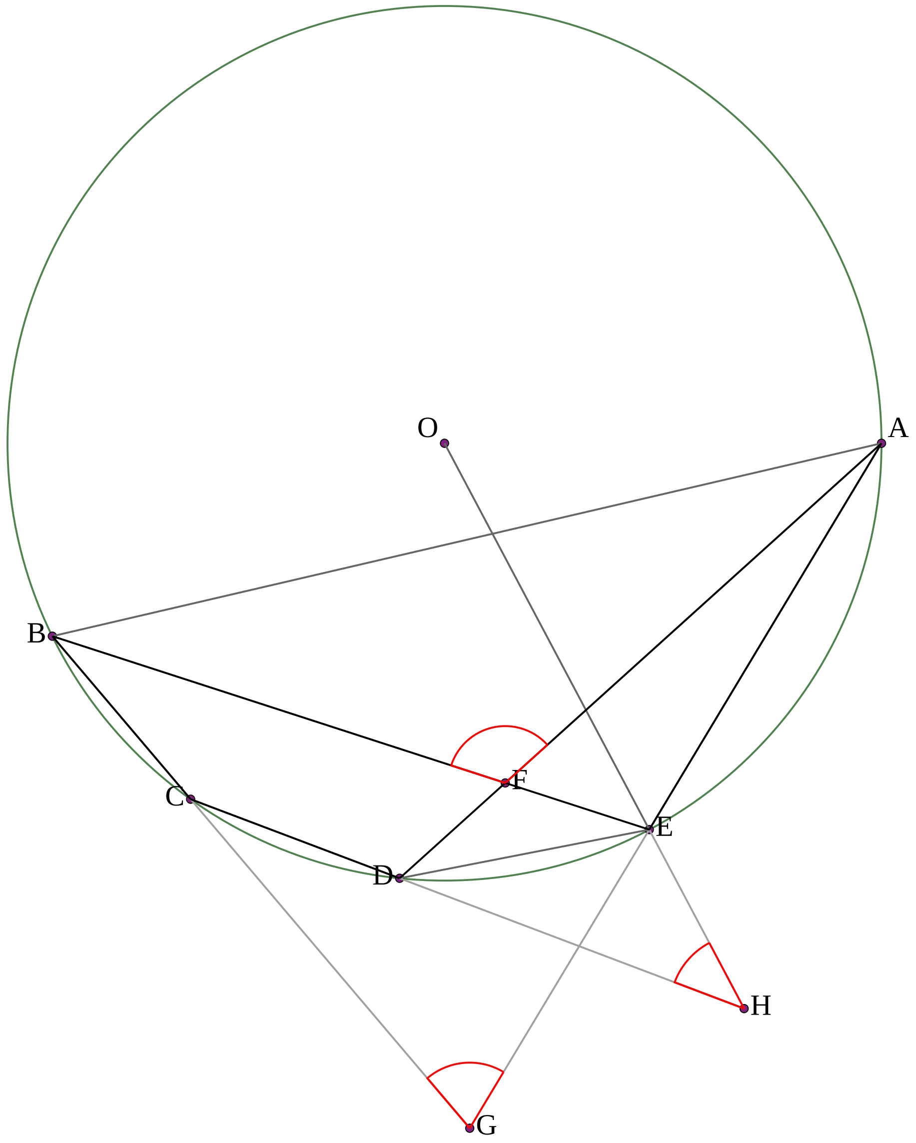
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and CA. Let G be the intersection of BD and AE. Let H be the intersection of DC and EO.
Prove that AFB+DHE = DGE+90

Example 102



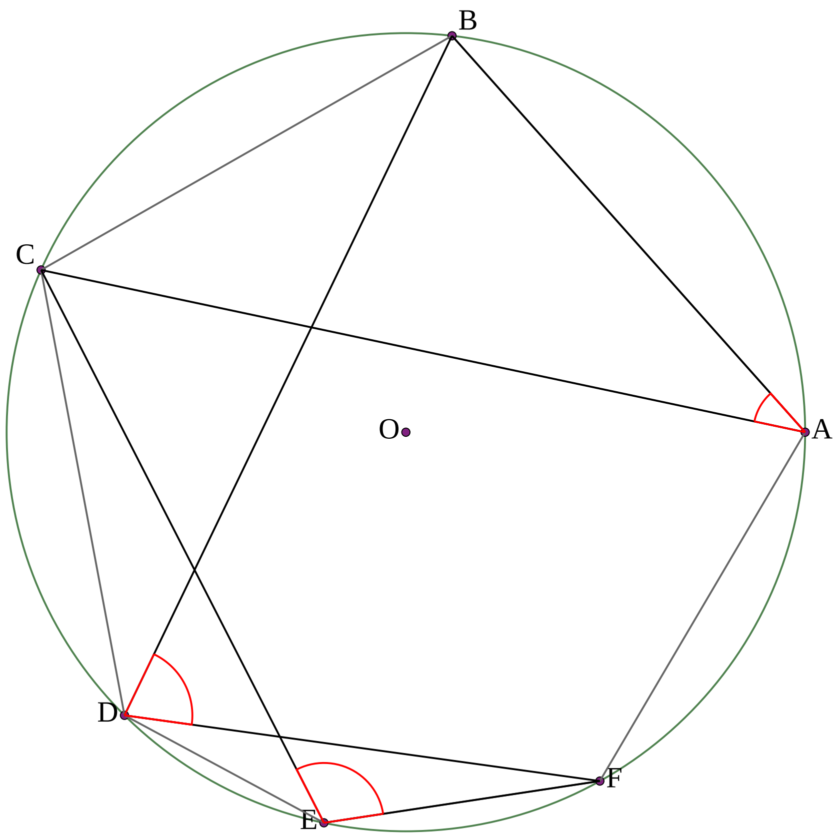
Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA. Let G be the intersection of CB and AE. Let H be the intersection of BD and EO.
Prove that AFC+AGB = DHE+90

Example 103



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA. Let G be the intersection of BC and AE. Let H be the intersection of CD and EO.
Prove that AFB+DHE = CGE+90

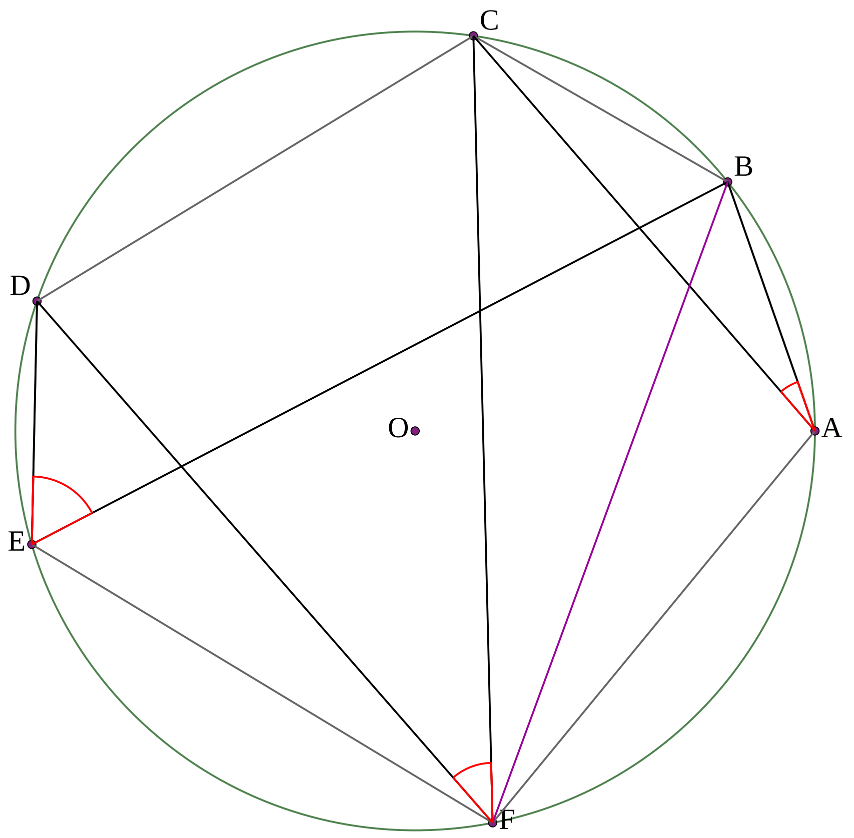
Solution to example 1



Let ABCDEF be a cyclic hexagon with center O.
Prove that CEF = BAC+BDF

Let BAC=x. Let CEF=y. Let BDF=z.
As BDFA is a cyclic quadrilateral, BAF=180-BDF, so BAF=180-z.
As CEFA is a cyclic quadrilateral, CAF=180-CEF, so CAF=180-y.
As BAC=x, BAF=x-y+180.
But BAF=180-z, so x-y+180=180-z, or x+z=y, or BAC+BDF=CEF.

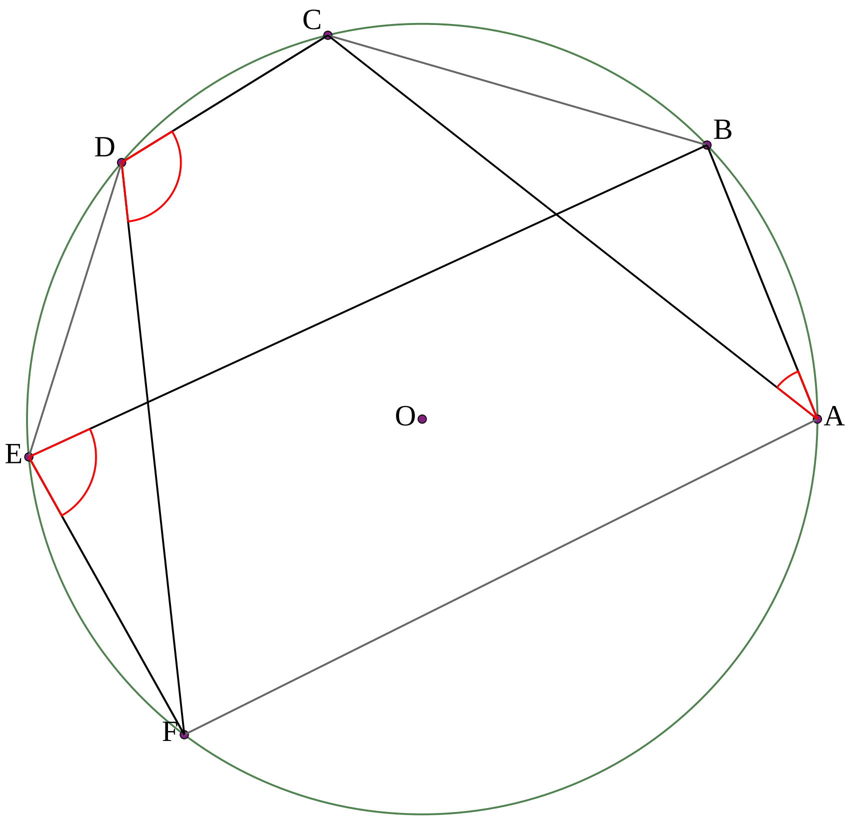
Solution to example 2



Let ABCDEF be a cyclic hexagon with center O.
Prove that BED = BAC+CFD

Draw line BF.
Let BAC=x. Let CFD=y. Let BED=z.
As BED and BFD stand on the same chord, BFD=BED, so BFD=z.
As BAC and BFC stand on the same chord, BFC=BAC, so BFC=x.
As CFD=y, DFB=x+y.
But BFD=z, so x+y=z, or BAC+CFD=BED.

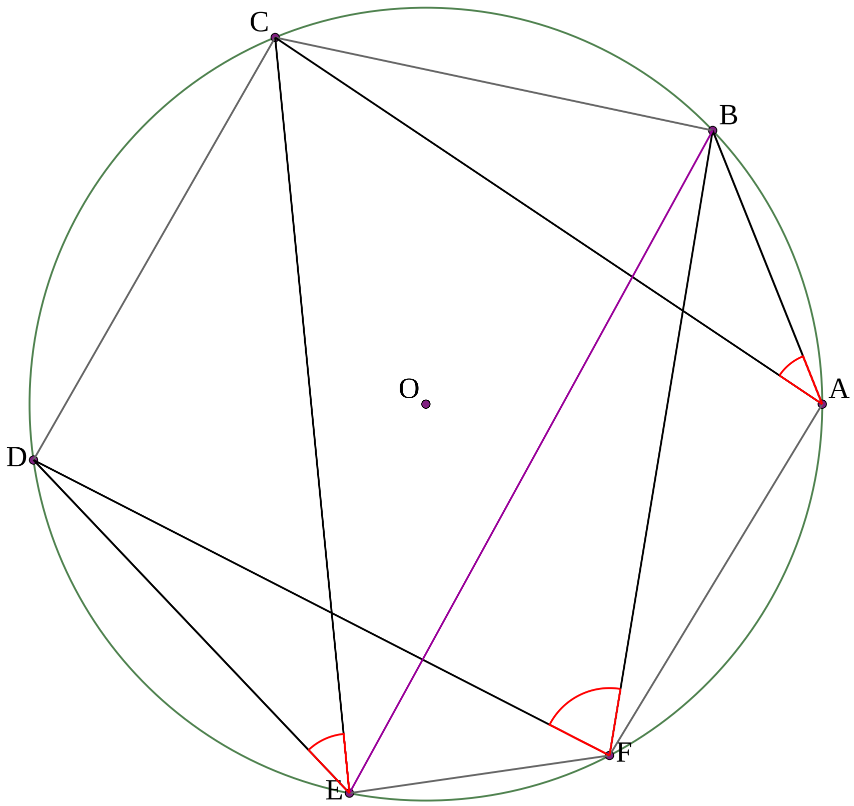
Solution to example 3



Let ABCDEF be a cyclic hexagon with center O.
Prove that CDF = BAC+BEF

Let BAC=x. Let CDF=y. Let BEF=z.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=180-z.
As CDFA is a cyclic quadrilateral, CAF=180-CDF, so CAF=180-y.
As BAC=x, BAF=x-y+180.
But BAF=180-z, so x-y+180=180-z, or x+z=y, or BAC+BEF=CDF.

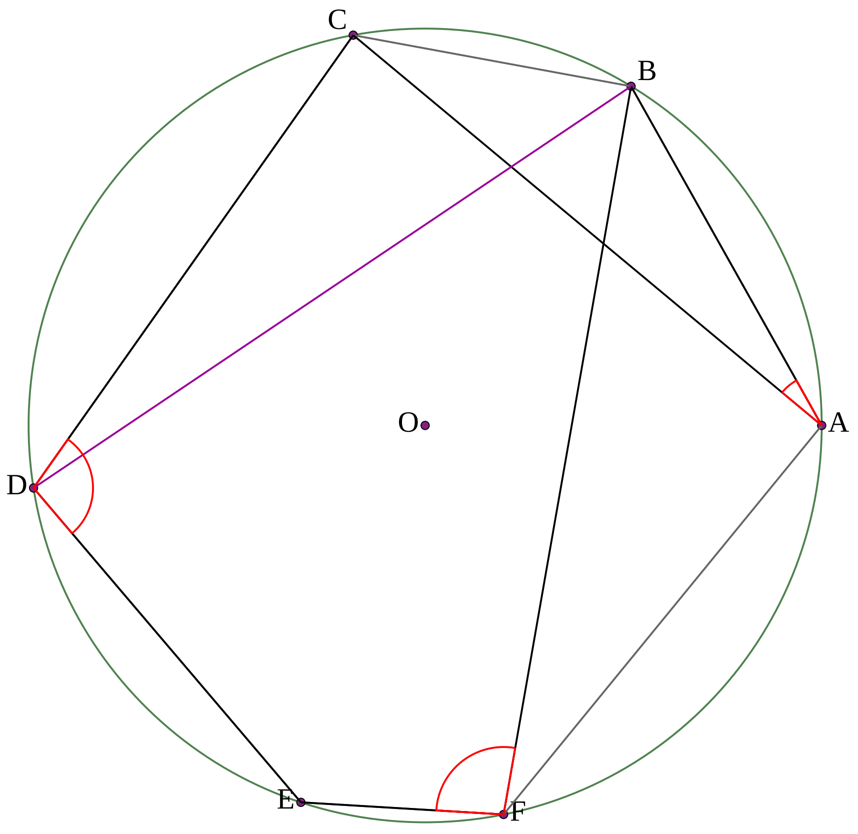
Solution to example 4



Let ABCDEF be a cyclic hexagon with center O.
Prove that BFD = BAC+CED

Draw line BE.
Let BAC=x. Let CED=y. Let BFD=z.
As BFD and BED stand on the same chord, BED=BFD, so BED=z.
As BAC and BEC stand on the same chord, BEC=BAC, so BEC=x.
As CED=y, DEB=x+y.
But BED=z, so x+y=z, or BAC+CED=BFD.

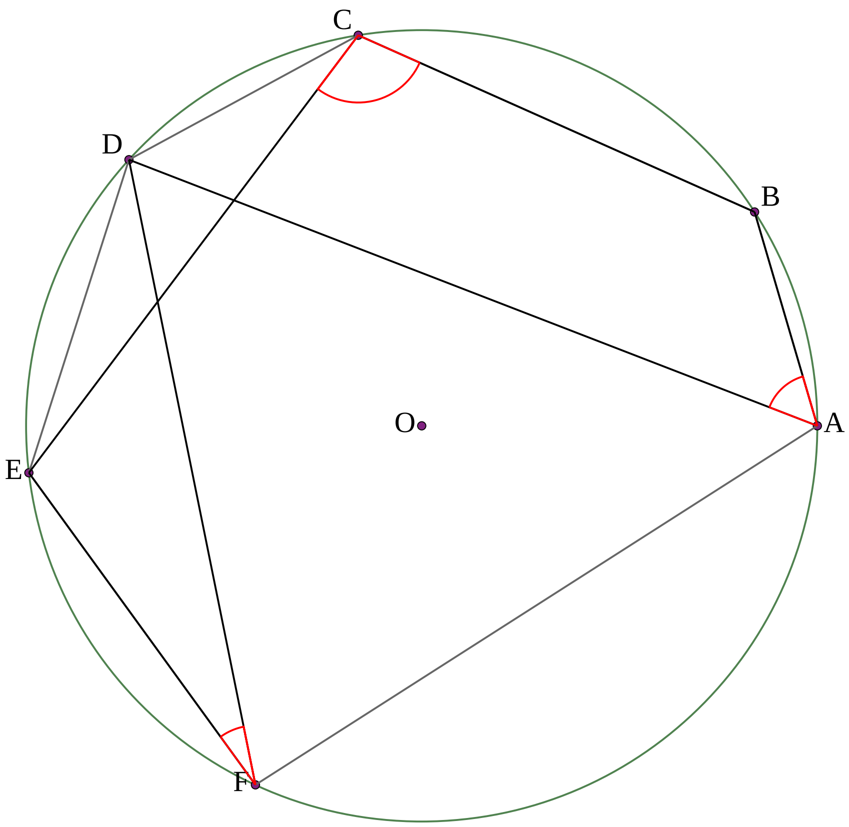
Solution to example 5



Let ABCDEF be a cyclic hexagon with center O.
Prove that CDE+BFE = BAC+180

Draw line BD.
Let BAC=x. Let CDE=y. Let BFE=z.
As BFED is a cyclic quadrilateral, BDE=180-BFE, so BDE=180-z.
As BAC and BDC stand on the same chord, BDC=BAC, so BDC=x.
As CDE=y, EDB=y-x.
But BDE=180-z, so y-x=180-z, or y+z=x+180, or CDE+BFE=BAC+180.

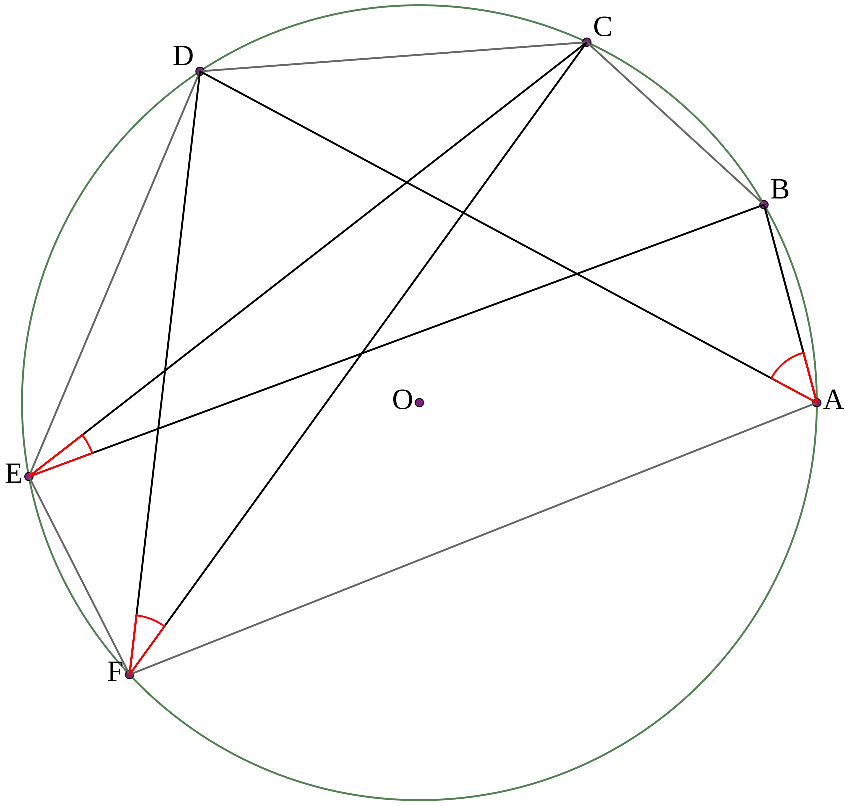
Solution to example 6



Let ABCDEF be a cyclic hexagon with center O.
Prove that BAD+DFE+BCE = 180

Let BAD=x. Let DFE=y. Let BCE=z.
As DFE and DCE stand on the same chord, DCE=DFE, so DCE=y.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=180-x.
As BCE=z, ECD=180-x-z.
But DCE=y, so 180-x-z=y, or x+y+z=180, or BAD+DFE+BCE=180.

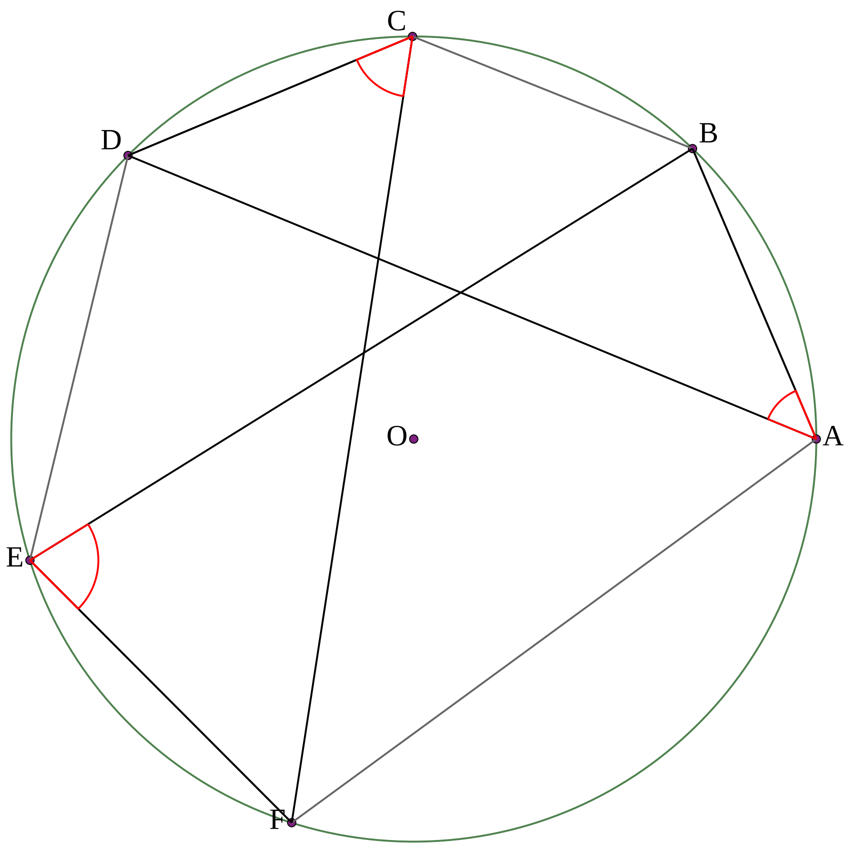
Solution to example 7



Let ABCDEF be a cyclic hexagon with center O.
Prove that CFD+BEC = BAD

Let BAD=x. Let CFD=y. Let BEC=z.
As CFD and CED stand on the same chord, CED=CFD, so CED=y.
As BAD and BED stand on the same chord, BED=BAD, so BED=x.
As BEC=z, CED=x-z.
But CED=y, so x-z=y, or x=y+z, or BAD=CFD+BEC.

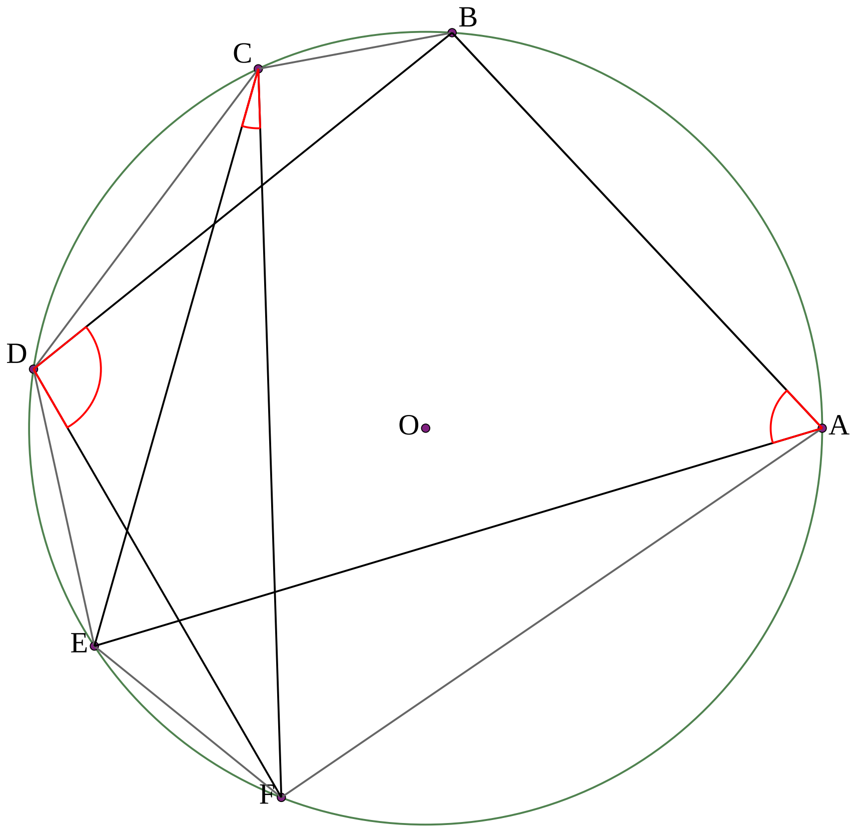
Solution to example 8



Let ABCDEF be a cyclic hexagon with center O.
Prove that BAD+DCF+BEF = 180

Let BAD=x. Let DCF=y. Let BEF=z.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=180-z.
As DCF and DAF stand on the same chord, DAF=DCF, so DAF=y.
As BAD=x, BAF=x+y.
But BAF=180-z, so x+y=180-z, or x+y+z=180, or BAD+DCF+BEF=180.

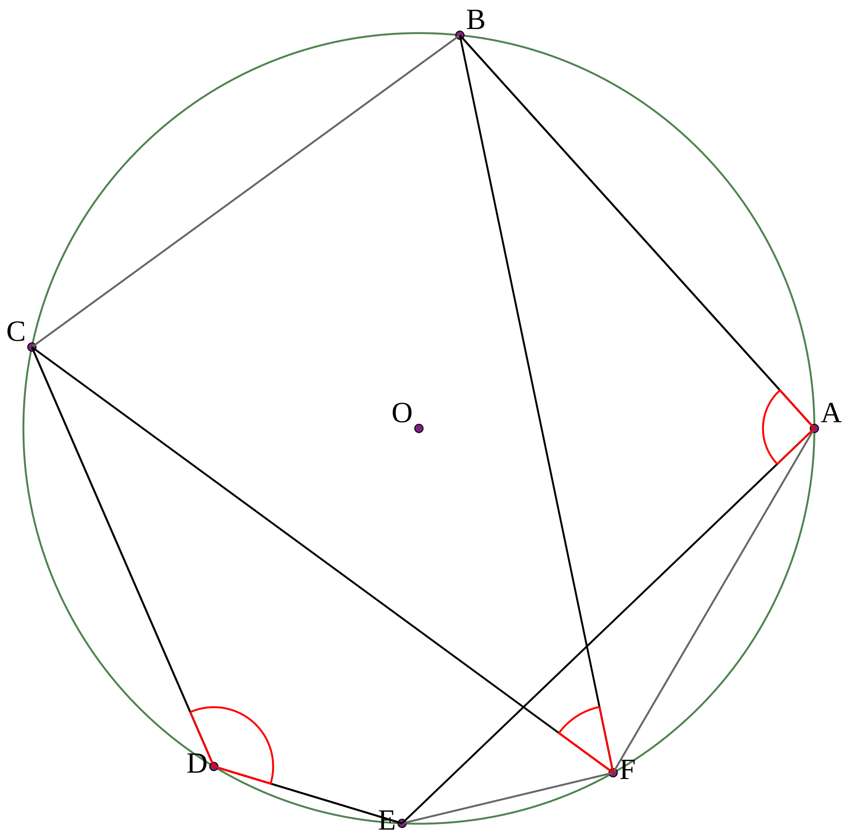
Solution to example 9



Let ABCDEF be a cyclic hexagon with center O.
Prove that BAE+ECF+BDF = 180

Let BAE=x. Let ECF=y. Let BDF=z.
As BDFA is a cyclic quadrilateral, BAF=180-BDF, so BAF=180-z.
As ECF and EAF stand on the same chord, EAF=ECF, so EAF=y.
As BAE=x, BAF=x+y.
But BAF=180-z, so x+y=180-z, or x+y+z=180, or BAE+ECF+BDF=180.

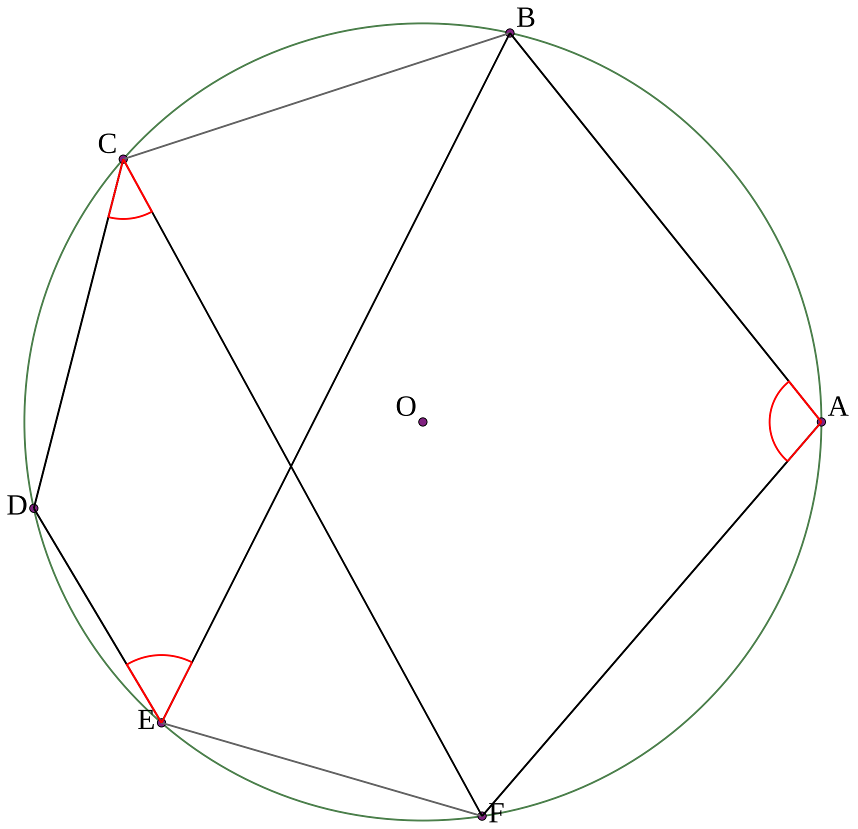
Solution to example 10



Let ABCDEF be a cyclic hexagon with center O.
Prove that BAE+CDE = BFC+180

Let BAE=x. Let CDE=y. Let BFC=z.
As CDEF is a cyclic quadrilateral, CFE=180-CDE, so CFE=180-y.
As BAE and BFE stand on the same chord, BFE=BAE, so BFE=x.
As BFC=z, CFE=x-z.
But CFE=180-y, so x-z=180-y, or x+y=z+180, or BAE+CDE=BFC+180.

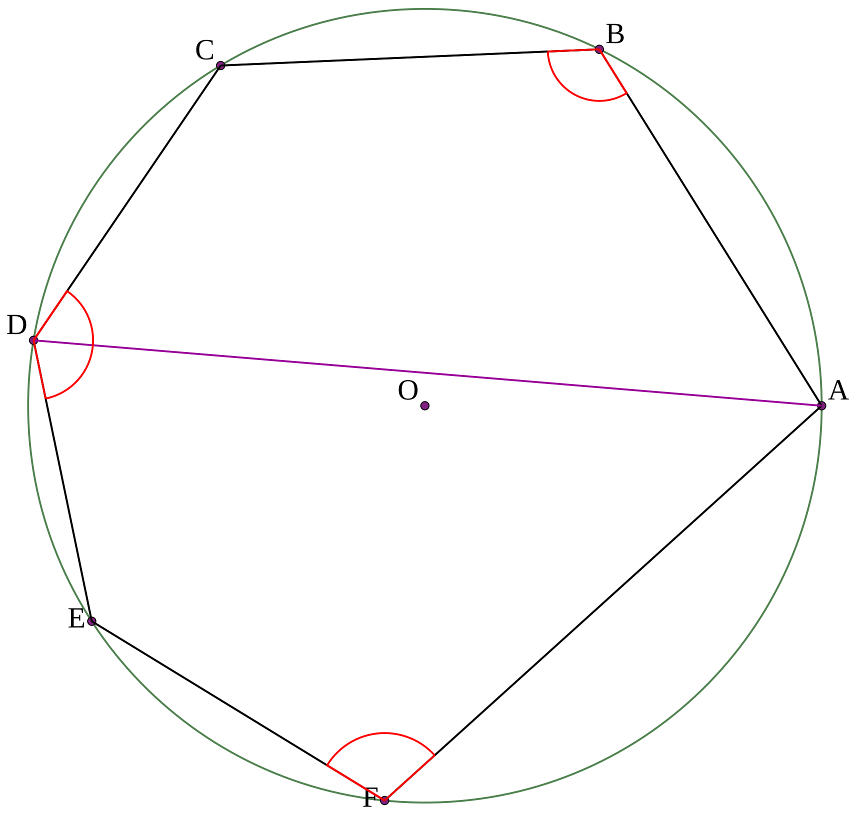
Solution to example 11



Let ABCDEF be a cyclic hexagon with center O.
Prove that DCF+BED = BAF

Let BAF=x. Let DCF=y. Let BED=z.
As BEDC is a cyclic quadrilateral, BCD=180-BED, so BCD=180-z.
As BAFC is a cyclic quadrilateral, BCF=180-BAF, so BCF=180-x.
As DCF=y, DCB=y-x+180.
But BCD=180-z, so y-x+180=180-z, or y+z=x, or DCF+BED=BAF.

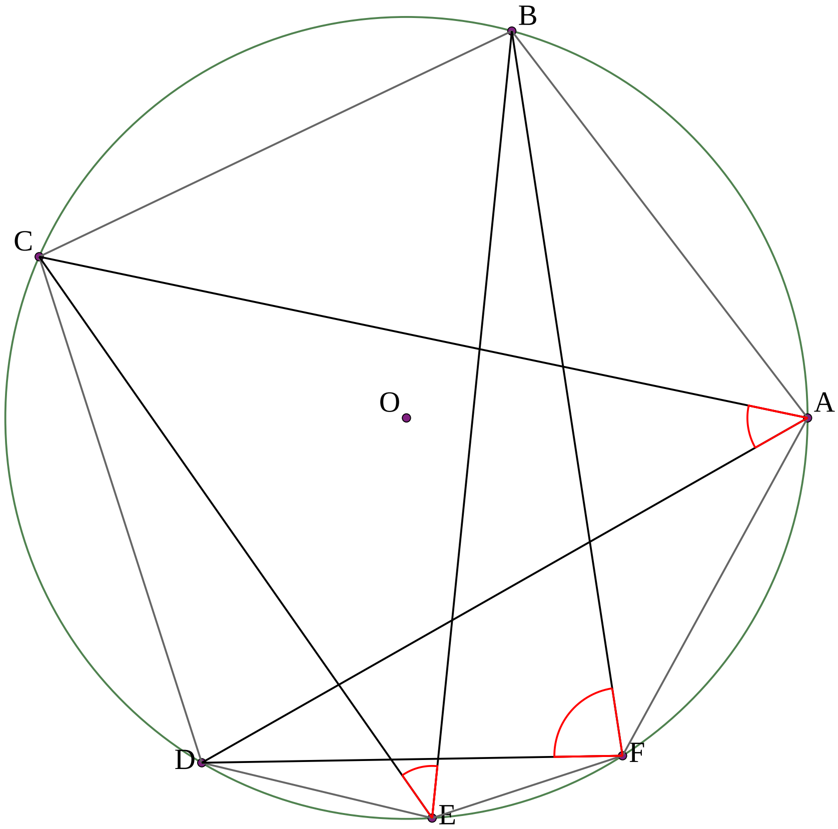
Solution to example 12



Let ABCDEF be a cyclic hexagon with center O.
Prove that ABC+CDE+AFE = 360

Draw line AD.
Let ABC=x. Let CDE=y. Let AFE=z.
As AFED is a cyclic quadrilateral, ADE=180-AFE, so ADE=180-z.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-x.
As CDE=y, EDA=x+y-180.
But ADE=180-z, so x+y-180=180-z, or x+y+z=360, or ABC+CDE+AFE=360.

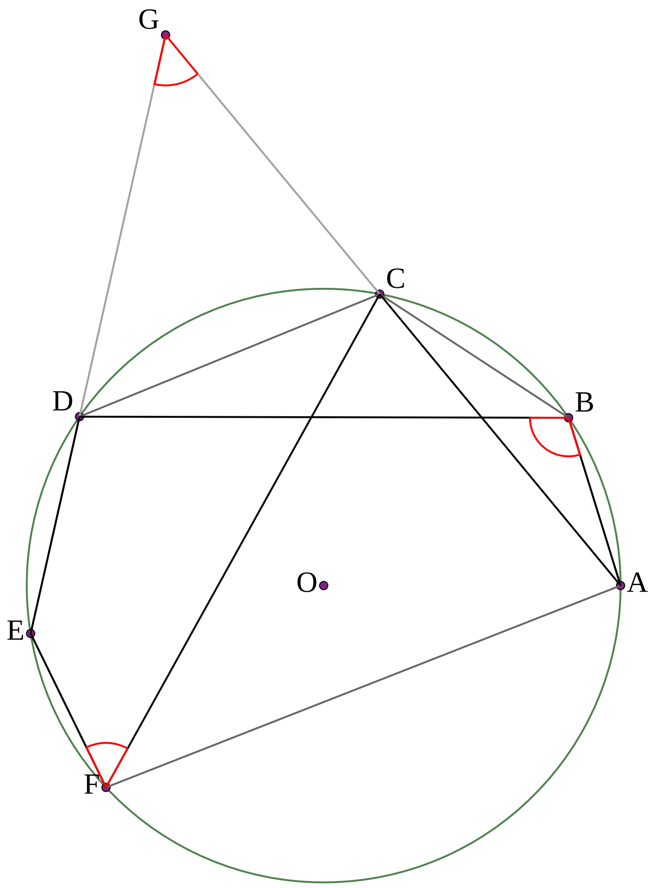
Solution to example 13



Let ABCDEF be a cyclic hexagon with center O.
Prove that BFD = BEC+CAD

Let BEC=x. Let CAD=y. Let BFD=z.
As BFD and BED stand on the same chord, BED=BFD, so BED=z.
As CAD and CED stand on the same chord, CED=CAD, so CED=y.
As BEC=x, BED=x+y.
But BED=z, so x+y=z, or BEC+CAD=BFD.

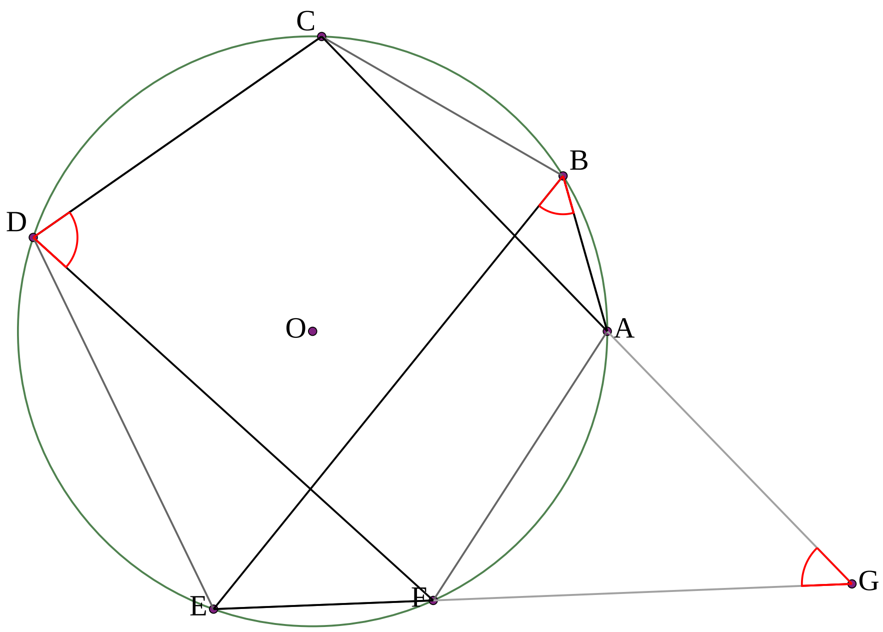
Solution to example 14



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DE and CA.
Prove that ABD = CFE+CGD

Let ABD=x. Let CFE=y. Let CGD=z.
As ABD and ACD stand on the same chord, ACD=ABD, so ACD=x.
As ACD=x, DCG=180-x.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=180-y.
As CDE=180-y, CDG=y.
As DCG=180-x, CGD=x-y.
But CGD=z, so x-y=z, or x=y+z, or ABD=CFE+CGD.

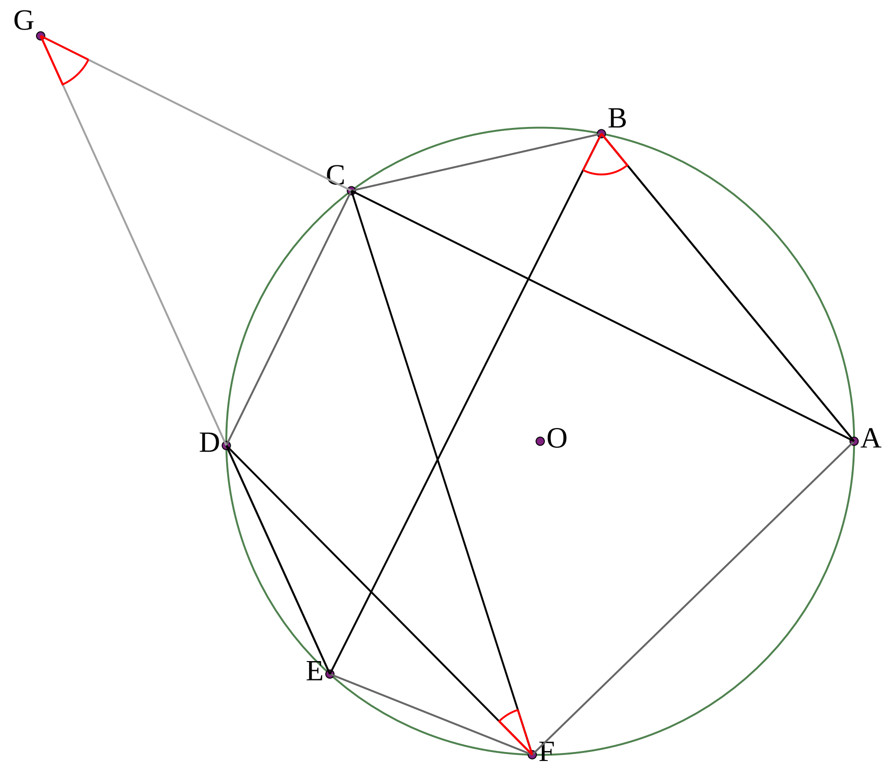
Solution to example 15



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and CA.
Prove that ABE+CDF+AGF = 180

Let ABE=x. Let CDF=y. Let AGF=z.
As ABEF is a cyclic quadrilateral, AFE=180-ABE, so AFE=180-x.
As AFE=180-x, AFG=x.
As CDFA is a cyclic quadrilateral, CAF=180-CDF, so CAF=180-y.
As CAF=180-y, FAG=y.
As AFG=x, AGF=180-x-y.
But AGF=z, so 180-x-y=z, or x+y+z=180, or ABE+CDF+AGF=180.

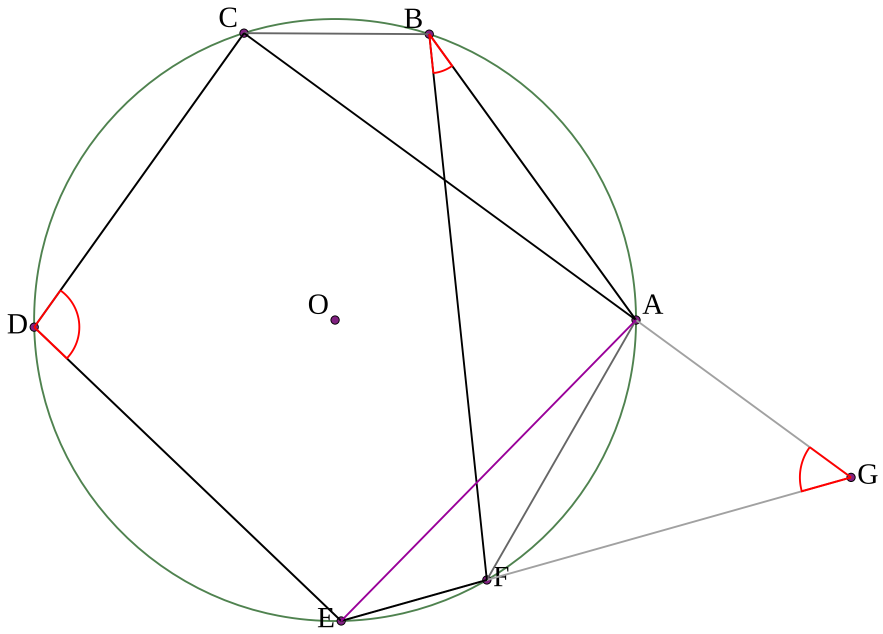
Solution to example 16



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and CA.
Prove that ABE = CFD+CGD

Let ABE=x. Let CFD=y. Let CGD=z.
Let CDG=w.
As CGD=z, DCG=180-z-w.
As DCG=180-z-w, DCA=z+w.
As ACDF is a cyclic quadrilateral, AFD=180-ACD, so AFD=180-z-w.
As ABEF is a cyclic quadrilateral, AFE=180-ABE, so AFE=180-x.
As CDG=w, CDE=180-w.
As CDEF is a cyclic quadrilateral, CFE=180-CDE, so CFE=w.
As AFE=180-x, AFC=180-x-w.
As AFC=180-x-w, AFD=y-x-w+180.
But AFD=180-z-w, so y-x-w+180=180-z-w, or y+z=x, or CFD+CGD=ABE.

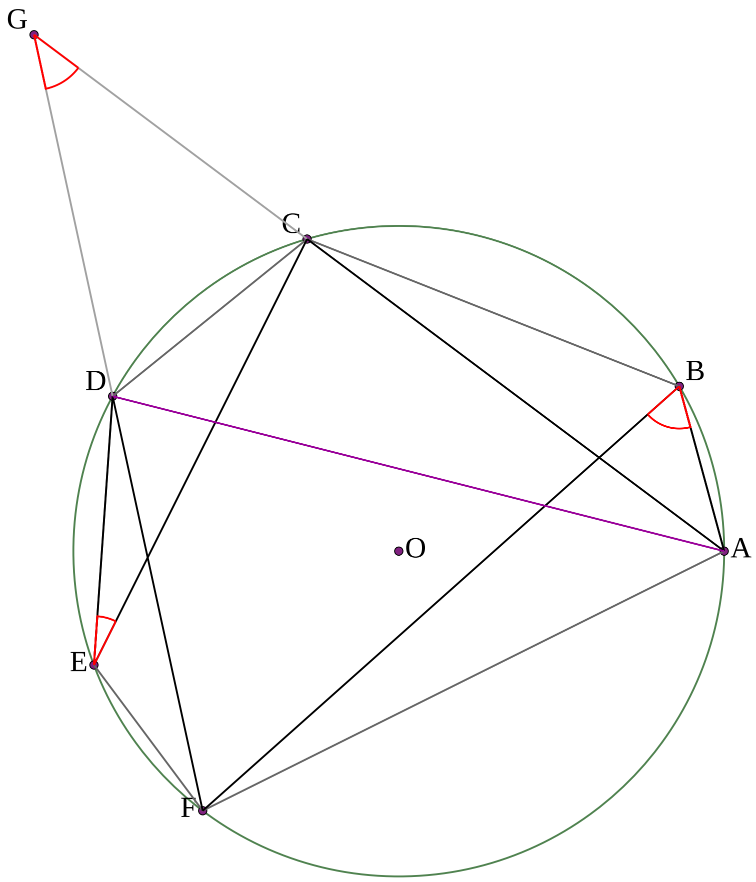
Solution to example 17



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FE and CA.
Prove that ABF+CDE+AGF = 180

Draw line AE.
Let ABF=x. Let CDE=y. Let AGF=z.
As ABF and AEF stand on the same chord, AEF=ABF, so AEF=x.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=180-y.
As CAE=180-y, EAG=y.
As AEG=x, AGE=180-x-y.
But AGE=z, so 180-x-y=z, or x+y+z=180, or ABF+CDE+AGF=180.

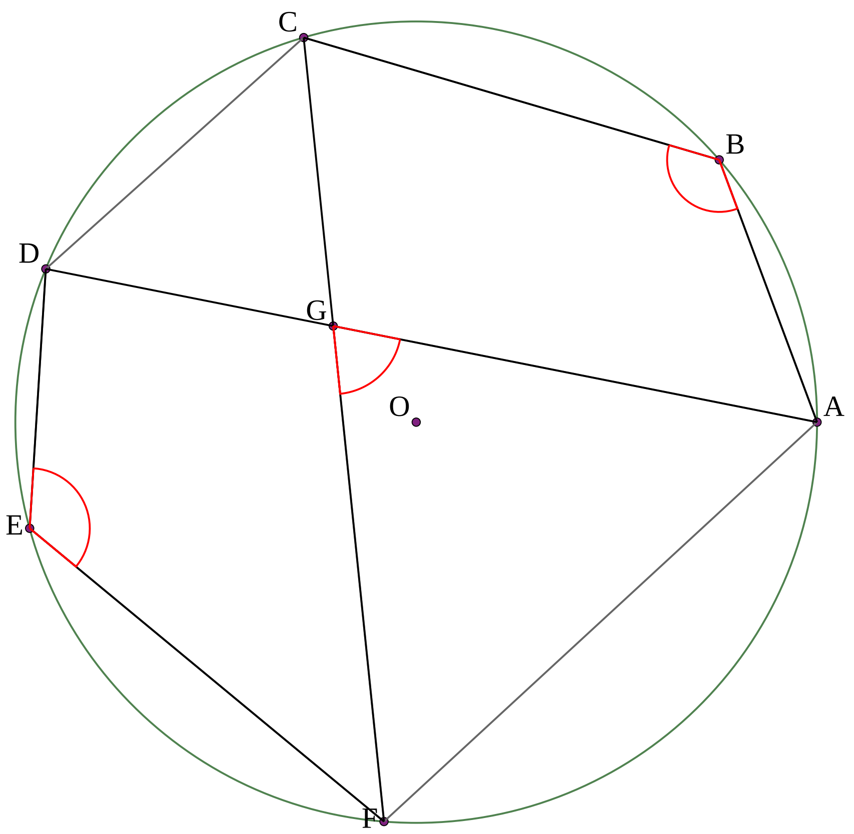
Solution to example 18



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and CA.
Prove that ABF = CED+CGD

Draw line AD.
Let ABF=x. Let CED=y. Let CGD=z.
As ABF and ADF stand on the same chord, ADF=ABF, so ADF=x.
As ADF=x, ADG=180-x.
As CED and CAD stand on the same chord, CAD=CED, so CAD=y.
As ADG=180-x, AGD=x-y.
But AGD=z, so x-y=z, or x=y+z, or ABF=CED+CGD.

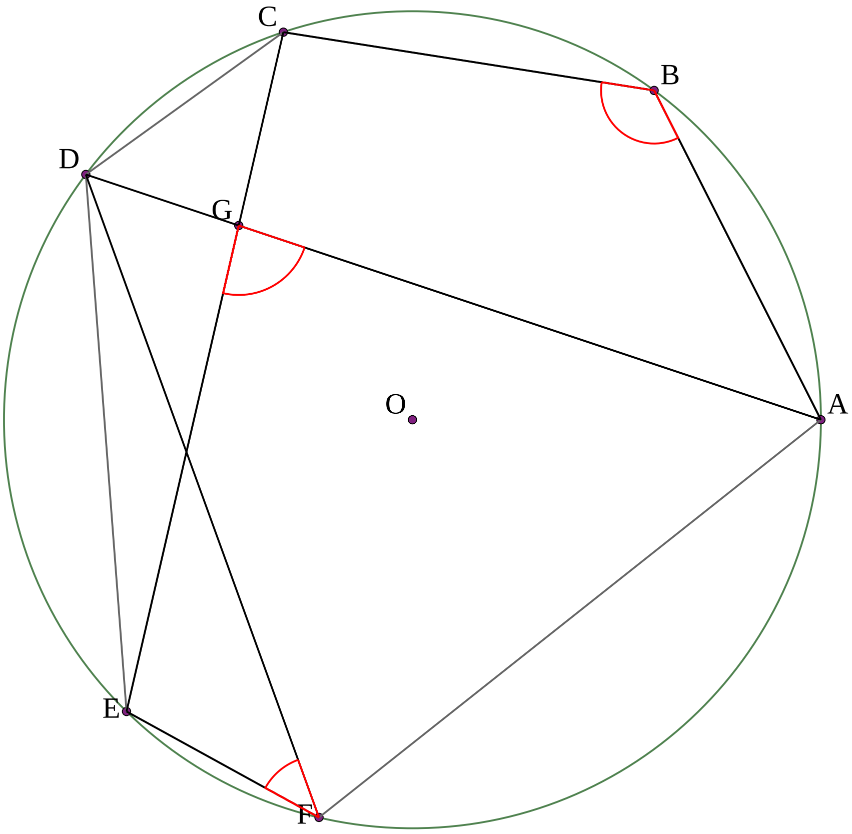
Solution to example 19



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CF and DA.
Prove that ABC+DEF = AGF+180

Let ABC=x. Let DEF=y. Let AGF=z.
Let AFG=w.
As AGF=z, FAG=180-z-w.
As DEFA is a cyclic quadrilateral, DAF=180-DEF, so DAF=180-y.
But FAG=180-z-w, so 180-y=180-z-w, or z+w=y.
As ABCF is a cyclic quadrilateral, AFC=180-ABC, so AFC=180-x.
But AFC=180-x, so w=180-x, or x+w=180.
We have these equations: y-z-w=0 (E1), x+w=180 (E2).
Hence x+y-z=180 (E2-E1), or z+180=x+y, or AGF+180=ABC+DEF.

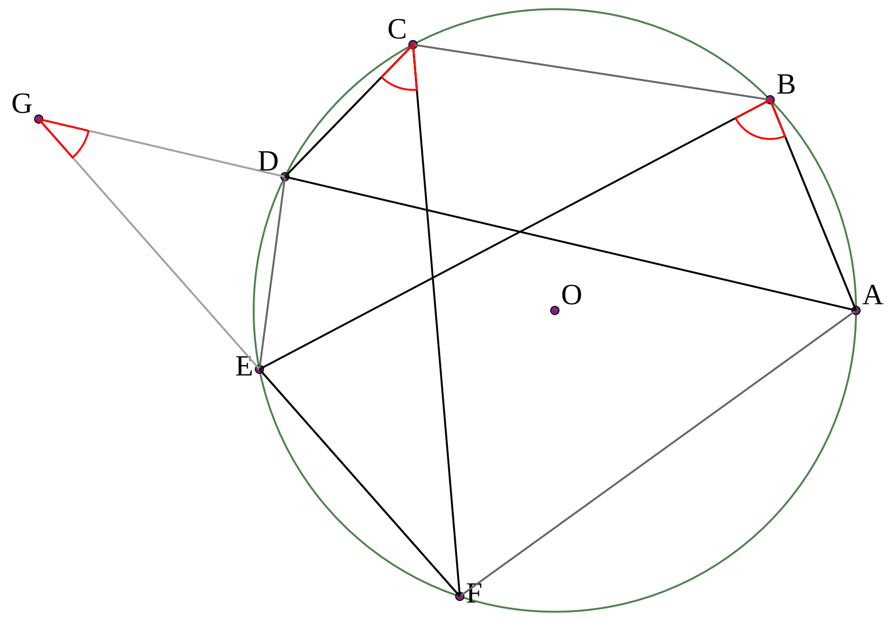
Solution to example 20



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CE and DA.
Prove that ABC = DFE+AGE

Let ABC=x. Let DFE=y. Let AGE=z.
As AGE=z, AGC=180-z.
As AGC=180-z, CGD=z.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-x.
As DFE and DCE stand on the same chord, DCE=DFE, so DCE=y.
As CDG=180-x, CGD=x-y.
But CGD=z, so x-y=z, or x=y+z, or ABC=DFE+AGE.

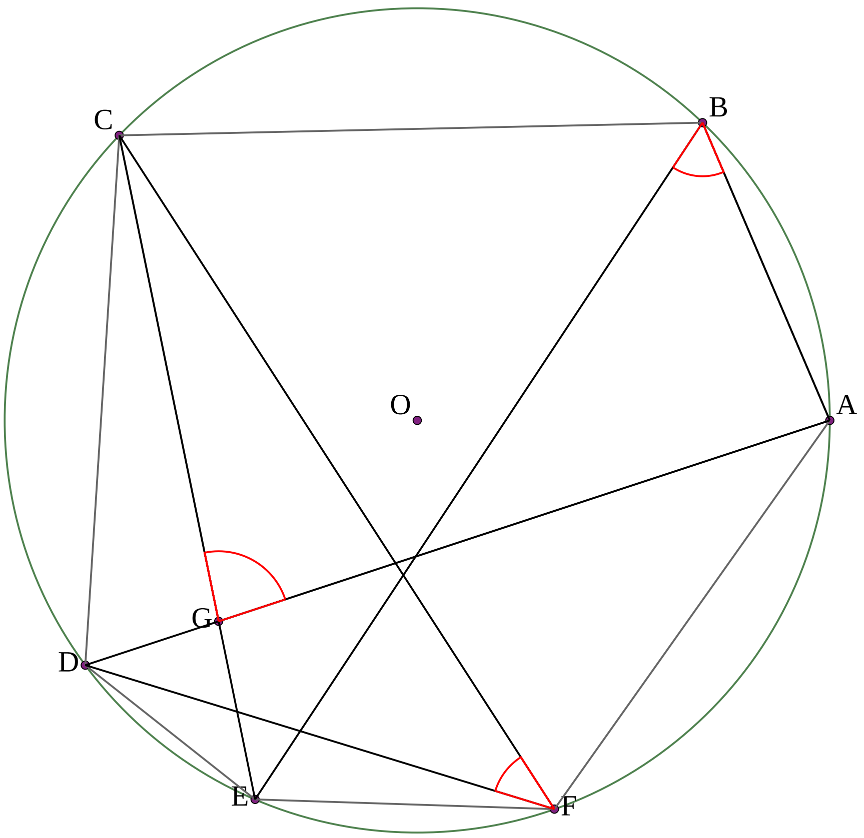
Solution to example 21



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and DA.
Prove that ABE = DCF+DGE

Let ABE=x. Let DCF=y. Let DGE=z.
As ABE and ADE stand on the same chord, ADE=ABE, so ADE=x.
As ADE=x, EDG=180-x.
As DCFE is a cyclic quadrilateral, DEF=180-DCF, so DEF=180-y.
As DEF=180-y, DEG=y.
As EDG=180-x, DGE=x-y.
But DGE=z, so x-y=z, or x=y+z, or ABE=DCF+DGE.

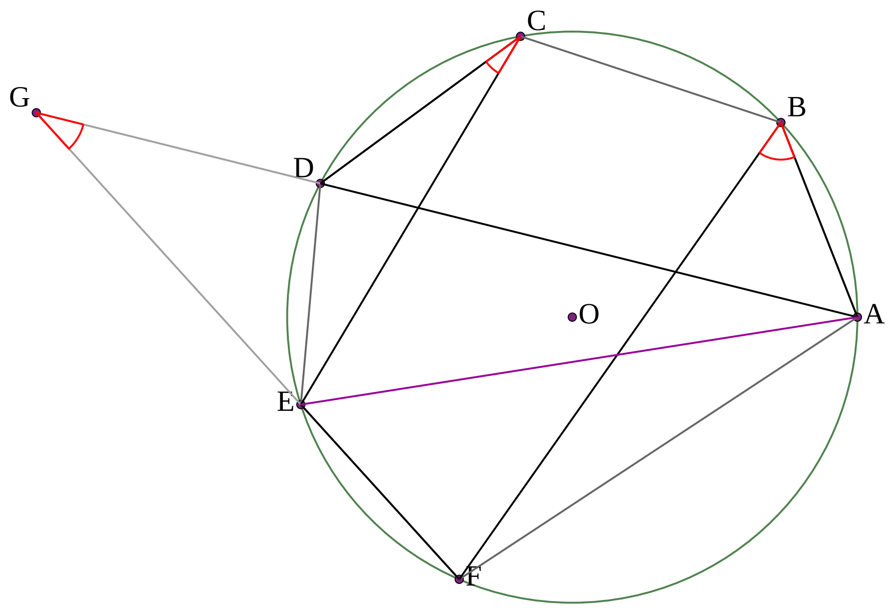
Solution to example 22



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and DA.
Prove that ABE+CFD+AGC = 180

Let ABE=x. Let CFD=y. Let AGC=z.
As AGC=z, AGE=180-z.
As AGE=180-z, EGD=z.
As ABE and ADE stand on the same chord, ADE=ABE, so ADE=x.
As CFD and CED stand on the same chord, CED=CFD, so CED=y.
As EDG=x, DGE=180-x-y.
But DGE=z, so 180-x-y=z, or x+y+z=180, or ABE+CFD+AGC=180.

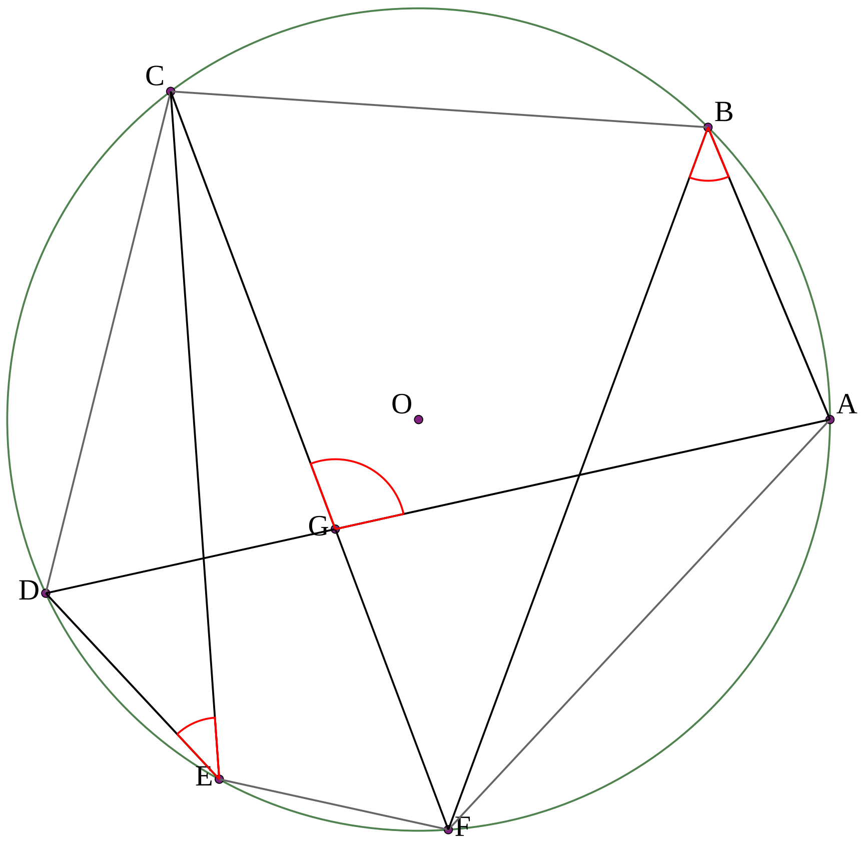
Solution to example 23



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FE and DA.
Prove that ABF = DCE+DGE

Draw line AE.
Let ABF=x. Let DCE=y. Let DGE=z.
As ABF and AEF stand on the same chord, AEF=ABF, so AEF=x.
As AEF=x, AEG=180-x.
As DCE and DAE stand on the same chord, DAE=DCE, so DAE=y.
As AEG=180-x, AGE=x-y.
But AGE=z, so x-y=z, or x=y+z, or ABF=DCE+DGE.

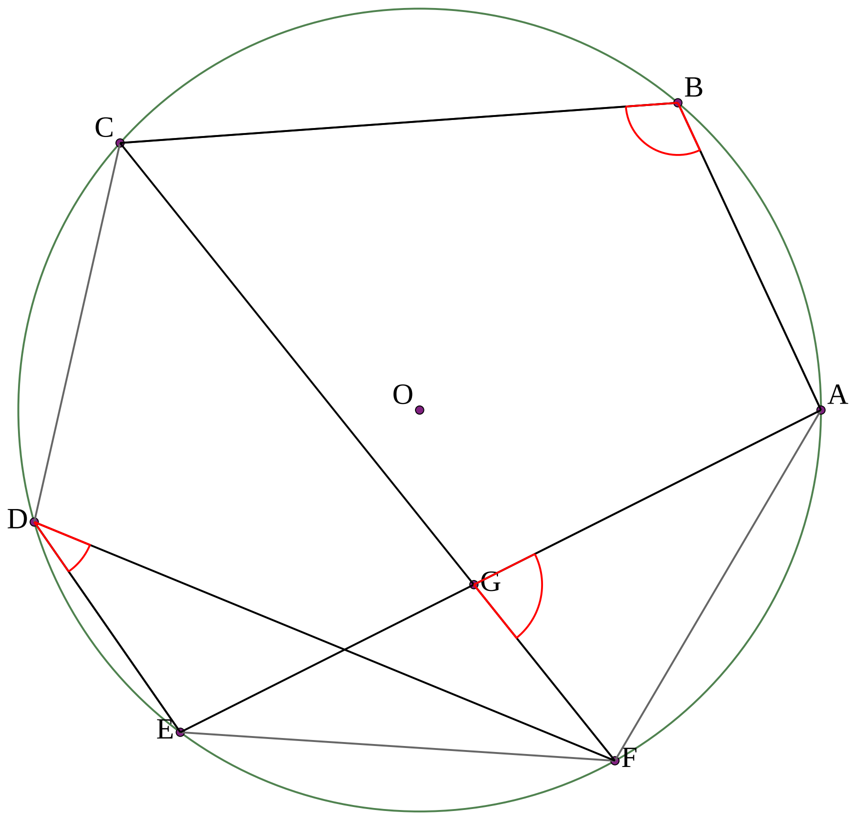
Solution to example 24



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FC and DA.
Prove that ABF+CED+AGC = 180

Let ABF=x. Let CED=y. Let AGC=z.
As AGC=z, CGD=180-z.
Let DCG=w.
As CGD=180-z, CDG=z-w.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=w-z+180.
As DCFE is a cyclic quadrilateral, DEF=180-DCF, so DEF=180-w.
As DEF=180-w, FEC=180-y-w.
As CEFB is a cyclic quadrilateral, CBF=180-CEF, so CBF=y+w.
As CBF=y+w, CBA=x+y+w.
But ABC=w-z+180, so x+y+w=w-z+180, or x+y+z=180, or ABF+CED+AGC=180.

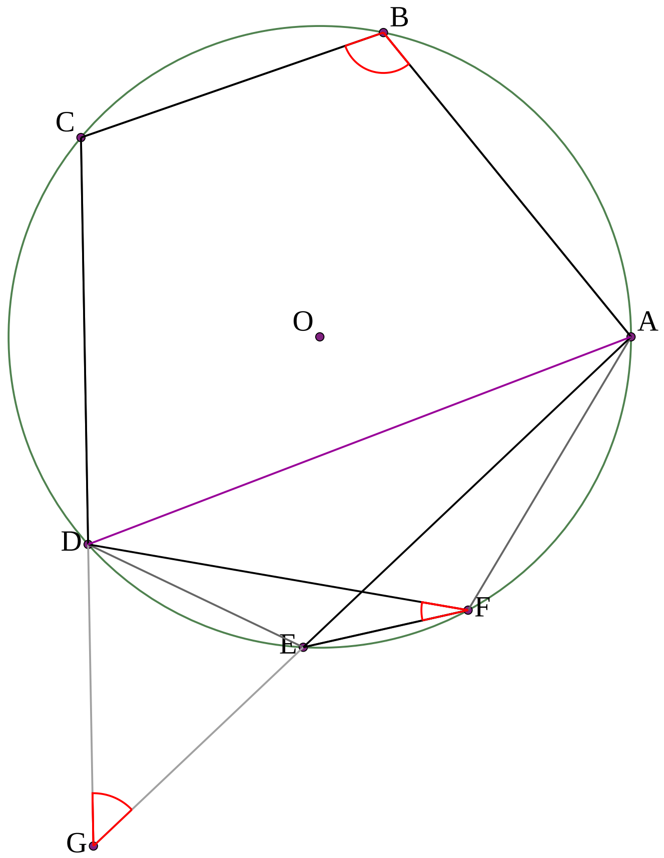
Solution to example 25



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CF and EA.
Prove that ABC = EDF+AGF

Let ABC=x. Let EDF=y. Let AGF=z.
As ABCF is a cyclic quadrilateral, AFC=180-ABC, so AFC=180-x.
As EDF and EAF stand on the same chord, EAF=EDF, so EAF=y.
As AFG=180-x, AGF=x-y.
But AGF=z, so x-y=z, or x=y+z, or ABC=EDF+AGF.

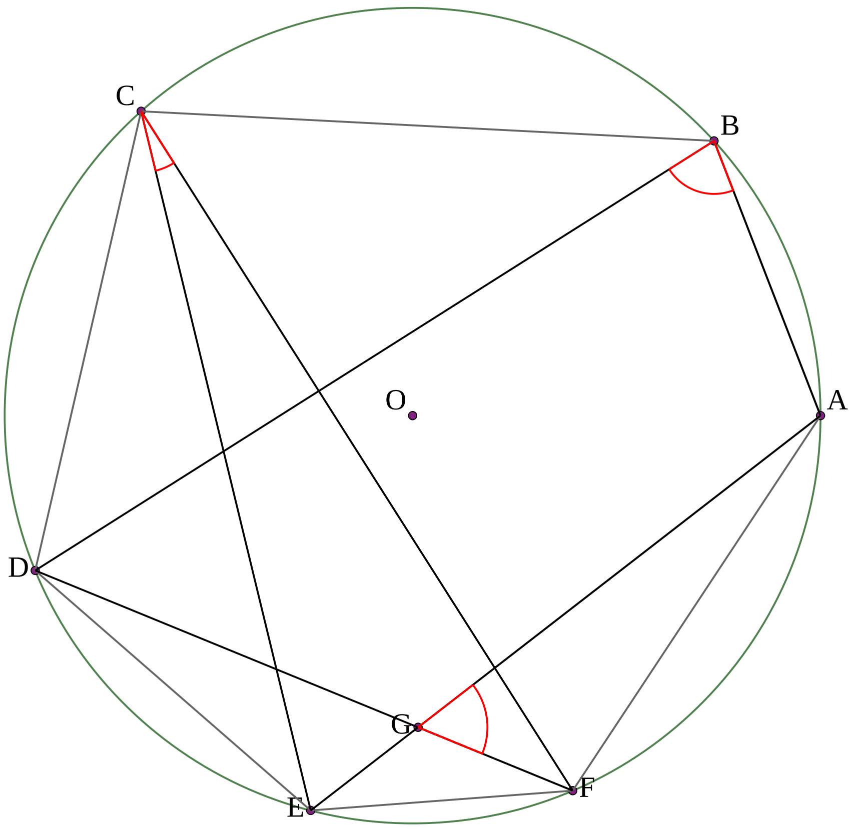
Solution to example 26



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CD and EA.
Prove that ABC+DFE+DGE = 180

Draw line AD.
Let ABC=x. Let DFE=y. Let DGE=z.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-x.
As ADC=180-x, ADG=x.
As DFE and DAE stand on the same chord, DAE=DFE, so DAE=y.
As ADG=x, AGD=180-x-y.
But AGD=z, so 180-x-y=z, or x+y+z=180, or ABC+DFE+DGE=180.

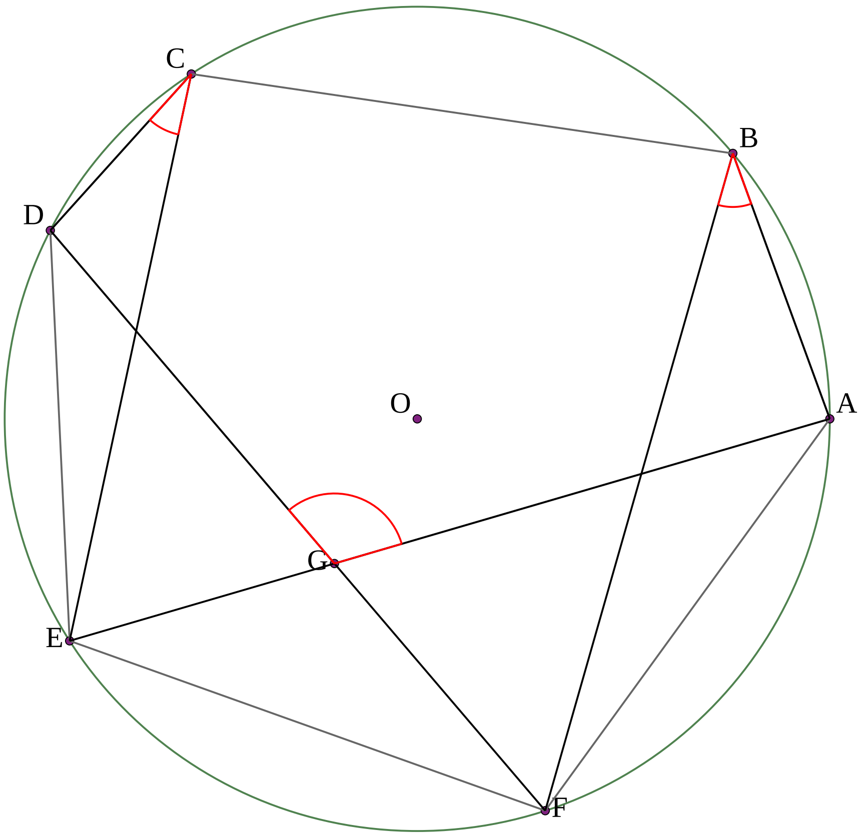
Solution to example 27



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DF and EA.
Prove that ABD = ECF+AGF

Let ABD=x. Let ECF=y. Let AGF=z.
Let AFG=w.
As AGF=z, FAG=180-z-w.
As ECF and EAF stand on the same chord, EAF=ECF, so EAF=y.
But FAG=180-z-w, so y=180-z-w, or y+z+w=180.
As ABDF is a cyclic quadrilateral, AFD=180-ABD, so AFD=180-x.
But AFD=180-x, so w=180-x, or x+w=180.
We have these equations: y+z+w=180 (E1), x+w=180 (E2).
Hence y+z-x=0 (E2-E1), or y+z=x, or ECF+AGF=ABD.

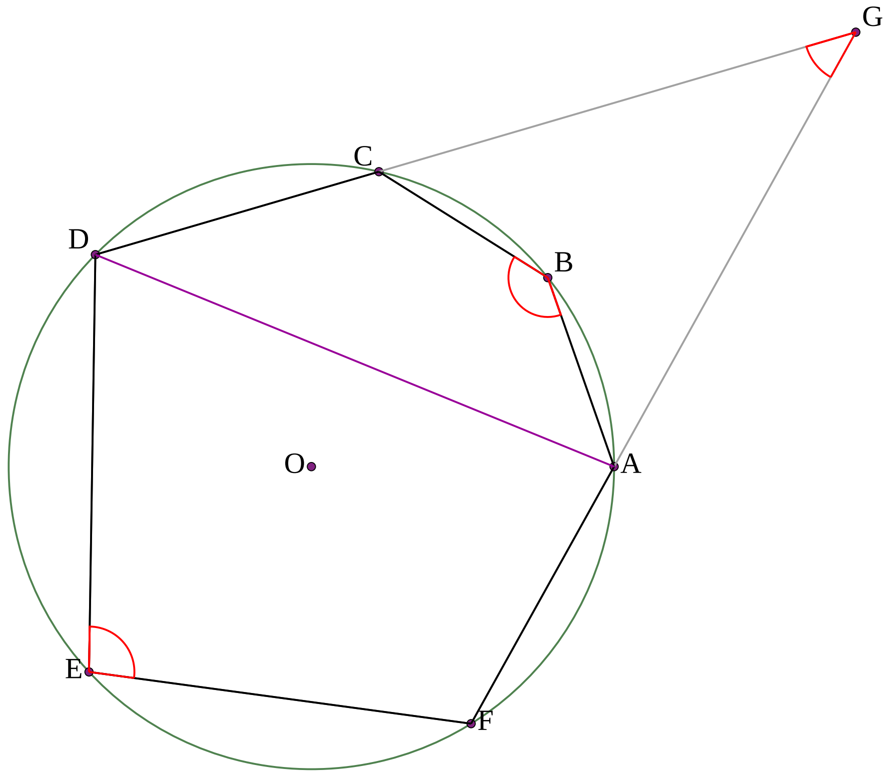
Solution to example 28



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and EA.
Prove that ABF+DCE+AGD = 180

Let ABF=x. Let DCE=y. Let AGD=z.
As AGD=z, AGF=180-z.
As AGF=180-z, FGE=z.
As ABF and AEF stand on the same chord, AEF=ABF, so AEF=x.
As DCE and DFE stand on the same chord, DFE=DCE, so DFE=y.
As FEG=x, EGF=180-x-y.
But EGF=z, so 180-x-y=z, or x+y+z=180, or ABF+DCE+AGD=180.

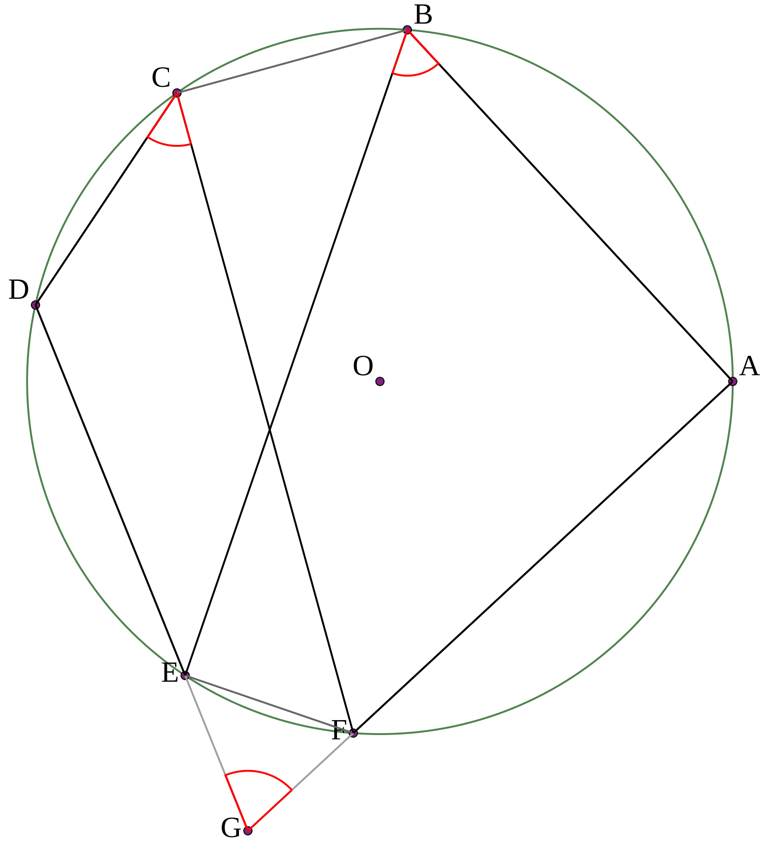
Solution to example 29



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CD and FA.
Prove that ABC = DEF+AGC

Draw line AD.
Let ABC=x. Let DEF=y. Let AGC=z.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-x.
As DEFA is a cyclic quadrilateral, DAF=180-DEF, so DAF=180-y.
As DAF=180-y, DAG=y.
As ADG=180-x, AGD=x-y.
But AGD=z, so x-y=z, or x=y+z, or ABC=DEF+AGC.

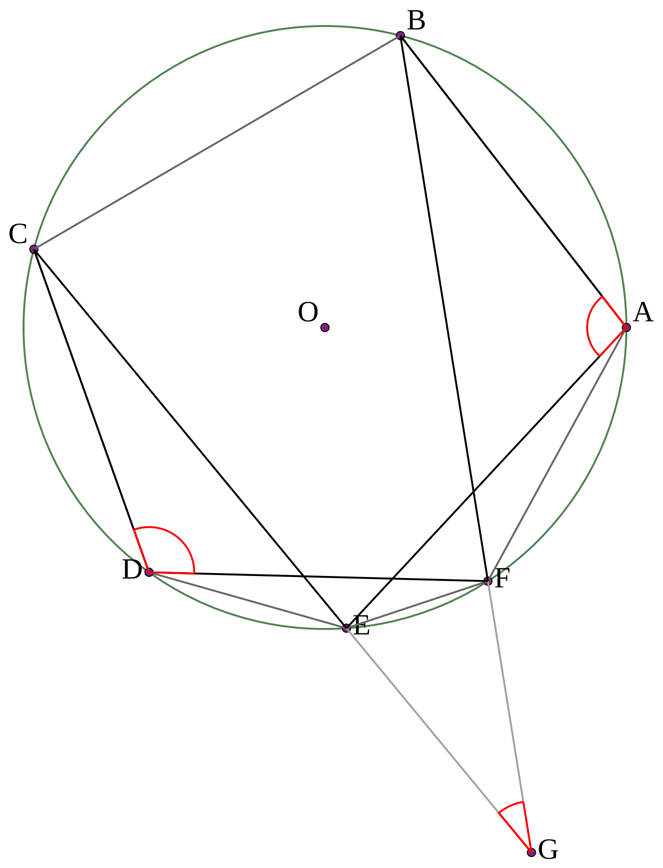
Solution to example 30



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and FA.
Prove that ABE+DCF+EGF = 180

Let ABE=x. Let DCF=y. Let EGF=z.
As ABEF is a cyclic quadrilateral, AFE=180-ABE, so AFE=180-x.
As AFE=180-x, EFG=x.
As DCFE is a cyclic quadrilateral, DEF=180-DCF, so DEF=180-y.
As DEF=180-y, FEG=y.
As EFG=x, EGF=180-x-y.
But EGF=z, so 180-x-y=z, or x+y+z=180, or ABE+DCF+EGF=180.

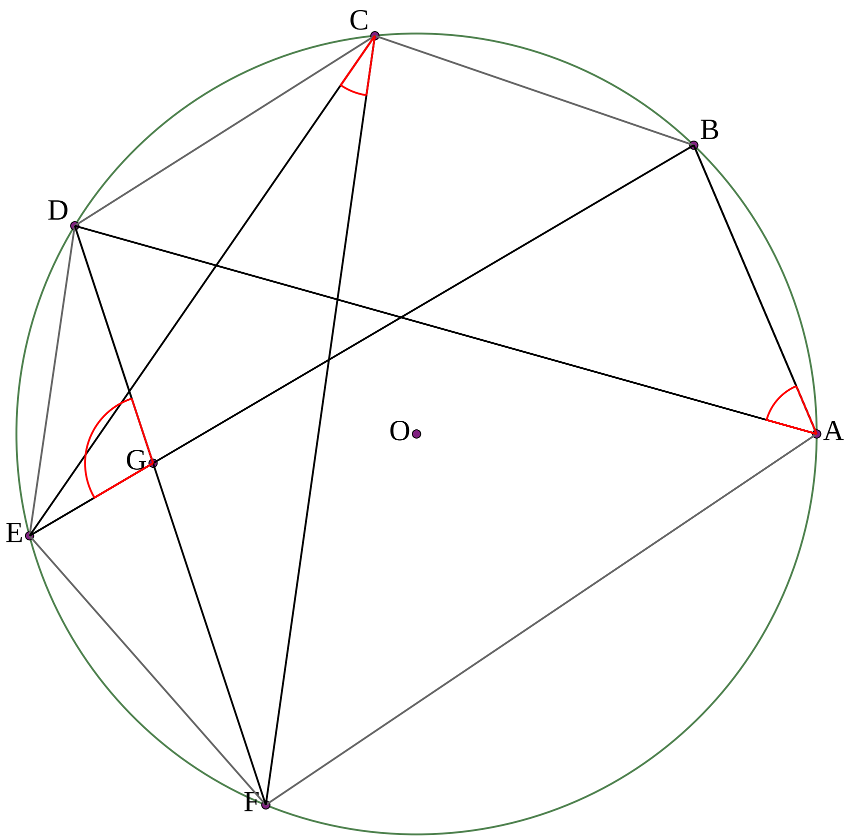
Solution to example 31



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and FB.
Prove that BAE+CDF = EGF+180

Let BAE=x. Let CDF=y. Let EGF=z.
As BAE and BFE stand on the same chord, BFE=BAE, so BFE=x.
As BFE=x, EFG=180-x.
As CDF and CEF stand on the same chord, CEF=CDF, so CEF=y.
As CEF=y, FEG=180-y.
As EFG=180-x, EGF=x+y-180.
But EGF=z, so x+y-180=z, or x+y=z+180, or BAE+CDF=EGF+180.

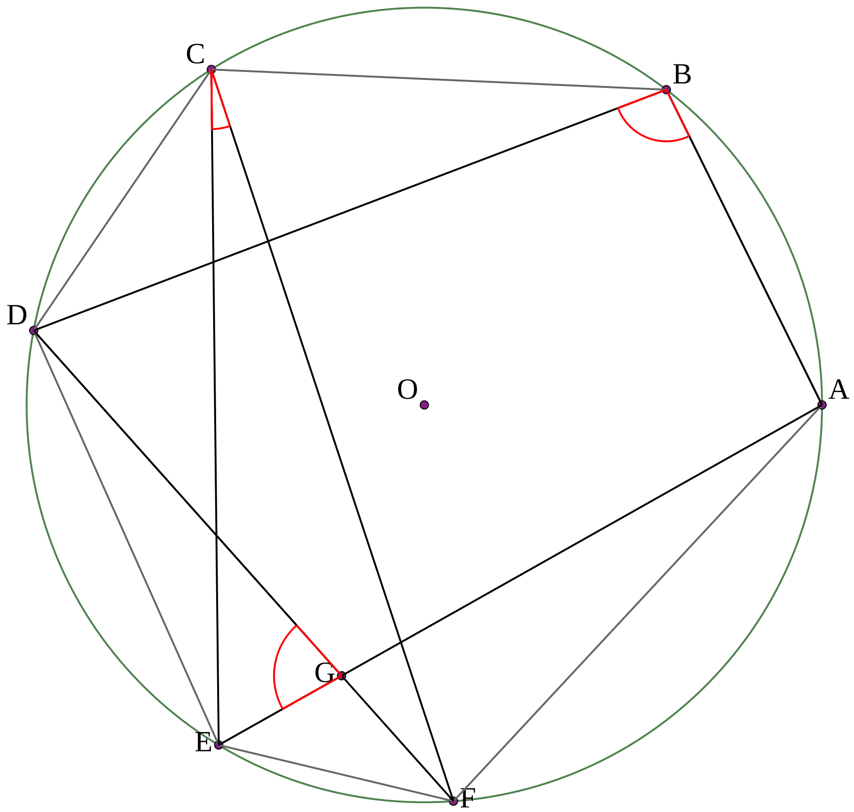
Solution to example 32



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BE and FD.
Prove that BAD+ECF+DGE = 180

Let BAD=x. Let ECF=y. Let DGE=z.
As BAD and BED stand on the same chord, BED=BAD, so BED=x.
As ECF and EDF stand on the same chord, EDF=ECF, so EDF=y.
As DEG=x, DGE=180-x-y.
But DGE=z, so 180-x-y=z, or x+y+z=180, or BAD+ECF+DGE=180.

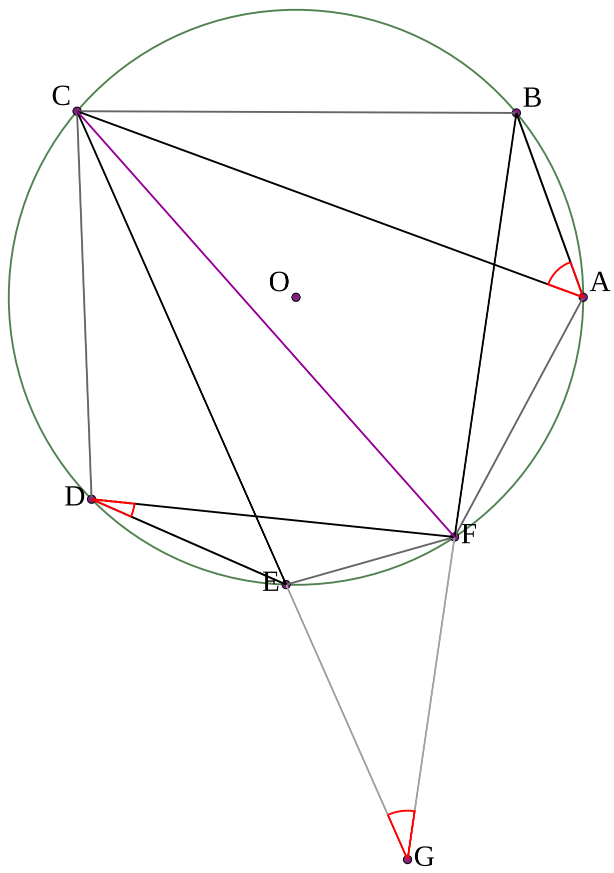
Solution to example 33



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AE and FD.
Prove that ECF+DGE = ABD

Let ABD=x. Let ECF=y. Let DGE=z.
As ABDE is a cyclic quadrilateral, AED=180-ABD, so AED=180-x.
As ECF and EDF stand on the same chord, EDF=ECF, so EDF=y.
As DEG=180-x, DGE=x-y.
But DGE=z, so x-y=z, or x=y+z, or ABD=ECF+DGE.

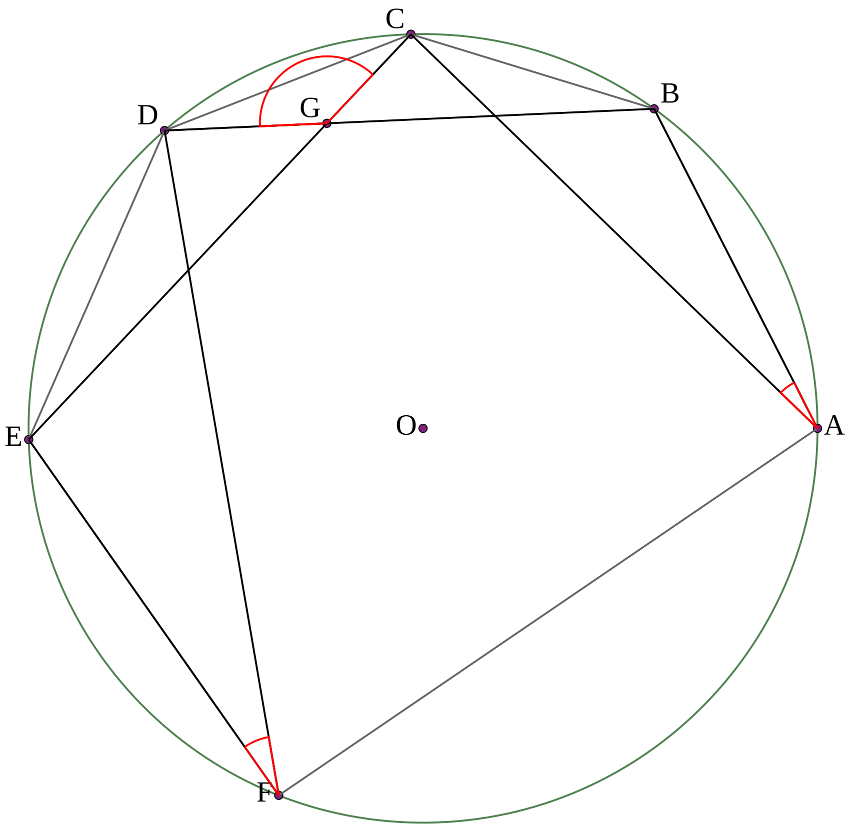
Solution to example 34



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BF and EC.
Prove that BAC = EDF+EGF

Draw line CF.
Let BAC=x. Let EDF=y. Let EGF=z.
As BAC and BFC stand on the same chord, BFC=BAC, so BFC=x.
As BFC=x, CFG=180-x.
As EDF and ECF stand on the same chord, ECF=EDF, so ECF=y.
As CFG=180-x, CGF=x-y.
But CGF=z, so x-y=z, or x=y+z, or BAC=EDF+EGF.

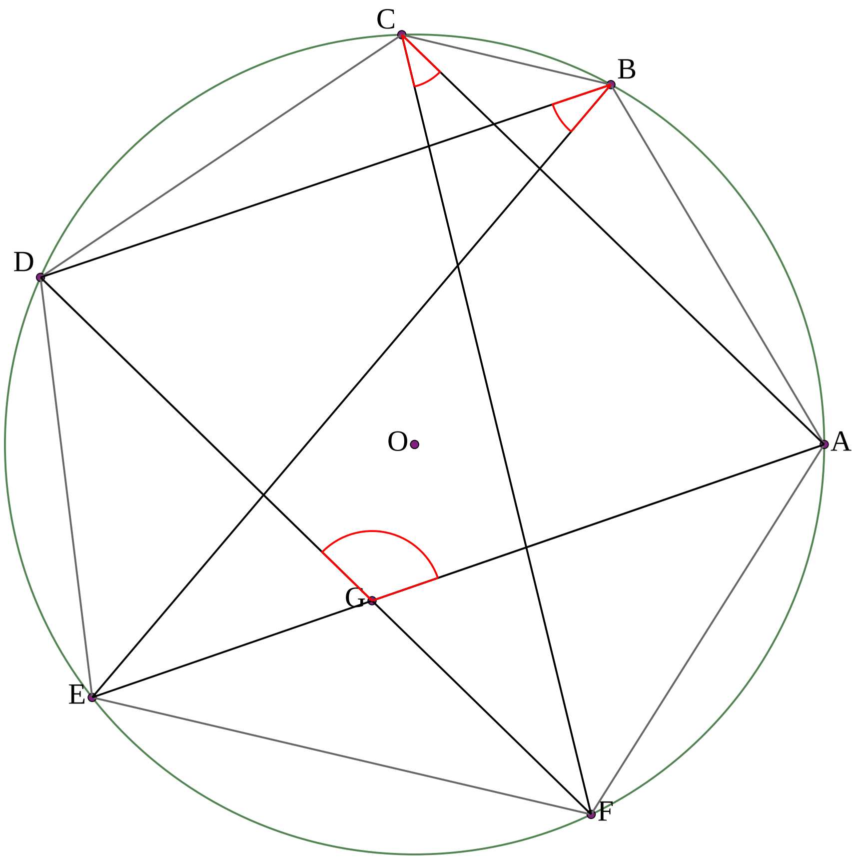
Solution to example 35



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BD and EC.
Prove that BAC+DFE+CGD = 180

Let BAC=x. Let DFE=y. Let CGD=z.
As BAC and BDC stand on the same chord, BDC=BAC, so BDC=x.
As DFE and DCE stand on the same chord, DCE=DFE, so DCE=y.
As CDG=x, CGD=180-x-y.
But CGD=z, so 180-x-y=z, or x+y+z=180, or BAC+DFE+CGD=180.

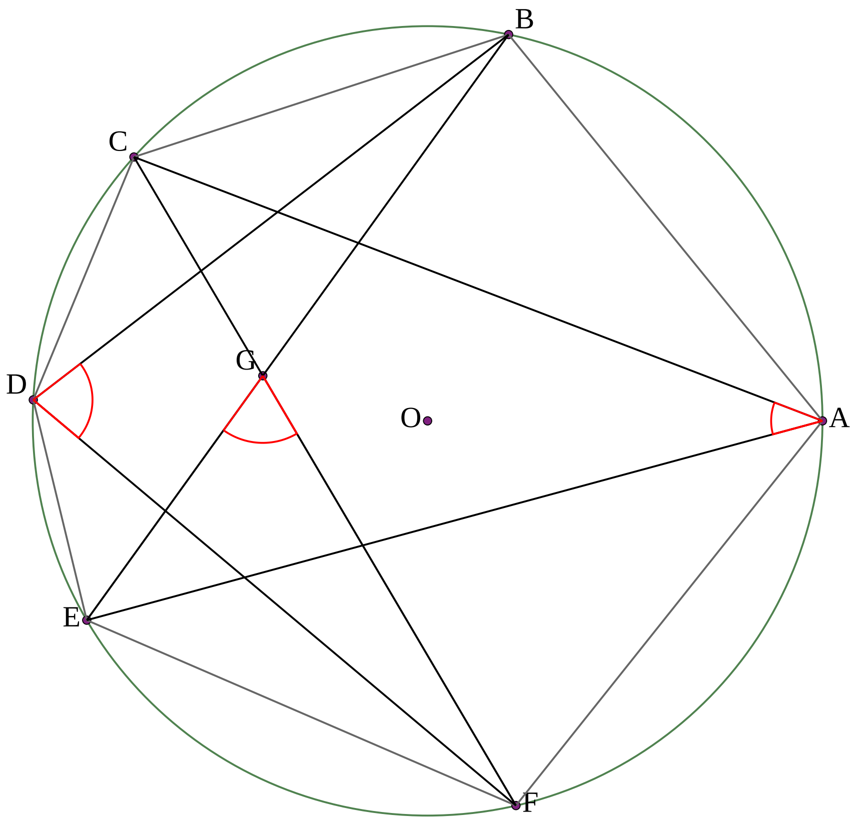
Solution to example 36



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and EA.
Prove that ACF+DBE+AGD = 180

Let ACF=x. Let DBE=y. Let AGD=z.
As AGD=z, AGF=180-z.
As AGF=180-z, FGE=z.
As ACF and AEF stand on the same chord, AEF=ACF, so AEF=x.
As DBE and DFE stand on the same chord, DFE=DBE, so DFE=y.
As FEG=x, EGF=180-x-y.
But EGF=z, so 180-x-y=z, or x+y+z=180, or ACF+DBE+AGD=180.

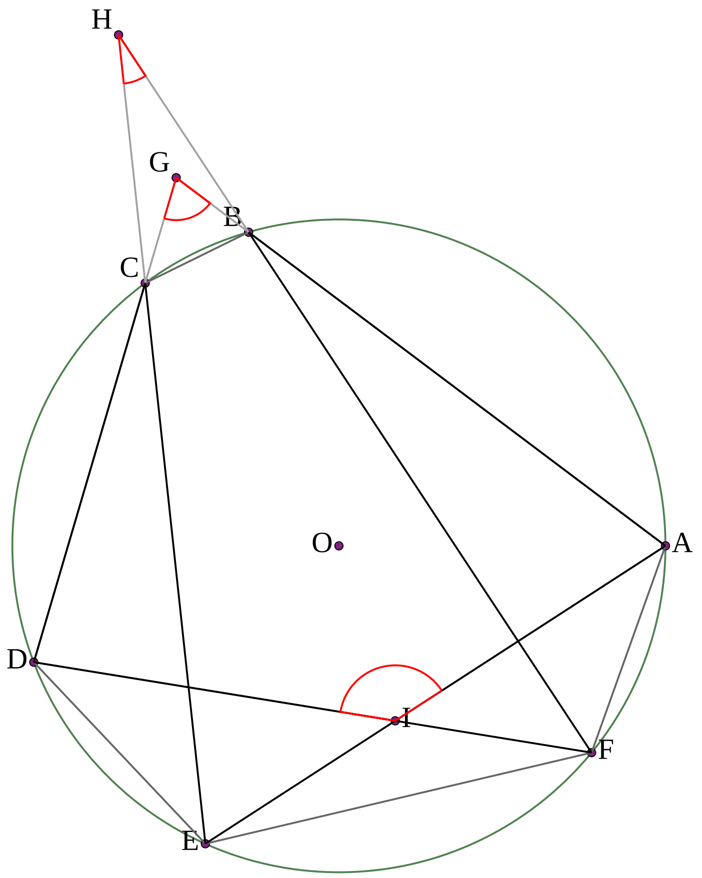
Solution to example 37



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CF and BE.
Prove that CAE+BDF+EGF = 180

Let CAE=x. Let BDF=y. Let EGF=z.
Let EFG=w.
As EGF=z, FEG=180-z-w.
As BDF and BEF stand on the same chord, BEF=BDF, so BEF=y.
But FEG=180-z-w, so y=180-z-w, or y+z+w=180.
As CAE and CFE stand on the same chord, CFE=CAE, so CFE=x.
But CFE=x, so w=x.
We have these equations: y+z+w=180 (E1), x-w=0 (E2).
Hence x+y+z=180 (E2-E1), or CAE+BDF+EGF=180.

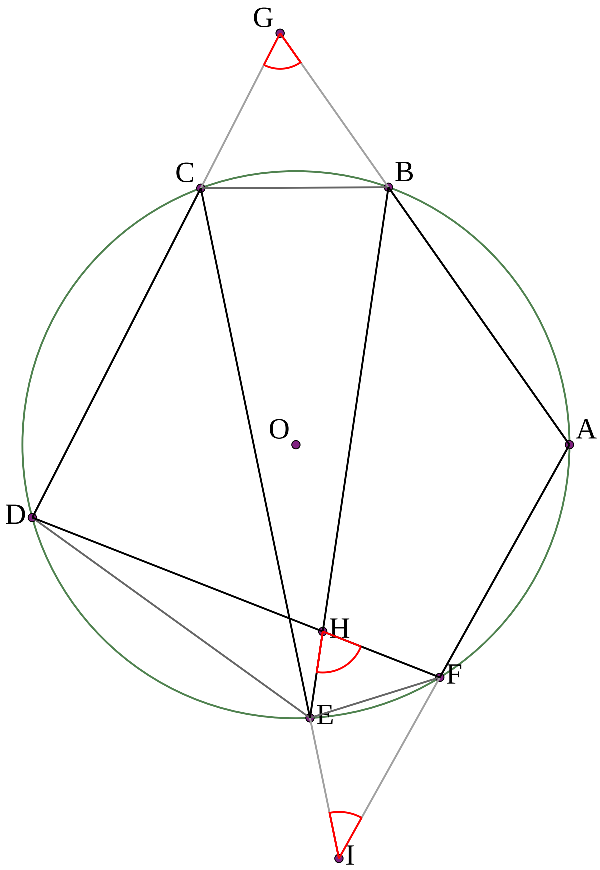
Solution to example 38



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and DC. Let H be the intersection of BF and CE. Let I be the intersection of FD and EA.
Prove that BGC+AID = BHC+180

Let BGC=x. Let BHC=y. Let AID=z.
Let EFI=u.
As DFE and DCE stand on the same chord, DCE=DFE, so DCE=u.
As DCE=u, ECG=180-u.
Let CBH=w.
As BHC=y, BCH=180-y-w.
As BCH=180-y-w, BCE=y+w.
As ECG=180-u, GCB=180-y-w-u.
As BCG=180-y-w-u, CBG=y+w+u-x.
As CBH=w, CBF=180-w.
As CBFE is a cyclic quadrilateral, CEF=180-CBF, so CEF=w.
As AID=z, AIF=180-z.
As AIF=180-z, FIE=z.
As EIF=z, FEI=180-z-u.
As CEF=w, CEI=z+w+u-180.
As AECB is a cyclic quadrilateral, ABC=180-AEC, so ABC=360-z-w-u.
As ABC=360-z-w-u, CBG=z+w+u-180.
But CBG=y+w+u-x, so z+w+u-180=y+w+u-x, or x+z=y+180, or BGC+AID=BHC+180.

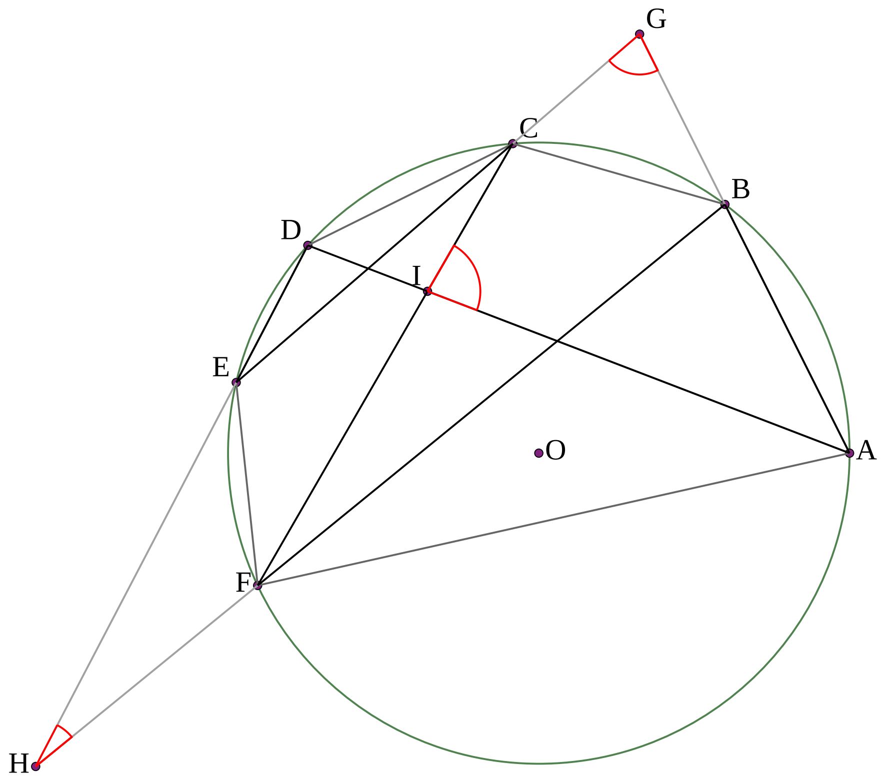
Solution to example 39



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CD. Let H be the intersection of BE and DF. Let I be the intersection of EC and FA.
Prove that BGC+EHF+EIF = 180

Let BGC=x. Let EHF=y. Let EIF=z.
Let FEI=u.
As FEI=u, FEC=180-u.
As CEF and CDF stand on the same chord, CDF=CEF, so CDF=180-u.
As EHF=y, EHD=180-y.
Let DEH=w.
As DHE=180-y, EDH=y-w.
As CDH=180-u, CDE=y-w-u+180.
As CDEB is a cyclic quadrilateral, CBE=180-CDE, so CBE=w+u-y.
As BEDC is a cyclic quadrilateral, BCD=180-BED, so BCD=180-w.
As BCD=180-w, BCG=w.
As BCG=w, CBG=180-x-w.
As EIF=z, EFI=180-z-u.
As EFI=180-z-u, EFA=z+u.
As AFEB is a cyclic quadrilateral, ABE=180-AFE, so ABE=180-z-u.
As ABE=180-z-u, EBG=z+u.
As CBG=180-x-w, CBE=x+z+w+u-180.
But CBE=w+u-y, so x+z+w+u-180=w+u-y, or x+y+z=180, or BGC+EHF+EIF=180.

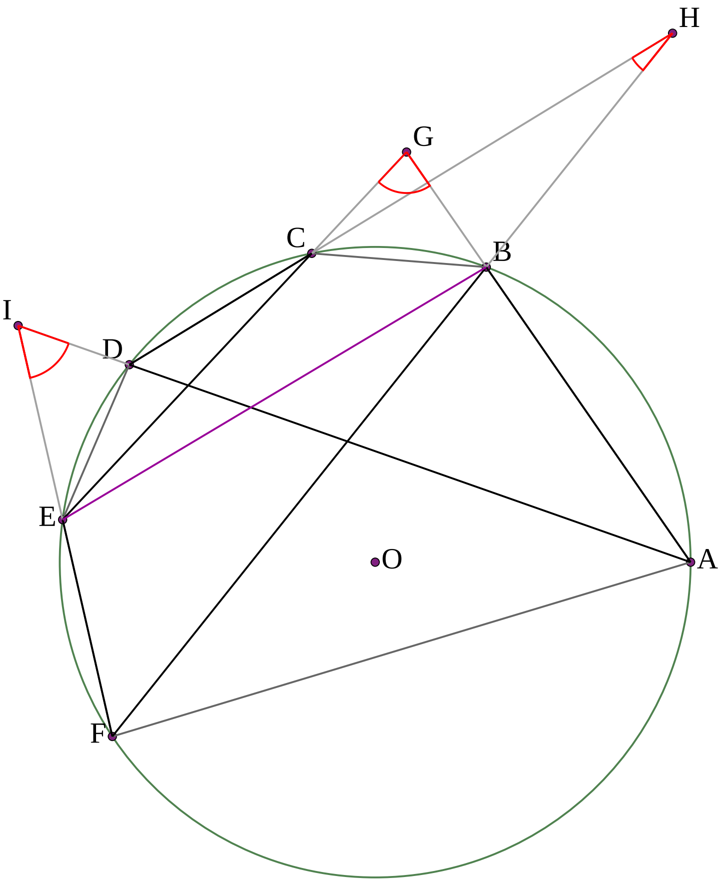
Solution to example 40



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CE. Let H be the intersection of BF and ED. Let I be the intersection of FC and DA.
Prove that BGC+EHF+AIC = 180

Let BGC=x. Let EHF=y. Let AIC=z.
As AIC=z, CID=180-z.
Let DCI=w.
As CID=180-z, CDI=z-w.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=w-z+180.
As ABC=w-z+180, CBG=z-w.
As CBG=z-w, BCG=w-x-z+180.
As DCFE is a cyclic quadrilateral, DEF=180-DCF, so DEF=180-w.
As DEF=180-w, FEH=w.
As FEH=w, EFH=180-y-w.
As EFH=180-y-w, EFB=y+w.
As BFEC is a cyclic quadrilateral, BCE=180-BFE, so BCE=180-y-w.
As BCE=180-y-w, BCG=y+w.
But BCG=w-x-z+180, so y+w=w-x-z+180, or x+y+z=180, or BGC+EHF+AIC=180.

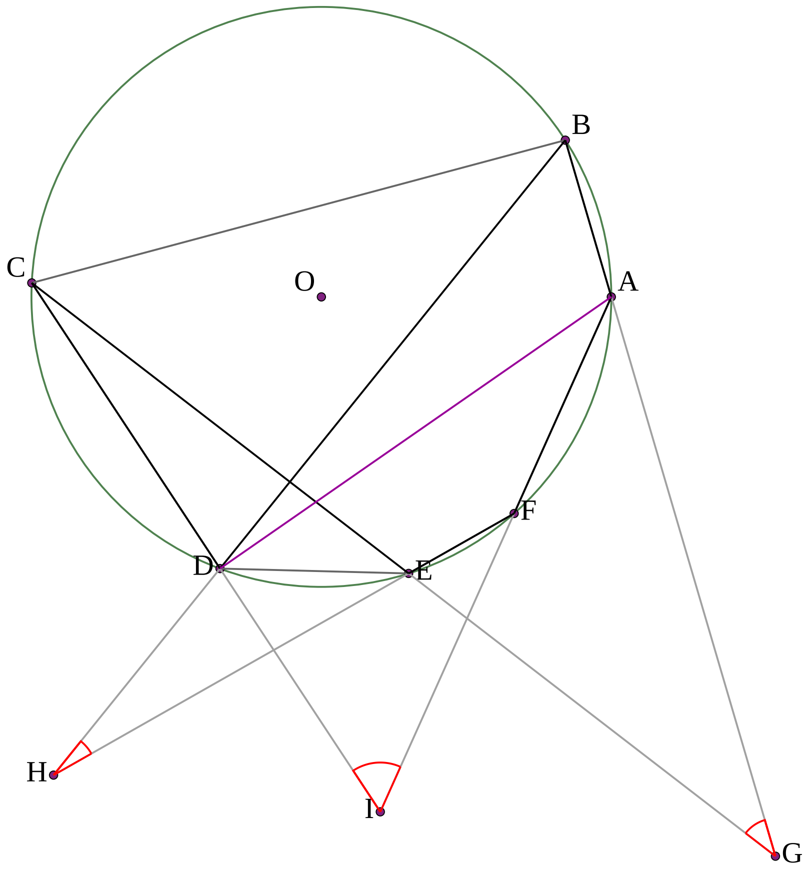
Solution to example 41



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and EC. Let H be the intersection of BF and CD. Let I be the intersection of FE and DA.
Prove that BGC = BHC+DIE

Draw line BE.
Let BGC=x. Let BHC=y. Let DIE=z.
Let CBH=w.
As BHC=y, BCH=180-y-w.
As BCH=180-y-w, BCD=y+w.
As BCDE is a cyclic quadrilateral,
BED=180-BCD, so BED=180-y-w.
Let DEI=u.
As DEI=u, DEF=180-u.
As CBH=w, CBF=180-w.
As CBFE is a cyclic quadrilateral, CEF=180-CBF,
so CEF=w.
As DEF=180-u, DEC=180-w-u.
As BED=180-y-w, BEG=u-y.
As DIE=z, EDI=180-z-u.
As EDI=180-z-u, EDA=z+u.
As ADE and ABE stand on the same chord,
ABE=ADE, so ABE=z+u.
As ABE=z+u, EBG=180-z-u.
As BEG=u-y, BGE=y+z.
But BGE=x, so y+z=x, or BHC+DIE=BGC.

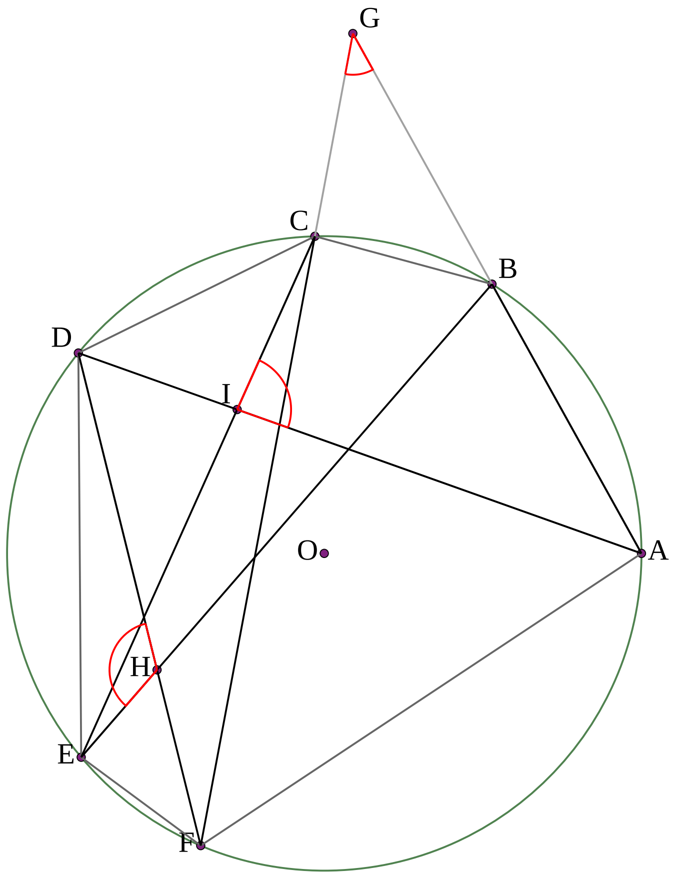
Solution to example 42



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CE. Let H be the intersection of BD and EF. Let I be the intersection of DC and FA.
Prove that DIF = AGE+DHE

Draw line AD.
Let AGE=x. Let DHE=y. Let DIF=z.
Let ADI=w.
As AID=z, DAI=180-z-w.
As DAFE is a cyclic quadrilateral, DEF=180-DAF, so DEF=z+w.
As DEF=z+w, DEH=180-z-w.
As DEH=180-z-w, EDH=z+w-y.
As ADI=w, ADC=180-w.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=w.
As CBG=w, BCG=180-x-w.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=180-x-w.
As BDE=180-x-w, EDH=x+w.
But EDH=z+w-y, so x+w=z+w-y, or x+y=z, or AGE+DHE=DIF.

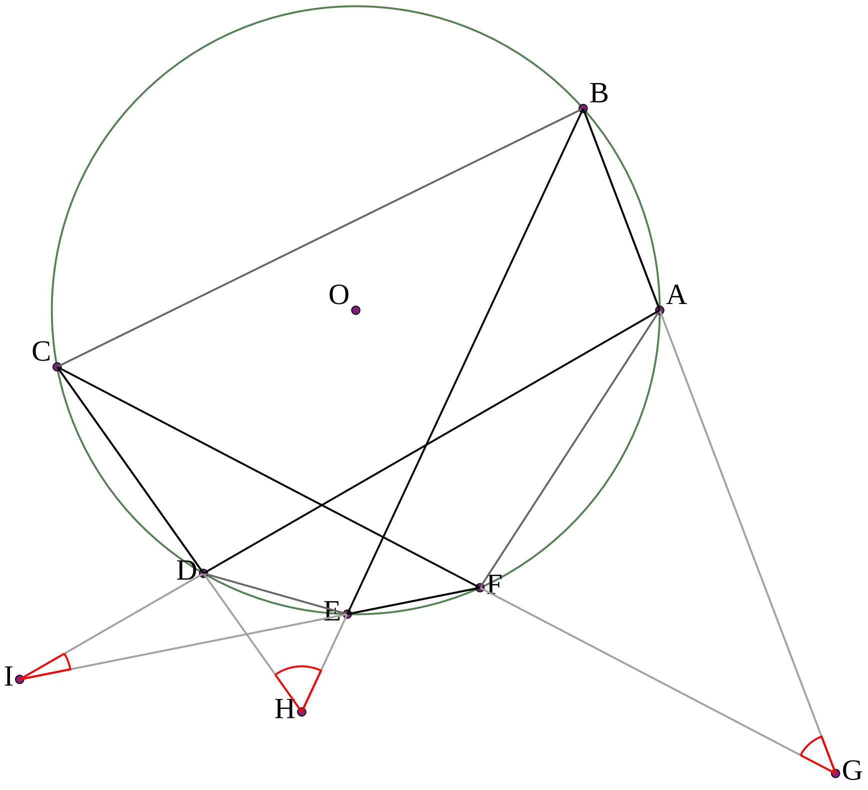
Solution to example 43



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and CF. Let H be the intersection of BE and FD. Let I be the intersection of EC and DA.
Prove that BGC+AIC = DHE

Let BGC=x. Let DHE=y. Let AIC=z.
Let DCI=u.
Let DEH=w.
As BEDC is a cyclic quadrilateral, BCD=180-BED, so BCD=180-w.
As DCE=u, ECB=180-w-u.
As DHE=y, EDH=180-y-w.
As EDF and ECF stand on the same chord, ECF=EDF, so ECF=180-y-w.
As BCE=180-w-u, BCF=y-u.
As BCF=y-u, BCG=u-y+180.
As BCG=u-y+180, CBG=y-x-u.
As AIC=z, CID=180-z.
As CID=180-z, CDI=z-u.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=u-z+180.
As ABC=u-z+180, CBG=z-u.
But CBG=y-x-u, so z-u=y-x-u, or x+z=y, or BGC+AIC=DHE.

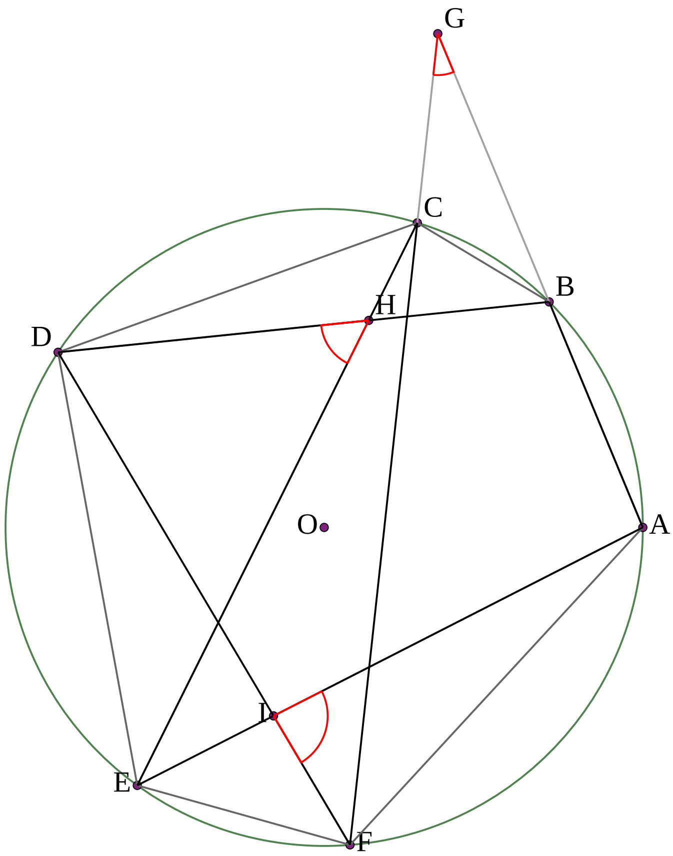
Solution to example 44



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and FC. Let H be the intersection of BE and CD. Let I be the intersection of EF and DA.
Prove that DHE = AGF+DIE

Let AGF=x. Let DHE=y. Let DIE=z.
Let DEH=w.
As DEH=w, DEB=180-w.
Let DEI=u.
As DEI=u, DEF=180-u.
As BED=180-w, BEF=w-u.
As BEF and BCF stand on the same chord, BCF=BEF, so BCF=w-u.
As BCG=w-u, CBG=u-x-w+180.
As DHE=y, EDH=180-y-w.
As EDH=180-y-w, EDC=y+w.
As DIE=z, EDI=180-z-u.
As EDI=180-z-u, EDA=z+u.
As CDE=y+w, CDA=y+w-z-u.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=z+u-y-w+180.
But CBG=u-x-w+180, so z+u-y-w+180=u-x-w+180, or x+z=y, or AGF+DIE=DHE.

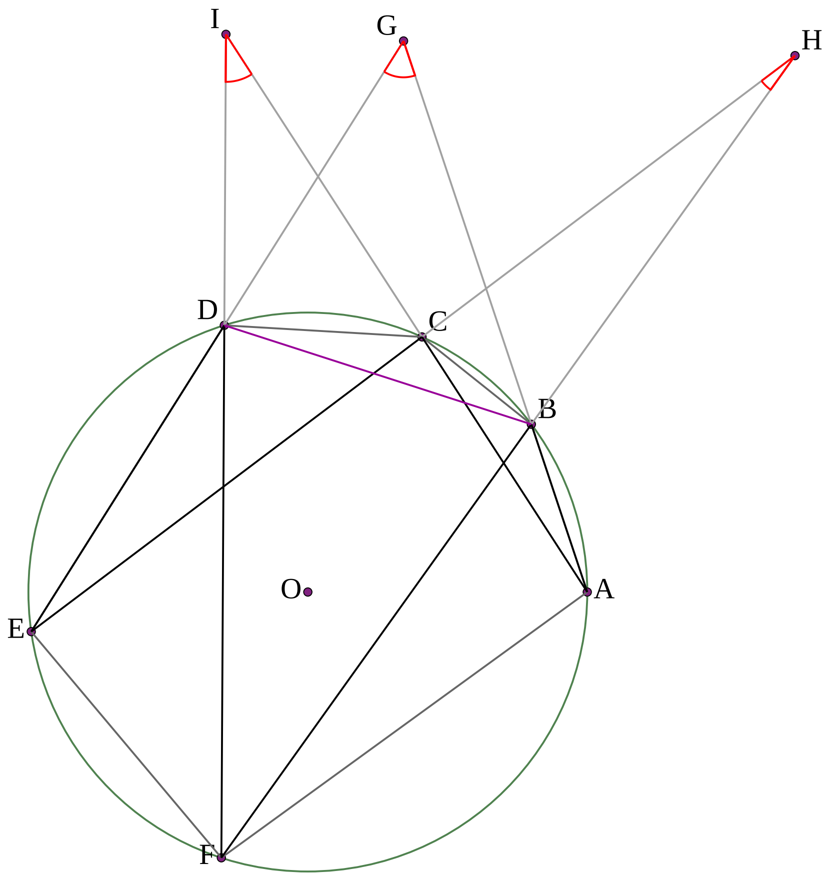
Solution to example 45



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and FC. Let H be the intersection of BD and CE. Let I be the intersection of DF and EA.
Prove that BGC+DHE = AIF

Let BGC=x. Let DHE=y. Let AIF=z.
Let EDH=w.
As DHE=y, DEH=180-y-w.
As CED and CBD stand on the same chord, CBD=CED, so CBD=180-y-w.
Let AFI=u.
As AFDB is a cyclic quadrilateral, ABD=180-AFD, so ABD=180-u.
As ABD=180-u, DBG=u.
As CBD=180-y-w, CBG=y+w+u-180.
As BDE and BCE stand on the same chord, BCE=BDE, so BCE=w.
As AIF=z, FAI=180-z-u.
As EAF and ECF stand on the same chord, ECF=EAF, so ECF=180-z-u.
As BCE=w, BCF=z+w+u-180.
As BCF=z+w+u-180, BCG=360-z-w-u.
As CBG=y+w+u-180, BGC=z-y.
But BGC=x, so z-y=x, or z=x+y, or AIF=BGC+DHE.

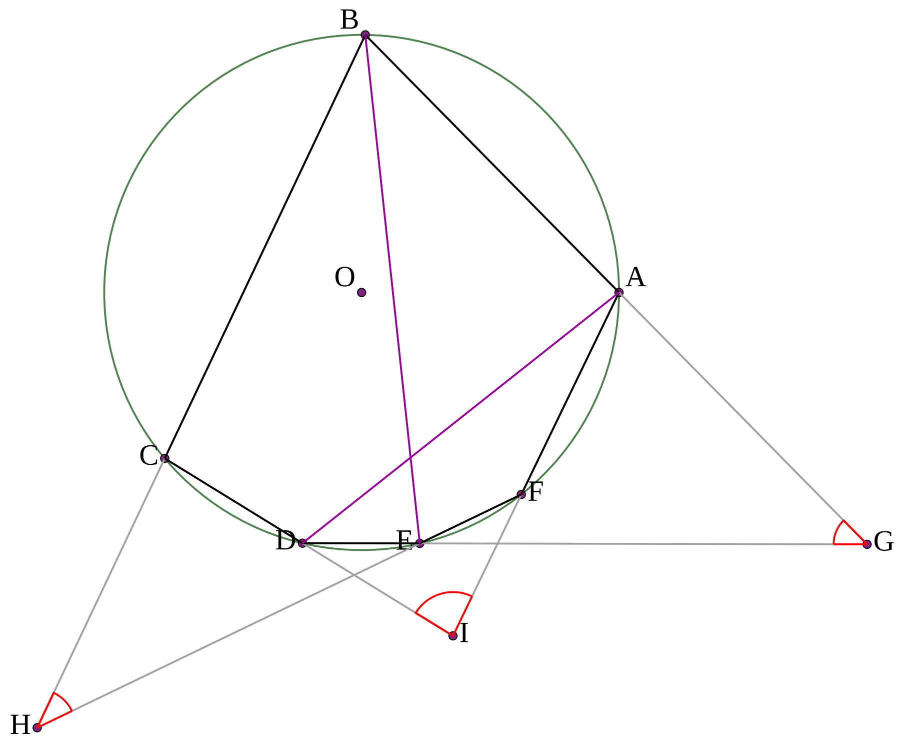
Solution to example 46



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and DE. Let H be the intersection of BF and EC. Let I be the intersection of FD and CA.
Prove that BGD = BHC+CID

Draw line BD.
Let BGD=x. Let BHC=y. Let CID=z.
Let CDI=w.
As CDI=w, CDF=180-w.
As CDFB is a cyclic quadrilateral, CBF=180-CDF, so CBF=w.
As CBF=w, CBH=180-w.
As CBH=180-w, BCH=w-y.
As BCH=w-y, BCE=y-w+180.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=y-w+180.
As CID=z, DCI=180-z-w.
As DCI=180-z-w, DCA=z+w.
As ACD and ABD stand on the same chord, ABD=ACD, so ABD=z+w.
As ABD=z+w, DBG=180-z-w.
As DBG=180-z-w, BDG=z+w-x.
As BDG=z+w-x, BDE=x-z-w+180.
But BDE=y-w+180, so x-z-w+180=y-w+180, or x=y+z, or BGD=BHC+CID.

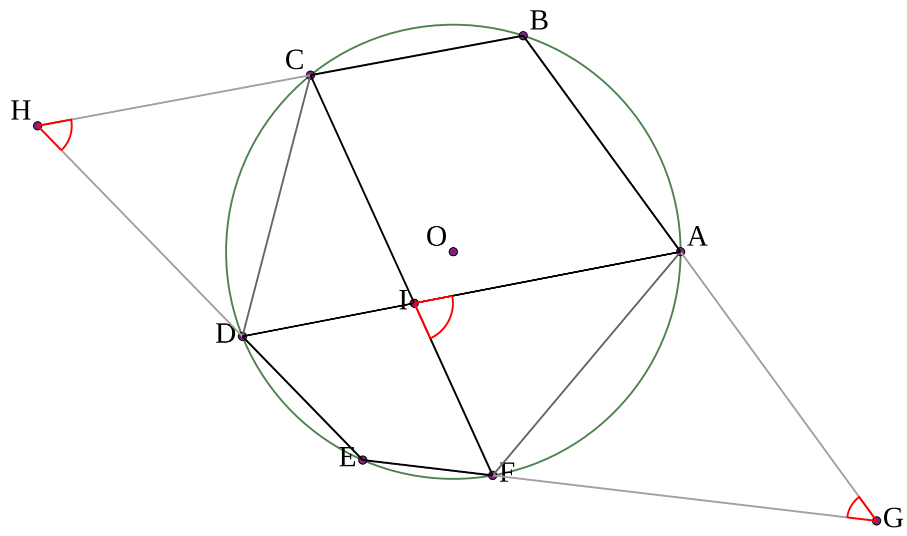
Solution to example 47



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and DE. Let H be the intersection of BC and EF. Let I be the intersection of CD and FA.
Prove that DIF = AGE+CHE

Draw lines AD and BE.
Let AGE=x. Let CHE=y. Let DIF=z.
Let ADI=u.
As AID=z, DAI=180-z-u.
As ADI=u, ADC=180-u.
Let EBH=w.
As CBED is a cyclic quadrilateral, CDE=180-CBE, so CDE=180-w.
As ADC=180-u, ADE=u-w.
As ADG=u-w, DAG=w-x-u+180.
As DAF=180-z-u, FAG=z+w-x.
As BHE=y, BEH=180-y-w.
As BEH=180-y-w, BEF=y+w.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=180-y-w.
As BAF=180-y-w, FAG=y+w.
But FAG=z+w-x, so y+w=z+w-x, or x+y=z, or AGE+CHE=DIF.

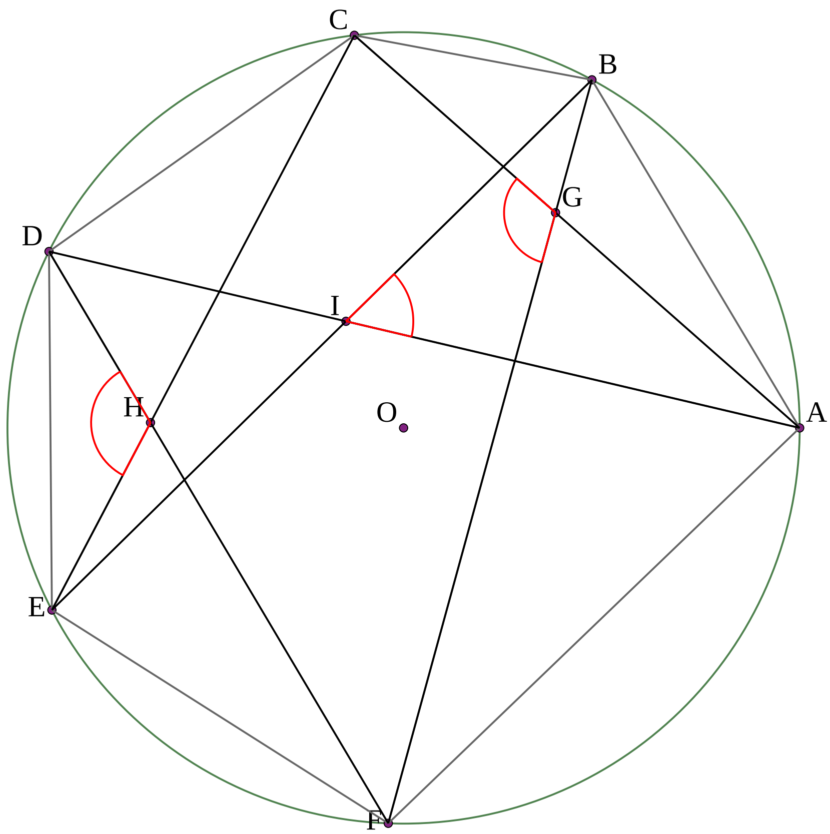
Solution to example 48



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AB and FE. Let H be the intersection of BC and ED. Let I be the intersection of CF and DA.
Prove that AGF+CHD+AIF = 180

Let AGF=x. Let CHD=y. Let AIF=z.
Let DCH=w.
As DCH=w, DCB=180-w.
As BCDA is a cyclic quadrilateral, BAD=180-BCD, so BAD=w.
As BAD=w, DAG=180-w.
Let AFI=u.
As AIF=z, FAI=180-z-u.
As DAG=180-w, GAF=z+u-w.
As FAG=z+u-w, AFG=w-x-z-u+180.
As AFC and ADC stand on the same chord, ADC=AFC, so ADC=u.
As CHD=y, CDH=180-y-w.
As CDH=180-y-w, CDE=y+w.
As ADC=u, ADE=y+w-u.
As ADEF is a cyclic quadrilateral, AFE=180-ADE, so AFE=u-y-w+180.
As AFE=u-y-w+180, AFG=y+w-u.
But AFG=w-x-z-u+180, so y+w-u=w-x-z-u+180, or x+y+z=180, or AGF+CHD+AIF=180.

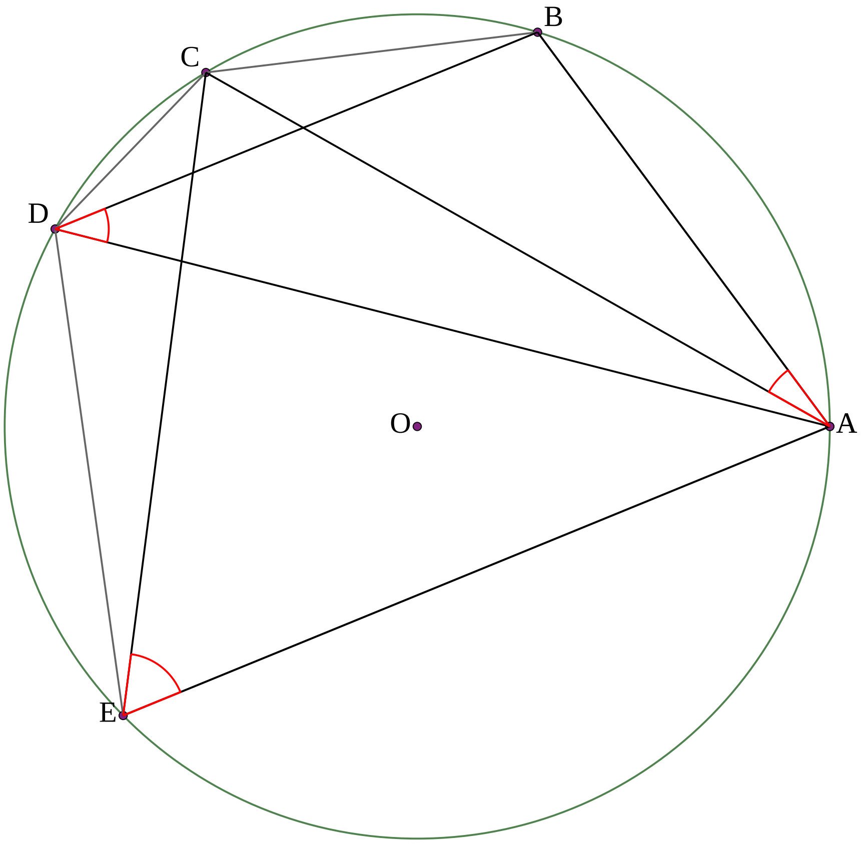
Solution to example 49



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AC and BF. Let H be the intersection of CE and FD. Let I be the intersection of EB and DA.
Prove that CGF+DHE = AIB+180

Let CGF=x. Let DHE=y. Let AIB=z.
Let ABI=u.
As AIB=z, BAI=180-z-u.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=z+u.
Let DCH=w.
As ABE and ACE stand on the same chord, ACE=ABE, so ACE=u.
As DCE=w, DCA=w+u.
As BCD=z+u, BCG=z-w.
As CGF=x, CGB=180-x.
As BCG=z-w, CBG=x+w-z.
As DHE=y, DHC=180-y.
As CHD=180-y, CDH=y-w.
As CDFB is a cyclic quadrilateral, CBF=180-CDF, so CBF=w-y+180.
But CBG=x+w-z, so w-y+180=x+w-z, or z+180=x+y, or AIB+180=CGF+DHE.

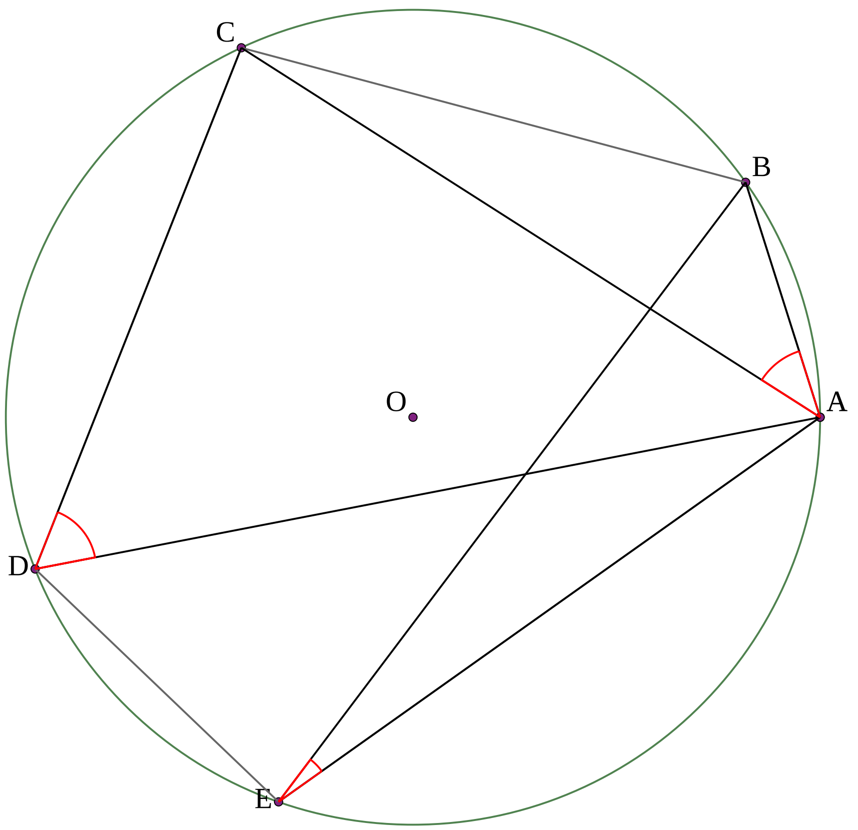
Solution to example 50



Let ABCDE be a cyclic pentagon with center O.
Prove that ADB+BAC = AEC

Let ADB=x. Let AEC=y. Let BAC=z.
As ADB and ACB stand on the same chord, ACB=ADB, so ACB=x.
As AECB is a cyclic quadrilateral, ABC=180-AEC, so ABC=180-y.
As BAC=z, ACB=y-z.
But ACB=x, so y-z=x, or y=x+z, or AEC=ADB+BAC.

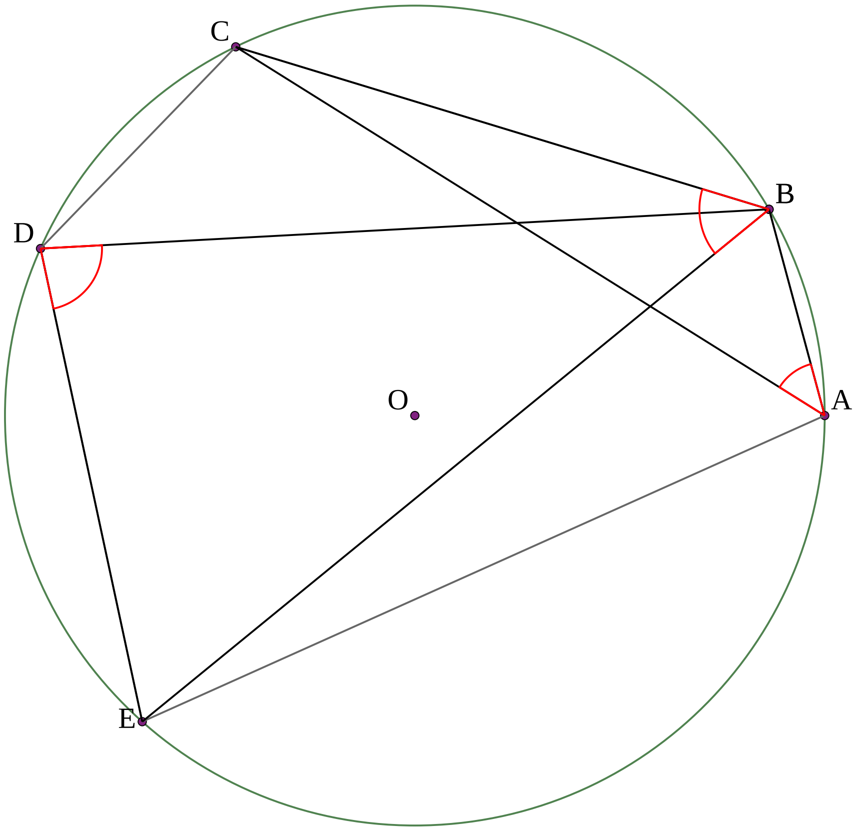
Solution to example 51



Let ABCDE be a cyclic pentagon with center O.
Prove that AEB+BAC = ADC

Let AEB=x. Let ADC=y. Let BAC=z.
As AEB and ACB stand on the same chord, ACB=AEB, so ACB=x.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=180-y.
As BAC=z, ACB=y-z.
But ACB=x, so y-z=x, or y=x+z, or ADC=AEB+BAC.

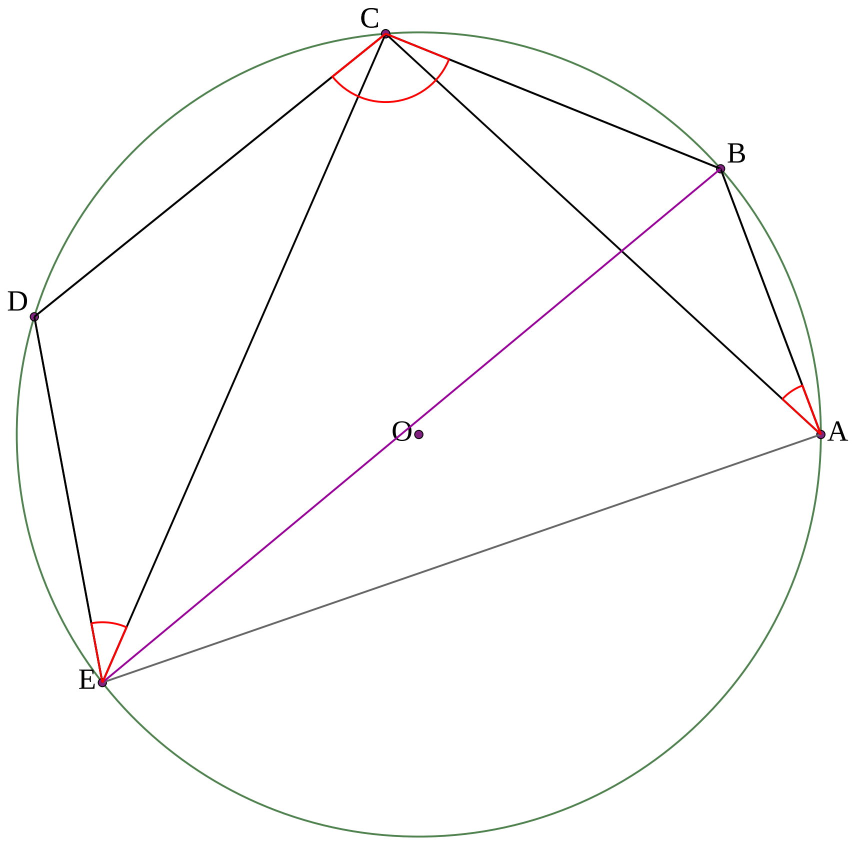
Solution to example 52



Let ABCDE be a cyclic pentagon with center O.
Prove that BAC+BDE+CBE = 180

Let BAC=x. Let BDE=y. Let CBE=z.
As CBE and CAE stand on the same chord, CAE=CBE, so CAE=z.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=180-y.
As BAC=x, CAE=180-x-y.
But CAE=z, so 180-x-y=z, or x+y+z=180, or BAC+BDE+CBE=180.

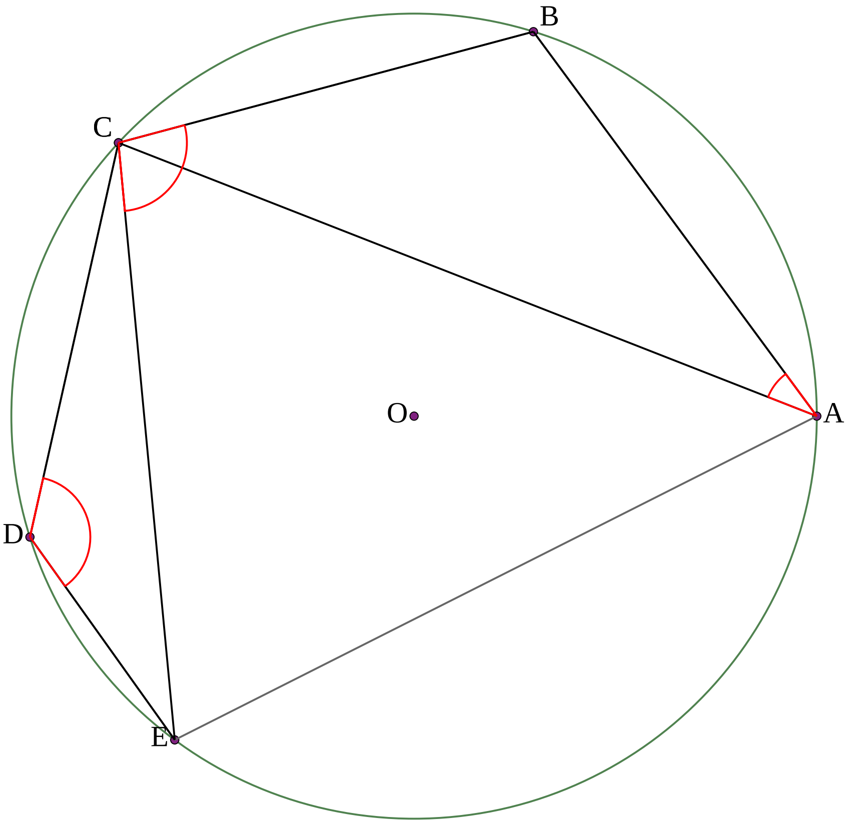
Solution to example 53



Let ABCDE be a cyclic pentagon with center O.
Prove that BAC+CED+BCD = 180

Draw line BE.
Let BAC=x. Let CED=y. Let BCD=z.
As BCDE is a cyclic quadrilateral, BED=180-BCD, so BED=180-z.
As BAC and BEC stand on the same chord, BEC=BAC, so BEC=x.
As CED=y, DEB=x+y.
But BED=180-z, so x+y=180-z, or x+y+z=180, or BAC+CED+BCD=180.

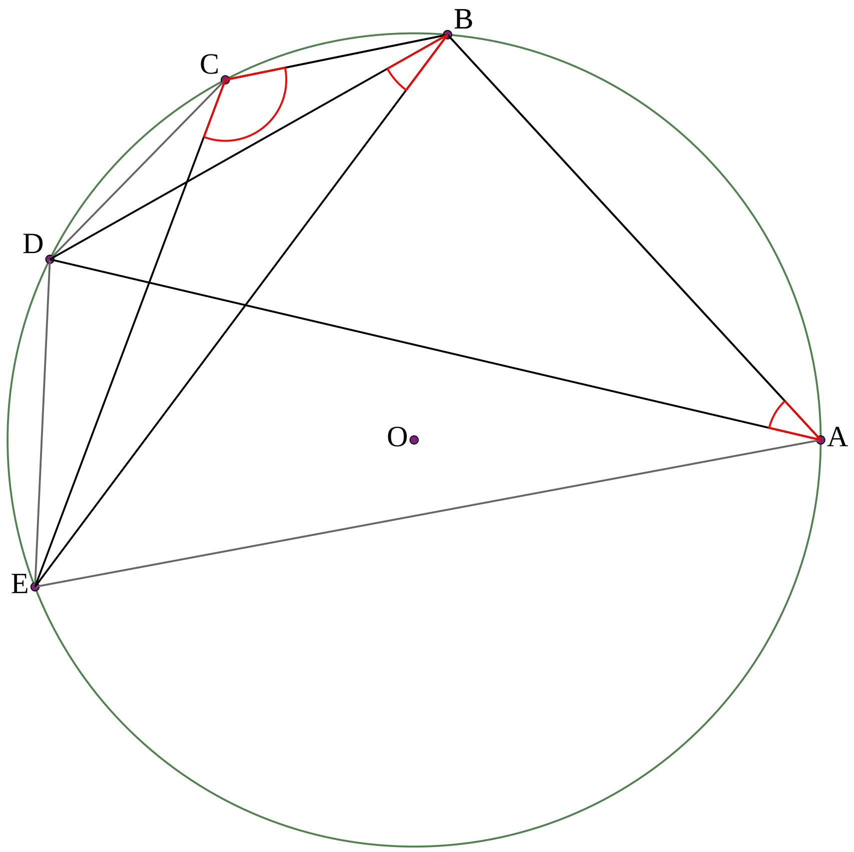
Solution to example 54



Let ABCDE be a cyclic pentagon with center O.
Prove that CDE = BAC+BCE

Let BAC=x. Let CDE=y. Let BCE=z.
As BCEA is a cyclic quadrilateral, BAE=180-BCE, so BAE=180-z.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=180-y.
As BAC=x, BAE=x-y+180.
But BAE=180-z, so x-y+180=180-z, or x+z=y, or BAC+BCE=CDE.

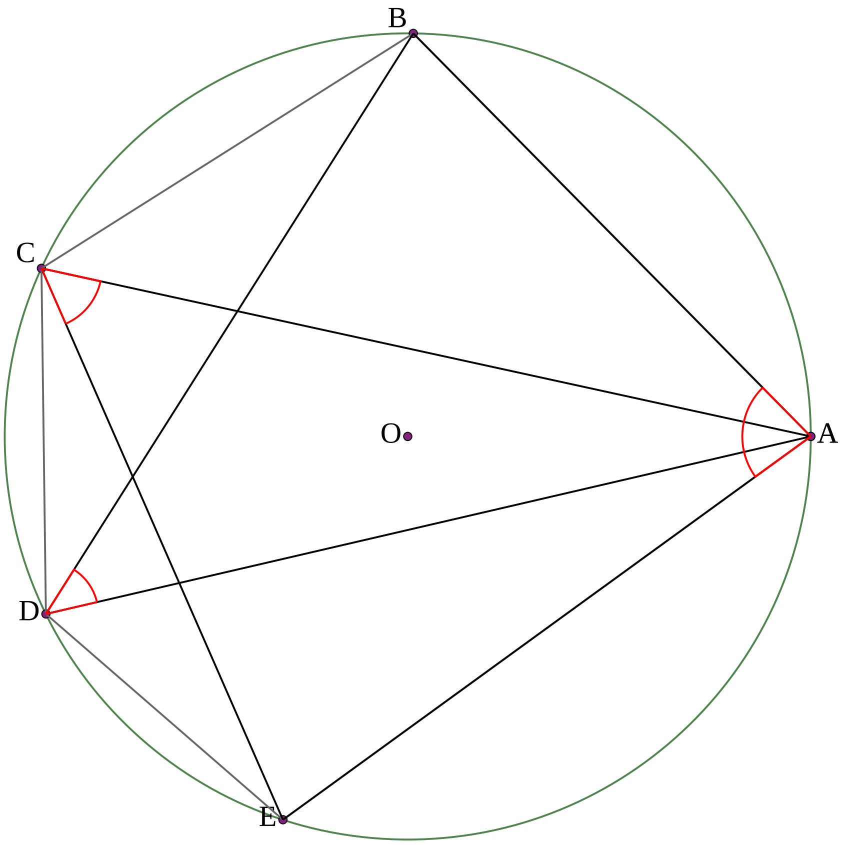
Solution to example 55



Let ABCDE be a cyclic pentagon with center O.
Prove that BAD+BCE+DBE = 180

Let BAD=x. Let BCE=y. Let DBE=z.
As BAD and BED stand on the same chord, BED=BAD, so BED=x.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=y.
As DBE=z, BED=180-y-z.
But BED=x, so 180-y-z=x, or x+y+z=180, or BAD+BCE+DBE=180.

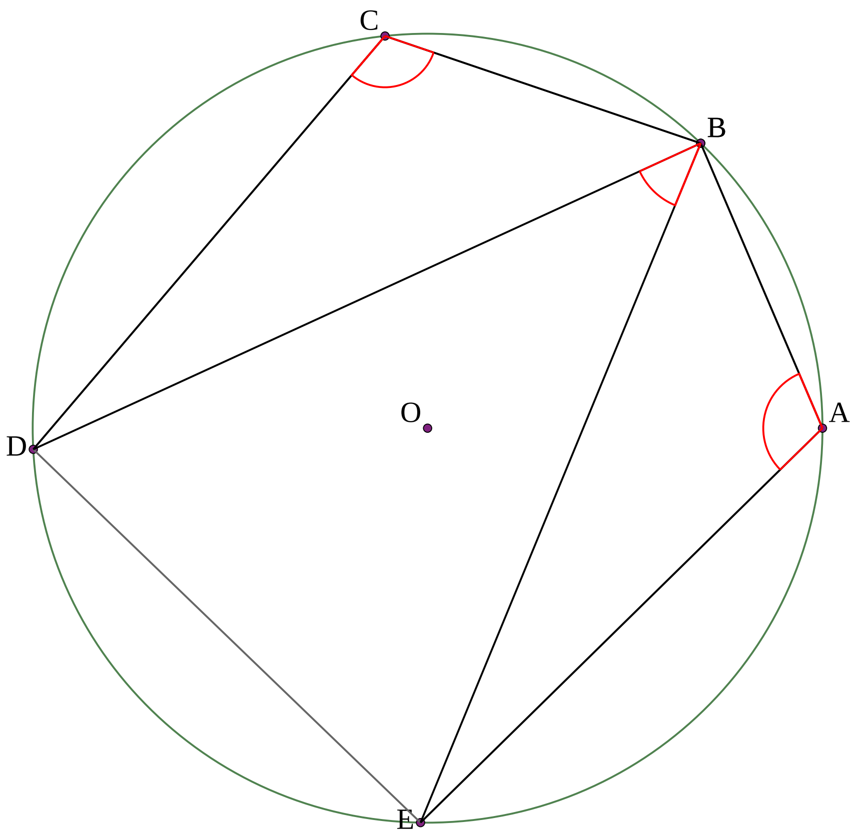
Solution to example 56



Let ABCDE be a cyclic pentagon with center O.
Prove that ADB+ACE+BAE = 180

Let ADB=x. Let ACE=y. Let BAE=z.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-z.
As ACE and ADE stand on the same chord, ADE=ACE, so ADE=y.
As ADB=x, BDE=x+y.
But BDE=180-z, so x+y=180-z, or x+y+z=180, or ADB+ACE+BAE=180.

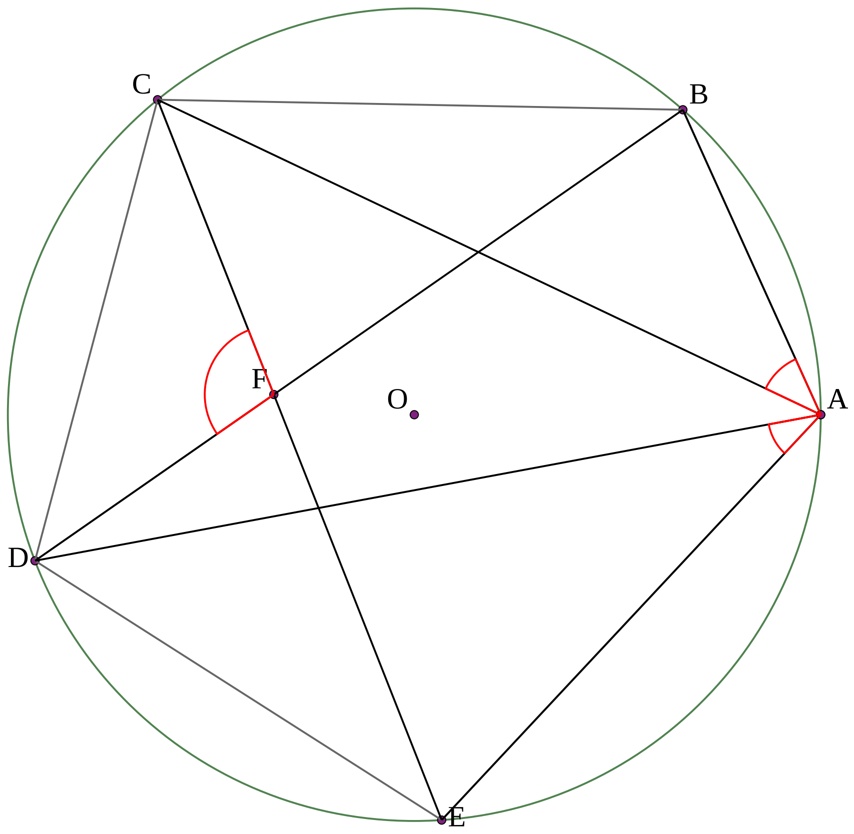
Solution to example 57



Let ABCDE be a cyclic pentagon with center O.
Prove that BCD+BAE = DBE+180

Let BCD=x. Let BAE=y. Let DBE=z.
As BCDE is a cyclic quadrilateral, BED=180-BCD, so BED=180-x.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-y.
As DBE=z, BED=y-z.
But BED=180-x, so y-z=180-x, or x+y=z+180, or BCD+BAE=DBE+180.

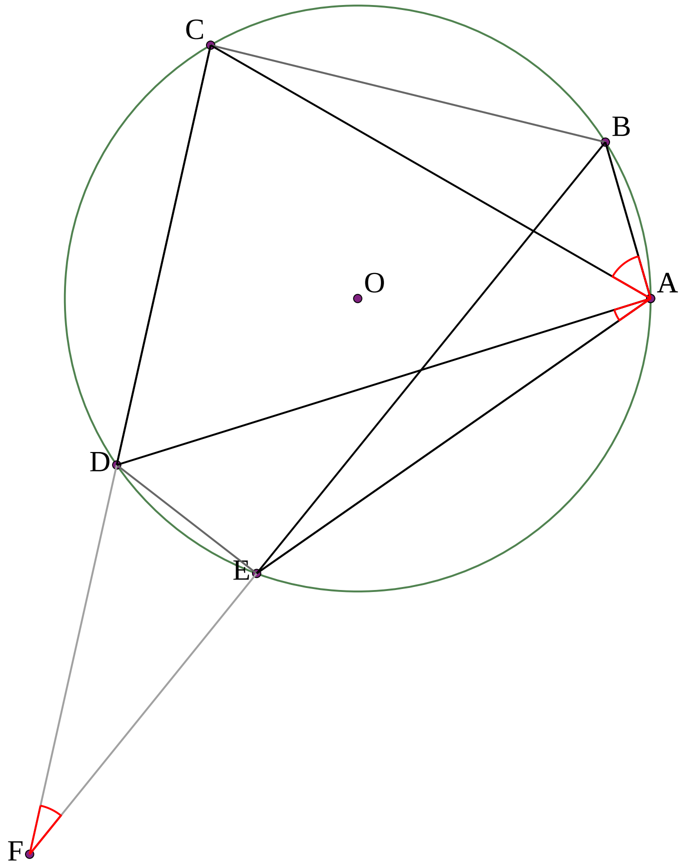
Solution to example 58



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EC.
Prove that DAE+BAC+CFD = 180

Let DAE=x. Let BAC=y. Let CFD=z.
As DAE and DCE stand on the same chord, DCE=DAE, so DCE=x.
As BAC and BDC stand on the same chord, BDC=BAC, so BDC=y.
As DCF=x, CFD=180-x-y.
But CFD=z, so 180-x-y=z, or x+y+z=180, or DAE+BAC+CFD=180.

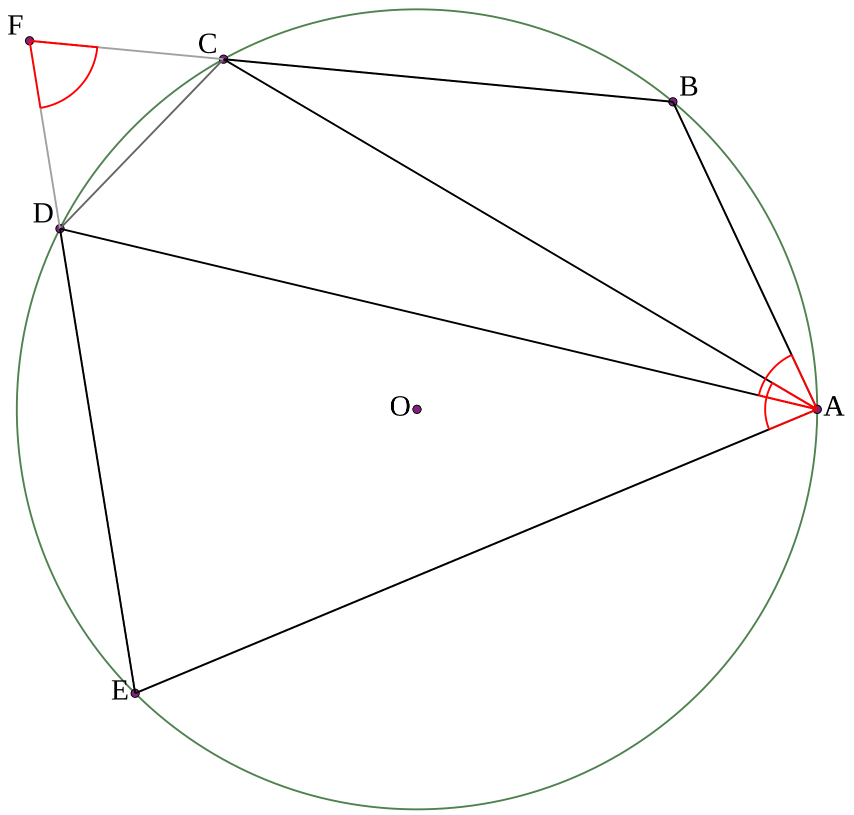
Solution to example 59



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BE and DC.
Prove that BAC = DAE+DFE

Let DAE=x. Let BAC=y. Let DFE=z.
Let DEF=w.
As DEF=w, DEB=180-w.
As BED and BAD stand on the same chord, BAD=BED, so BAD=180-w.
As BAD=180-w, DAC=180-y-w.
As DFE=z, EDF=180-z-w.
As EDF=180-z-w, EDC=z+w.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=180-z-w.
As CAE=180-z-w, CAD=180-x-z-w.
But CAD=180-y-w, so 180-x-z-w=180-y-w, or y=x+z, or BAC=DAE+DFE.

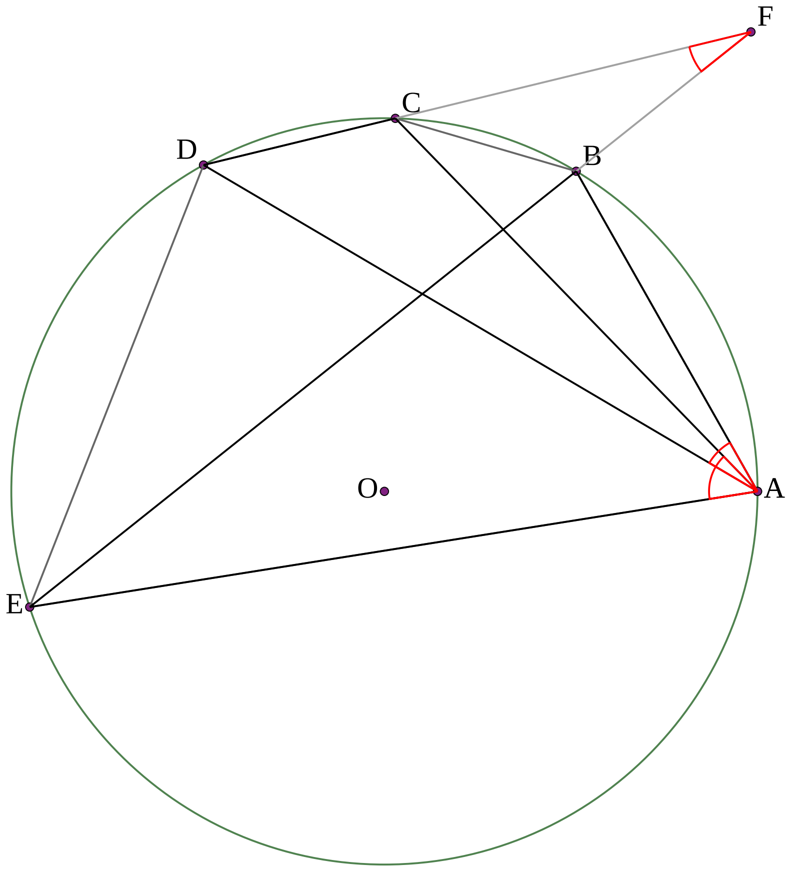
Solution to example 60



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BC and ED.
Prove that CAE+BAD+CFD = 180

Let CAE=x. Let BAD=y. Let CFD=z.
As CAED is a cyclic quadrilateral, CDE=180-CAE, so CDE=180-x.
As CDE=180-x, CDF=x.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=180-y.
As BCD=180-y, DCF=y.
As CDF=x, CFD=180-x-y.
But CFD=z, so 180-x-y=z, or x+y+z=180, or CAE+BAD+CFD=180.

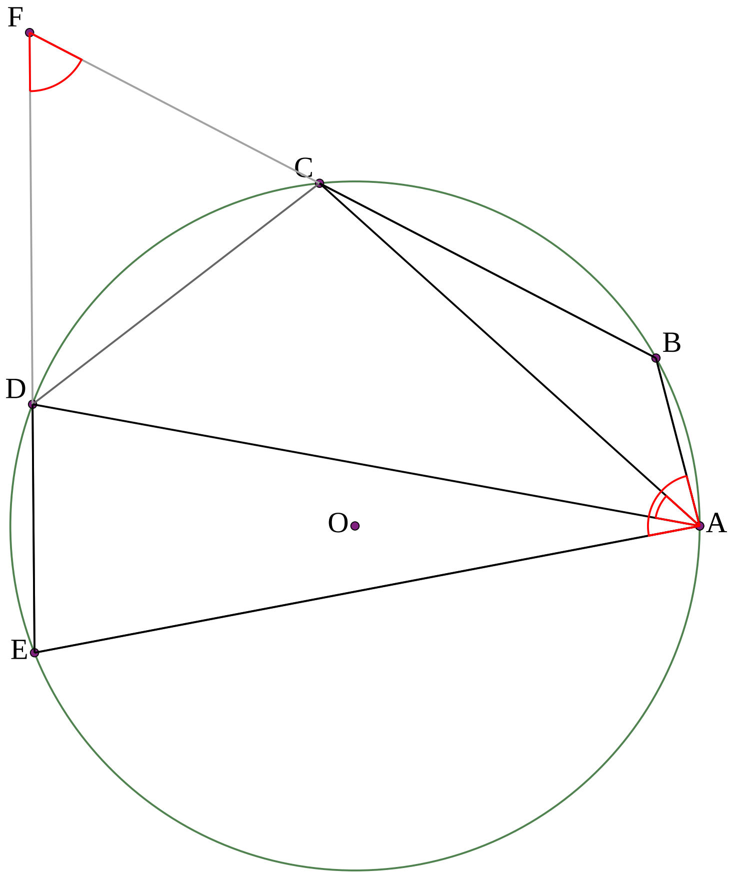
Solution to example 61



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BE and CD.
Prove that BAD+BFC = CAE

Let CAE=x. Let BAD=y. Let BFC=z.
As CAE and CBE stand on the same chord, CBE=CAE, so CBE=x.
As CBE=x, CBF=180-x.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=180-y.
As BCD=180-y, BCF=y.
As CBF=180-x, BFC=x-y.
But BFC=z, so x-y=z, or x=y+z, or CAE=BAD+BFC.

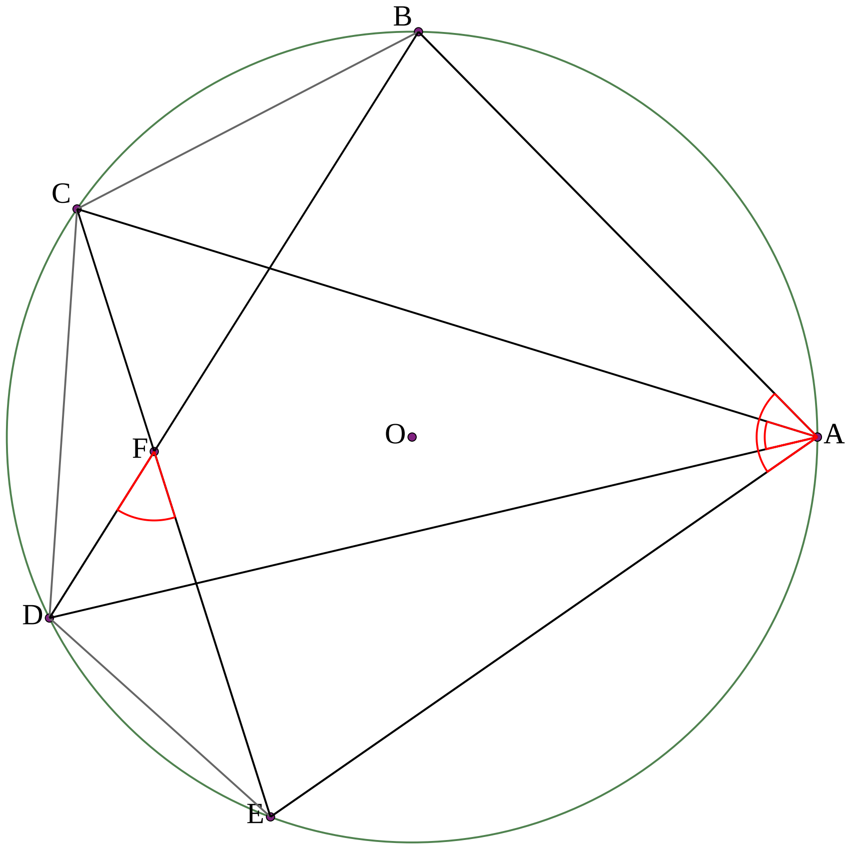
Solution to example 62



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BC and DE.
Prove that CAD+BAE+CFD = 180

Let CAD=x. Let BAE=y. Let CFD=z.
Let DCF=w.
As DCF=w, DCB=180-w.
As BCDA is a cyclic quadrilateral, BAD=180-BCD, so BAD=w.
As BAD=w, DAE=y-w.
As CFD=z, CDF=180-z-w.
As CDF=180-z-w, CDE=z+w.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=180-z-w.
As CAE=180-z-w, EAD=180-x-z-w.
But DAE=y-w, so 180-x-z-w=y-w, or x+y+z=180, or CAD+BAE+CFD=180.

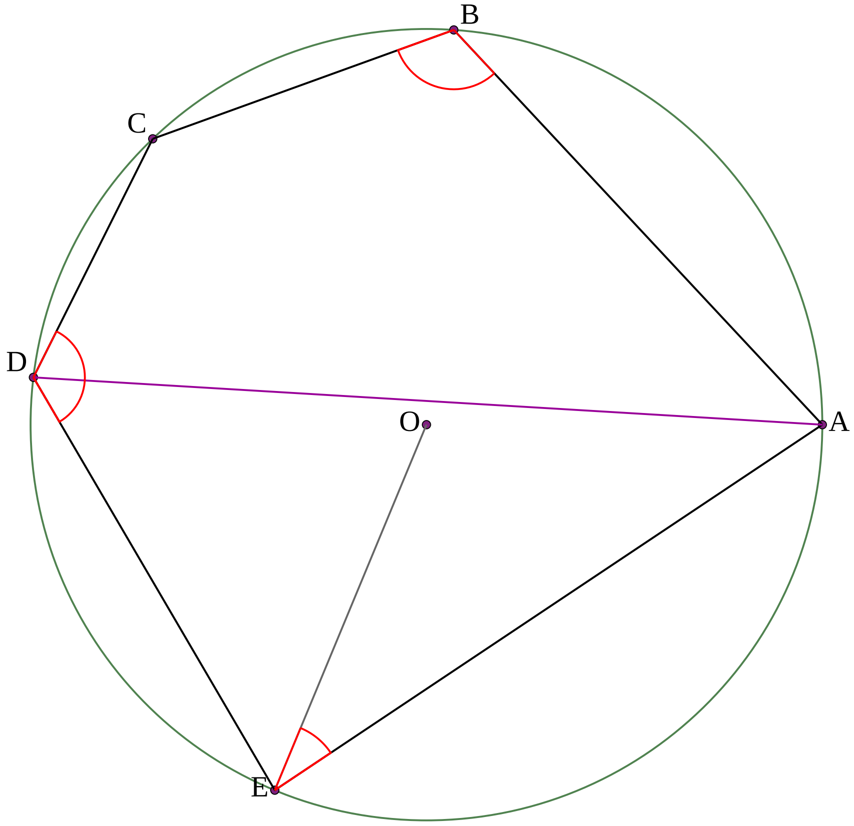
Solution to example 63



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and CE.
Prove that BAE = CAD+DFE

Let CAD=x. Let BAE=y. Let DFE=z.
As CAD and CED stand on the same chord, CED=CAD, so CED=x.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-y.
As DEF=x, DFE=y-x.
But DFE=z, so y-x=z, or y=x+z, or BAE=CAD+DFE.

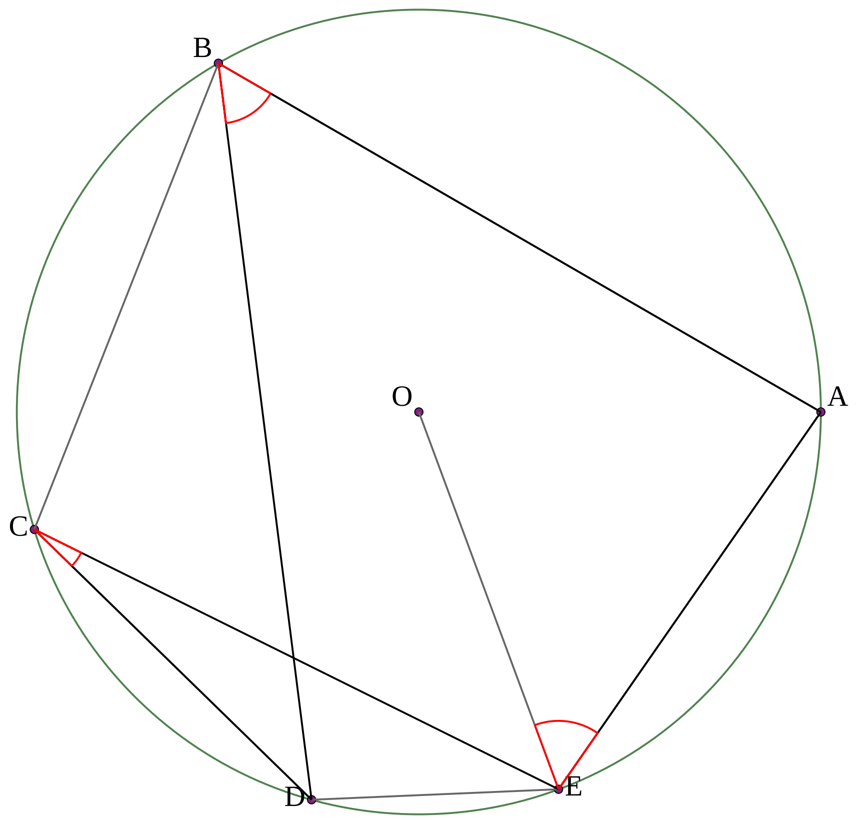
Solution to example 64



Let ABCDE be a cyclic pentagon with center O.
Prove that CDE+ABC+AEO = 270

Draw line AD.
Let CDE=x. Let ABC=y. Let AEO=z.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-y.
As CDE=x, EDA=x+y-180.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ADE, AOE=2ADE, so ADE=90-z.
But ADE=x+y-180, so 90-z=x+y-180, or x+y+z=270, or CDE+ABC+AEO=270.

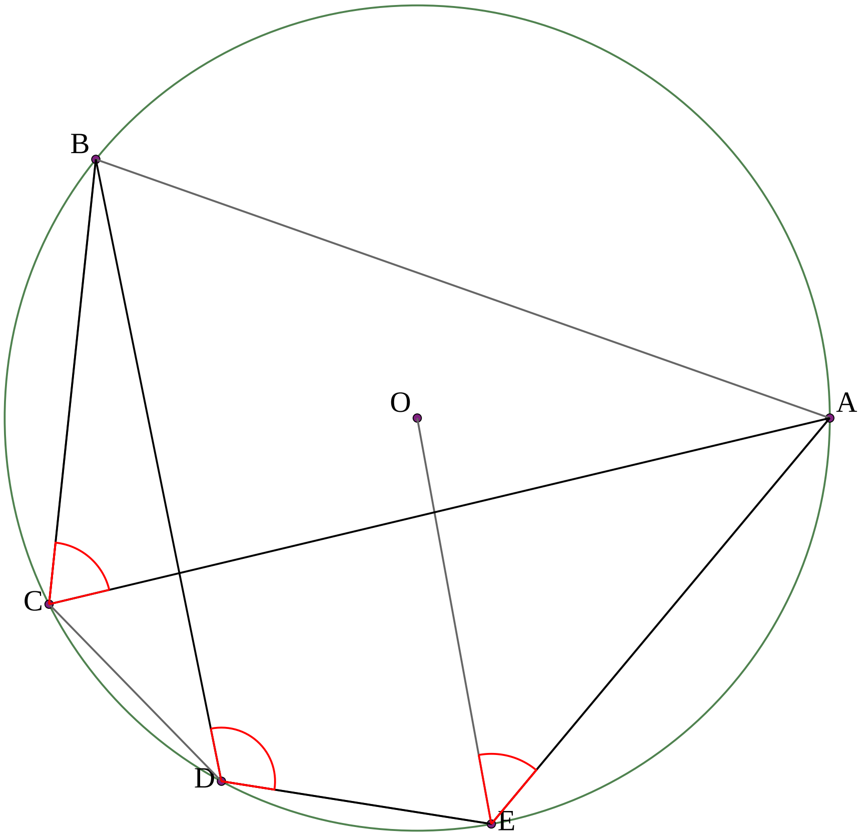
Solution to example 65



Let ABCDE be a cyclic pentagon with center O.
Prove that ABD+AEO = DCE+90

Let DCE=x. Let ABD=y. Let AEO=z.
As ABDE is a cyclic quadrilateral, AED=180-ABD, so AED=180-y.
As AEO=z, OED=180-y-z.
As triangle DEO is isosceles, DOE=2y+2z-180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=y+z-90.
But DCE=x, so y+z-90=x, or y+z=x+90, or ABD+AEO=DCE+90.

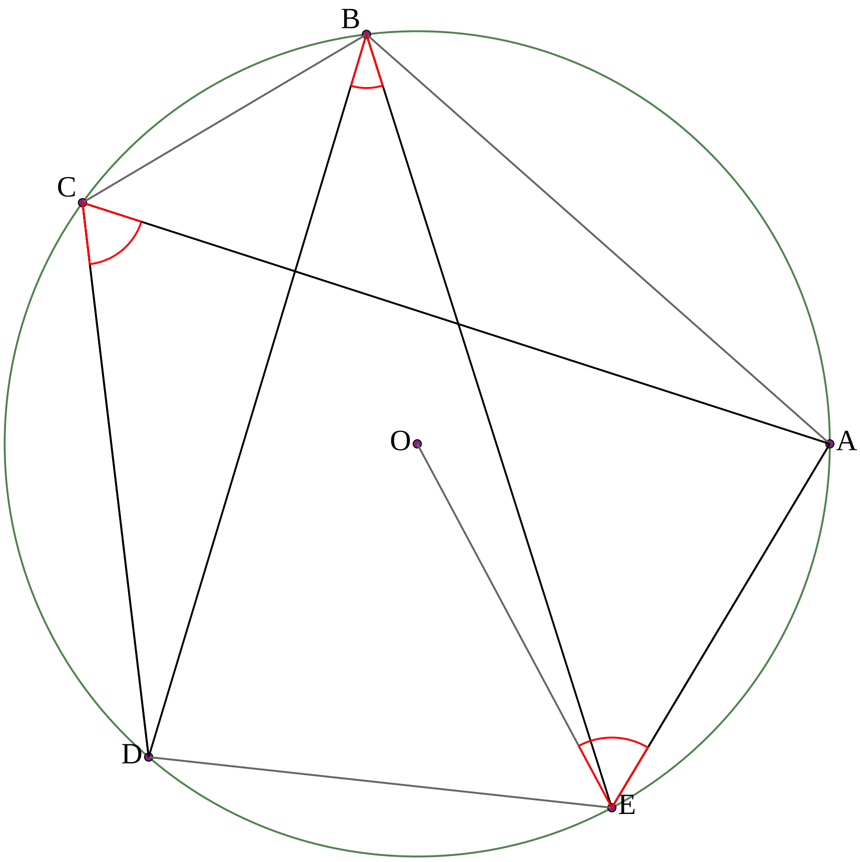
Solution to example 66



Let ABCDE be a cyclic pentagon with center O.
Prove that BDE+AEO = ACB+90

Let BDE=x. Let ACB=y. Let AEO=z.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=180-x.
As triangle AEO is isosceles, EAO=z.
As BAE=180-x, BAO=180-x-z.
As triangle BAO is isosceles, AOB=2x+2z-180.
As AOB is at the center of a circle on the same chord as ACB, AOB=2ACB, so ACB=x+z-90.
But ACB=y, so x+z-90=y, or x+z=y+90, or BDE+AEO=ACB+90.

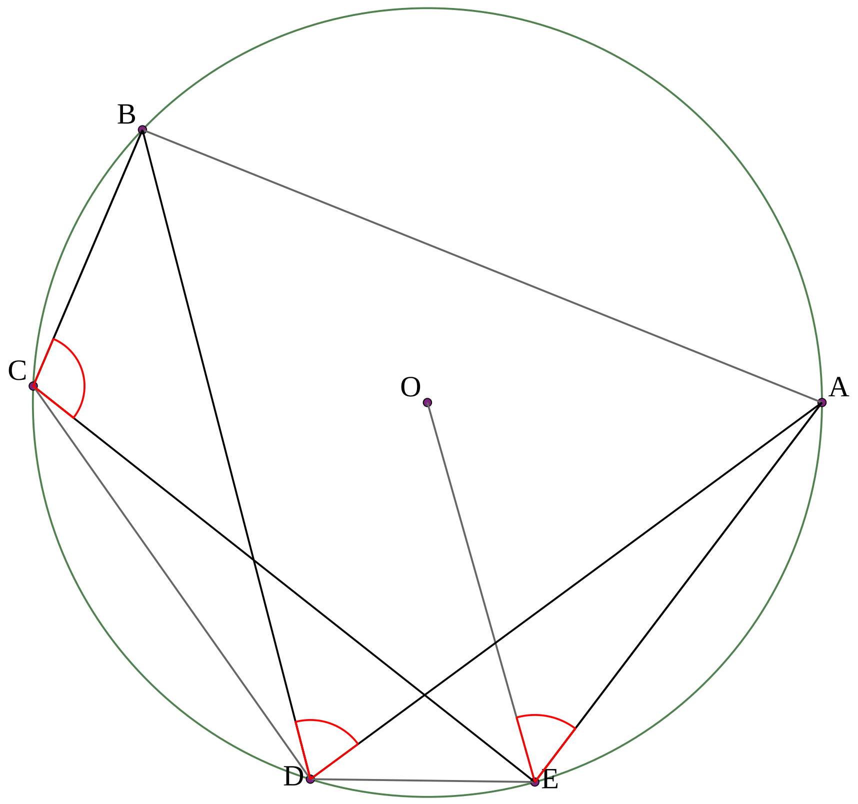
Solution to example 67



Let ABCDE be a cyclic pentagon with center O.
Prove that ACD+AEO = DBE+90

Let DBE=x. Let ACD=y. Let AEO=z.
As ACD and ABD stand on the same chord, ABD=ACD, so ABD=y.
As DBE=x, EBA=y-x.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-z.
But ABE=y-x, so 90-z=y-x, or x+90=y+z, or DBE+90=ACD+AEO.

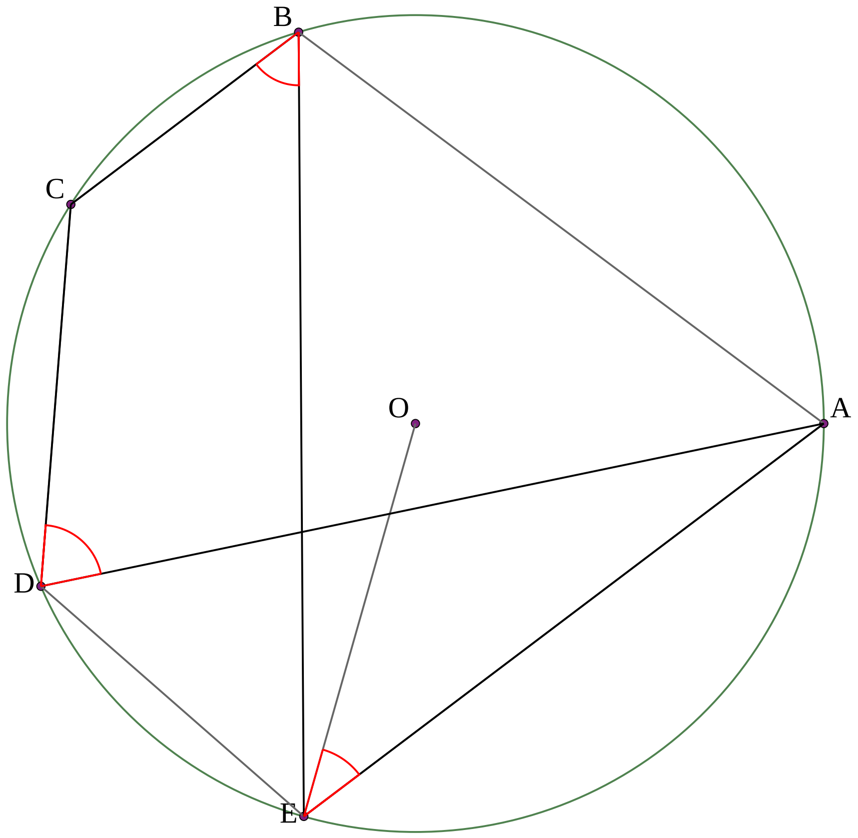
Solution to example 68



Let ABCDE be a cyclic pentagon with center O.
Prove that BCE+AEO = ADB+90

Let BCE=x. Let ADB=y. Let AEO=z.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=x.
As ADB=y, ADE=x-y.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ADE, AOE=2ADE, so ADE=90-z.
But ADE=x-y, so 90-z=x-y, or y+90=x+z, or ADB+90=BCE+AEO.

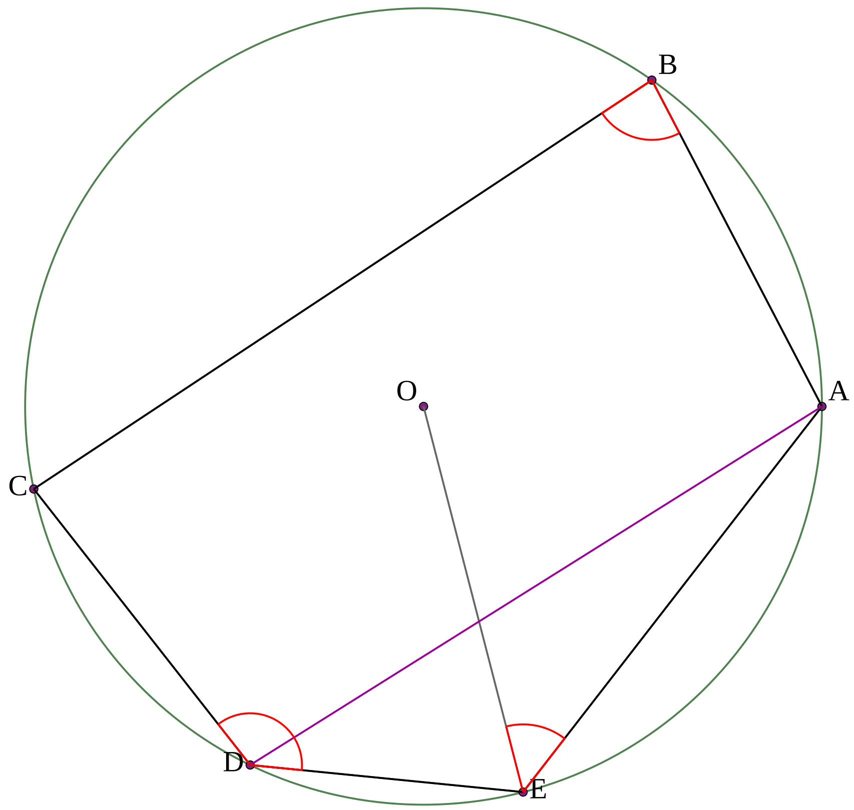
Solution to example 69



Let ABCDE be a cyclic pentagon with center O.
Prove that CBE+ADC = AEO+90

Let CBE=x. Let ADC=y. Let AEO=z.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=180-y.
As CBE=x, EBA=180-x-y.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-z.
But ABE=180-x-y, so 90-z=180-x-y, or x+y=z+90, or CBE+ADC=AEO+90.

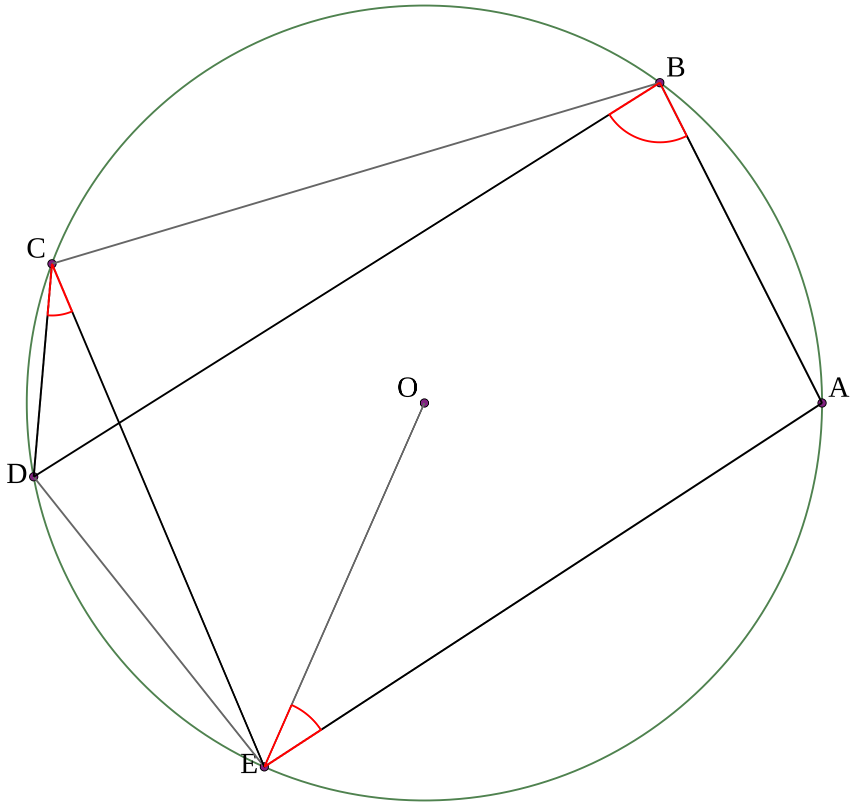
Solution to example 70



Let ABCDE be a cyclic pentagon with center O.
Prove that CDE+ABC+AEO = 270

Draw line AD.
Let CDE=x. Let ABC=y. Let AEO=z.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-y.
As CDE=x, EDA=x+y-180.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ADE, AOE=2ADE, so ADE=90-z.
But ADE=x+y-180, so 90-z=x+y-180, or x+y+z=270, or CDE+ABC+AEO=270.

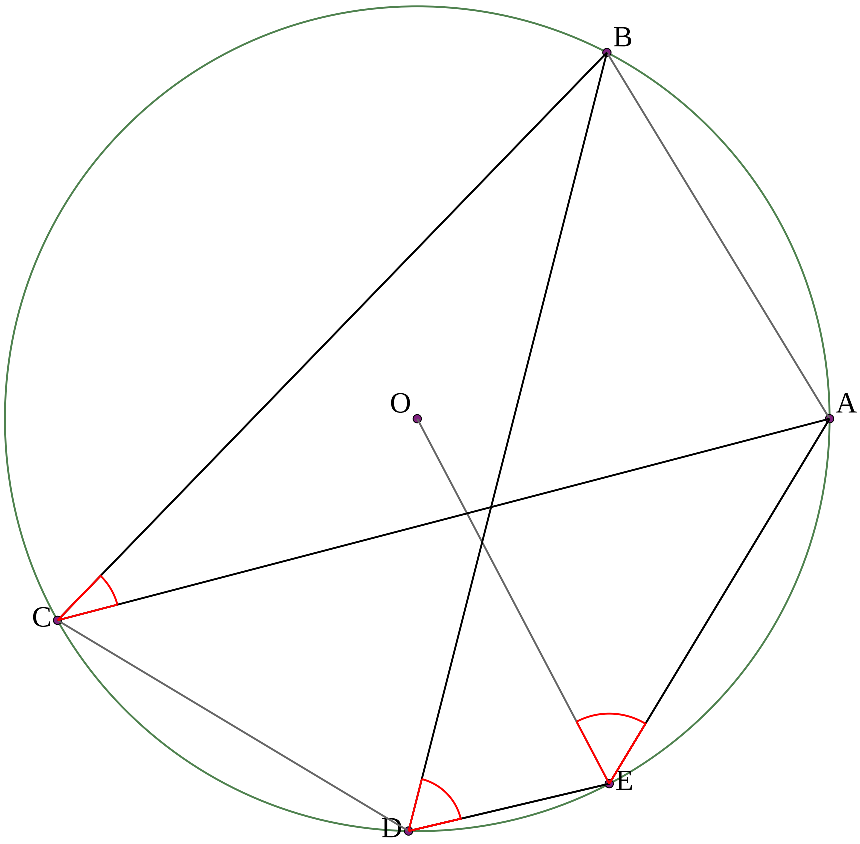
Solution to example 71



Let ABCDE be a cyclic pentagon with center O.
Prove that ABD+AEO = DCE+90

Let DCE=x. Let ABD=y. Let AEO=z.
As ABDE is a cyclic quadrilateral, AED=180-ABD, so AED=180-y.
As AEO=z, OED=180-y-z.
As triangle DEO is isosceles, DOE=2y+2z-180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=y+z-90.
But DCE=x, so y+z-90=x, or y+z=x+90, or ABD+AEO=DCE+90.

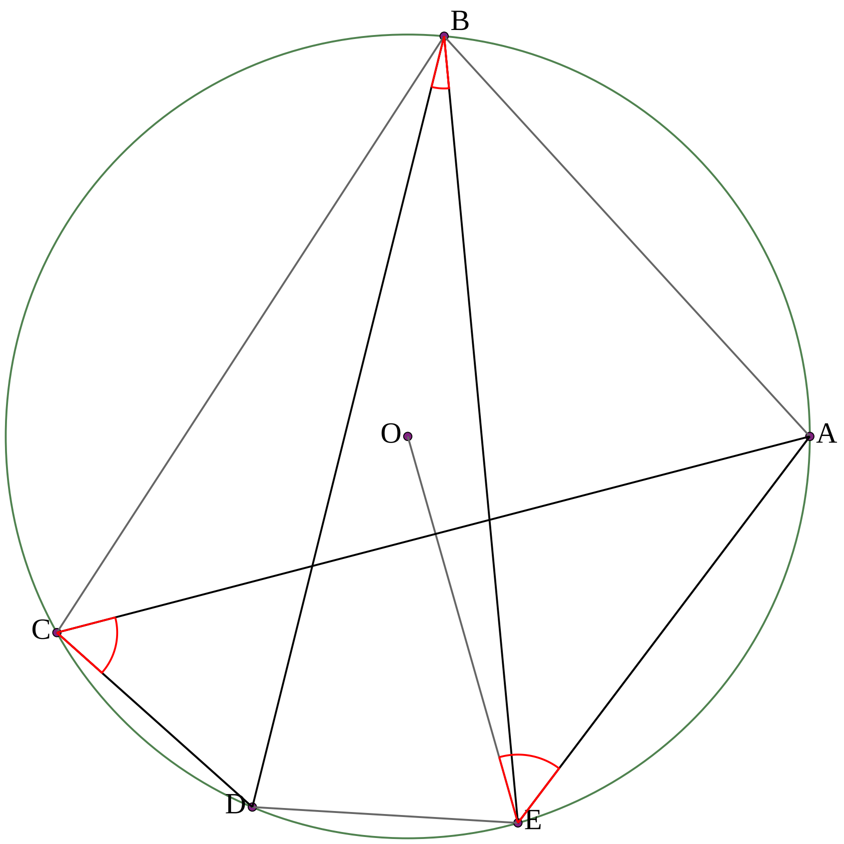
Solution to example 72



Let ABCDE be a cyclic pentagon with center O.
Prove that BDE+AEO = ACB+90

Let BDE=x. Let ACB=y. Let AEO=z.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=180-x.
As triangle AEO is isosceles, EAO=z.
As BAE=180-x, BAO=180-x-z.
As triangle BAO is isosceles, AOB=2x+2z-180.
As AOB is at the center of a circle on the same chord as ACB, AOB=2ACB, so ACB=x+z-90.
But ACB=y, so x+z-90=y, or x+z=y+90, or BDE+AEO=ACB+90.

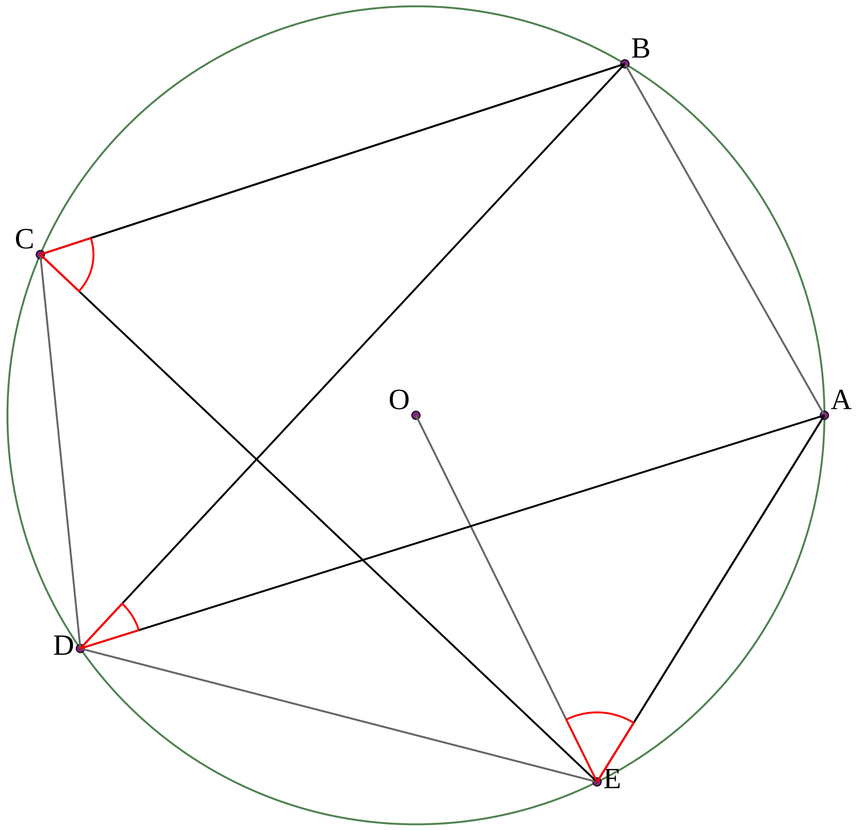
Solution to example 73



Let ABCDE be a cyclic pentagon with center O.
Prove that ACD+AEO = DBE+90

Let DBE=x. Let ACD=y. Let AEO=z.
As ACD and ABD stand on the same chord, ABD=ACD, so ABD=y.
As DBE=x, EBA=y-x.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-z.
But ABE=y-x, so 90-z=y-x, or x+90=y+z, or DBE+90=ACD+AEO.

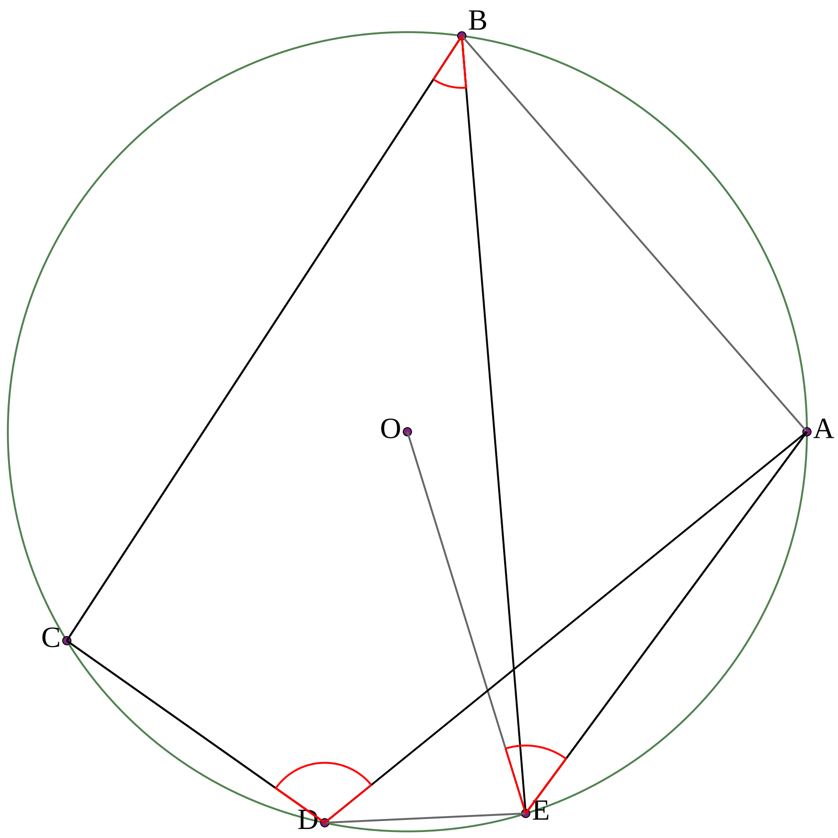
Solution to example 74



Let ABCDE be a cyclic pentagon with center O.
Prove that BCE+AEO = ADB+90

Let BCE=x. Let ADB=y. Let AEO=z.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=x.
As ADB=y, ADE=x-y.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ADE, AOE=2ADE, so ADE=90-z.
But ADE=x-y, so 90-z=x-y, or y+90=x+z, or ADB+90=BCE+AEO.

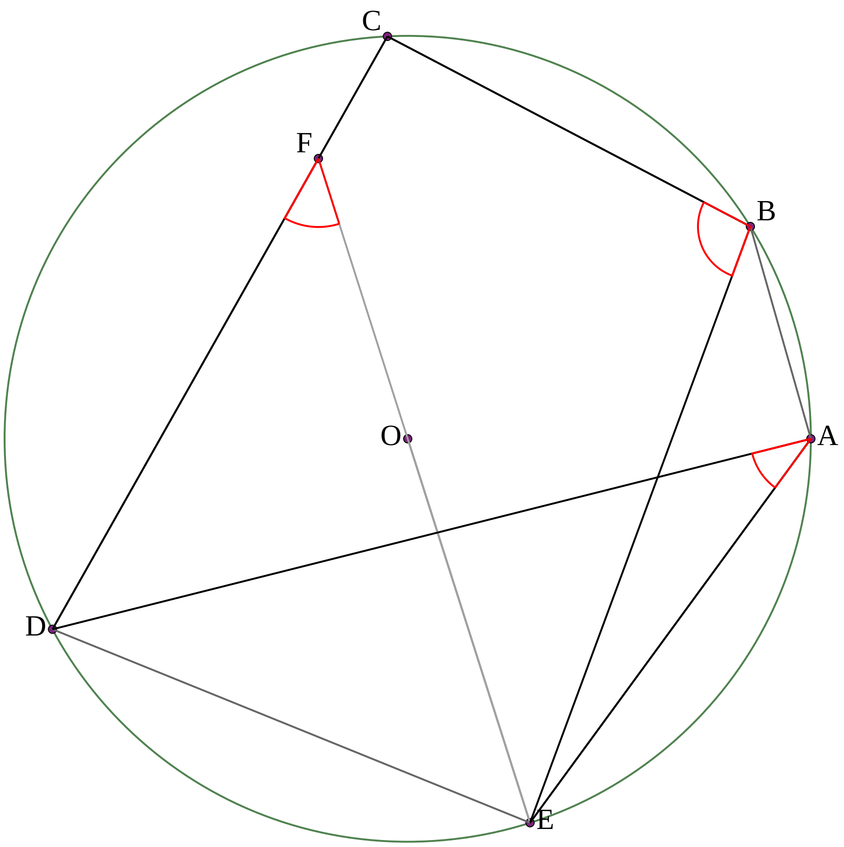
Solution to example 75



Let ABCDE be a cyclic pentagon with center O.
Prove that CBE+ADC = AEO+90

Let CBE=x. Let ADC=y. Let AEO=z.
As ADCB is a cyclic quadrilateral, ABC=180-ADC, so ABC=180-y.
As CBE=x, EBA=180-x-y.
As triangle AEO is isosceles, AOE=180-2z.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-z.
But ABE=180-x-y, so 90-z=180-x-y, or x+y=z+90, or CBE+ADC=AEO+90.

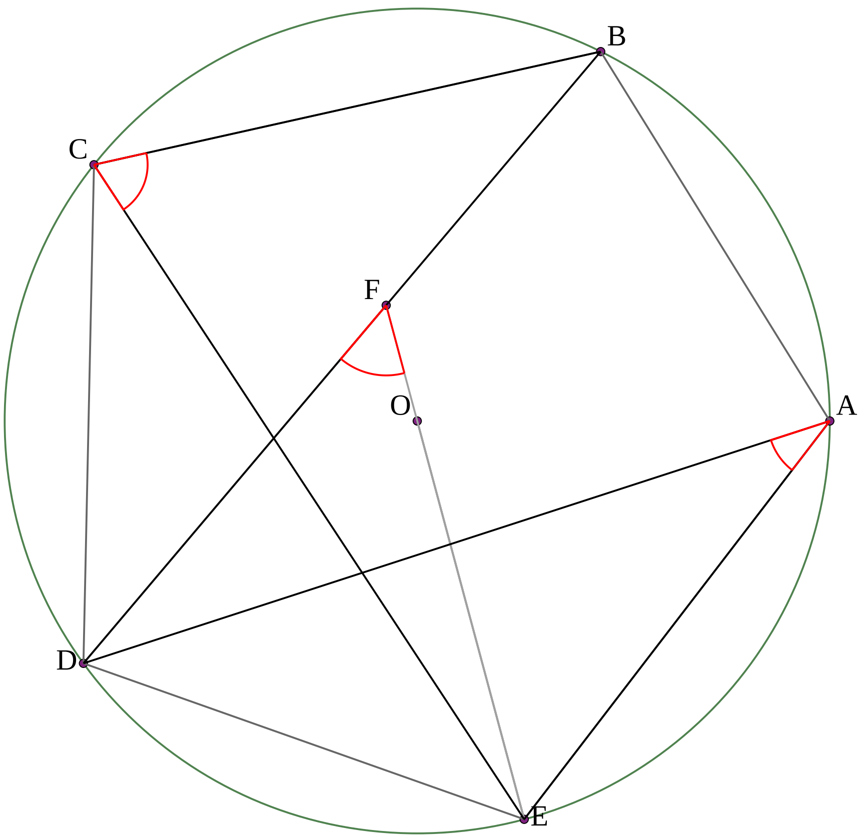
Solution to example 76



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of CD and EO.
Prove that CBE+DAE = DFE+90

Let CBE=x. Let DAE=y. Let DFE=z.
As CBED is a cyclic quadrilateral, CDE=180-CBE, so CDE=180-x.
As EDF=180-x, DEF=x-z.
As triangle DEO is isosceles, DOE=2z-2x+180.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DAE=z-x+90.
But DAE=y, so z-x+90=y, or z+90=x+y, or DFE+90=CBE+DAE.

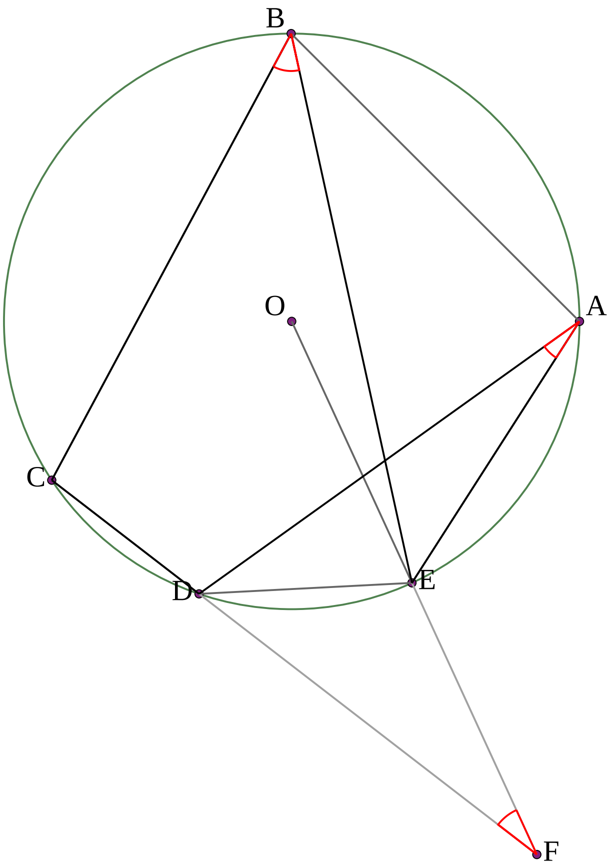
Solution to example 77



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EO.
Prove that BCE+DFE = DAE+90

Let BCE=x. Let DAE=y. Let DFE=z.
As DAE and DCE stand on the same chord, DCE=DAE, so DCE=y.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=x.
As EDF=x, DEF=180-x-z.
As triangle DEO is isosceles, DOE=2x+2z-180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=x+z-90.
But DCE=y, so x+z-90=y, or x+z=y+90, or BCE+DFE=DAE+90.

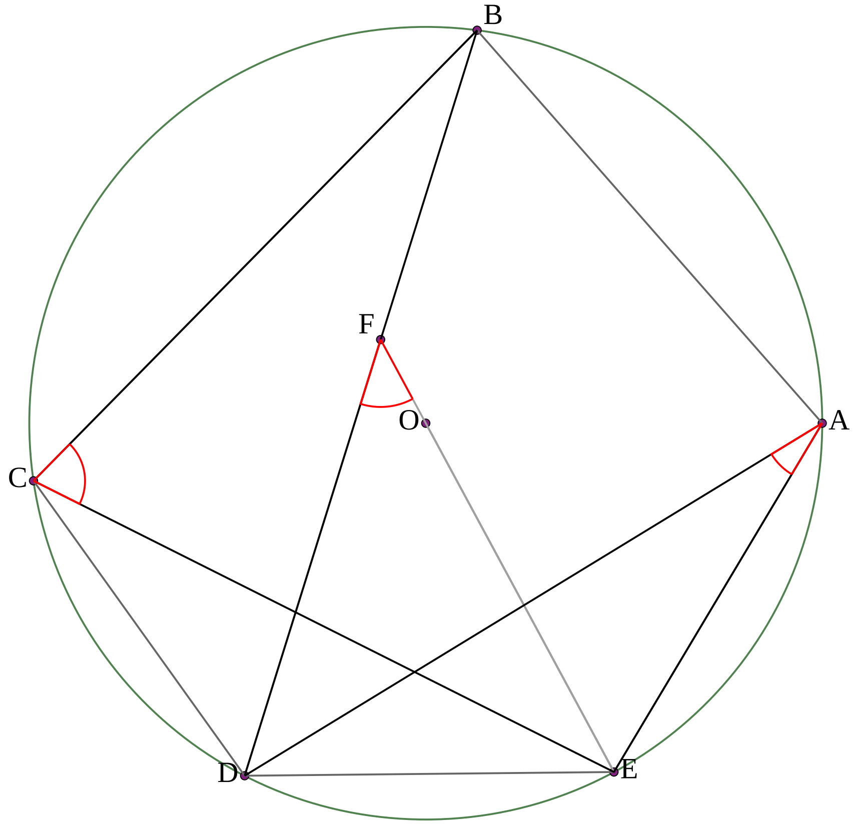
Solution to example 78



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of CD and EO.
Prove that CBE+DAE+DFE = 90

Let CBE=x. Let DAE=y. Let DFE=z.
As CBED is a cyclic quadrilateral, CDE=180-CBE, so CDE=180-x.
As CDE=180-x, EDF=x.
As EDF=x, DEF=180-x-z.
As DEF=180-x-z, DEO=x+z.
As triangle DEO is isosceles, DOE=180-2x-2z.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DAE=90-x-z.
But DAE=y, so 90-x-z=y, or x+y+z=90, or CBE+DAE+DFE=90.

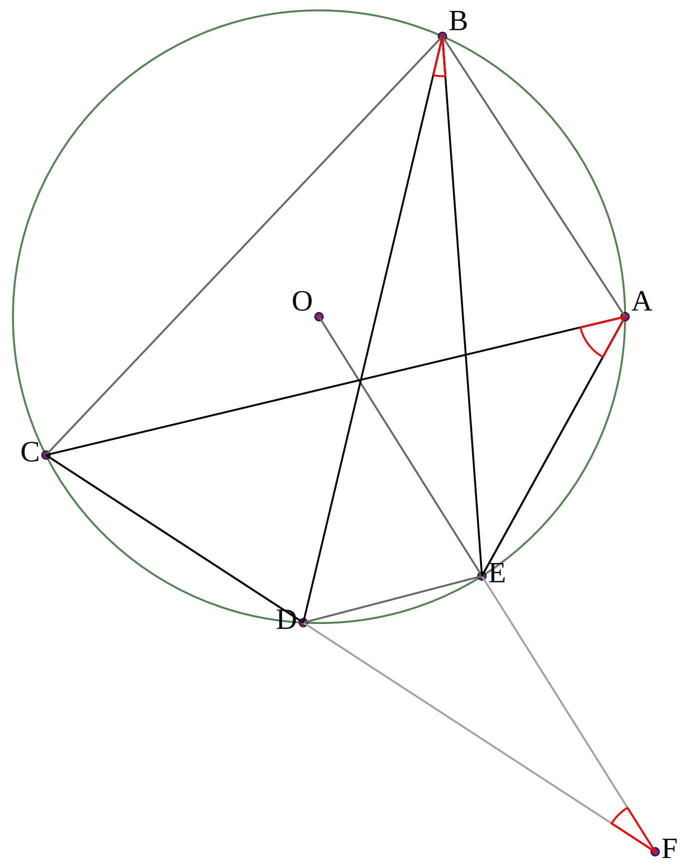
Solution to example 79



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EO.
Prove that BCE+DFE = DAE+90

Let BCE=x. Let DAE=y. Let DFE=z.
As DAE and DCE stand on the same chord, DCE=DAE, so DCE=y.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=x.
As EDF=x, DEF=180-x-z.
As triangle DEO is isosceles, DOE=2x+2z-180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=x+z-90.
But DCE=y, so x+z-90=y, or x+z=y+90, or BCE+DFE=DAE+90.

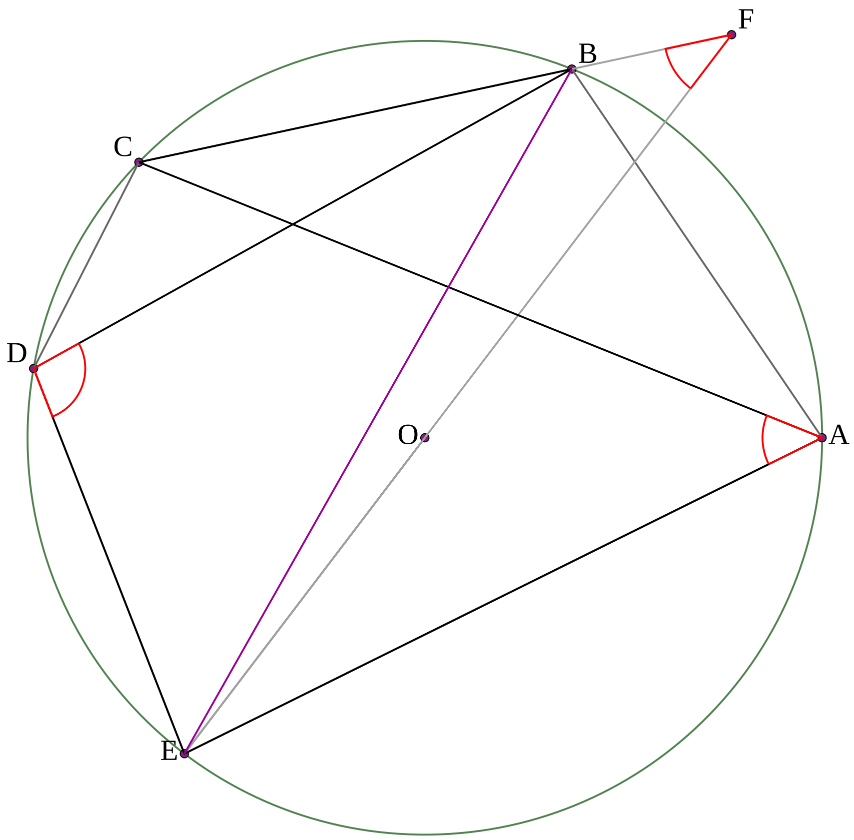
Solution to example 80



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of DC and EO.
Prove that DBE+CAE+DFE = 90

Let DBE=x. Let CAE=y. Let DFE=z.
As CAED is a cyclic quadrilateral, CDE=180-CAE, so CDE=180-y.
As CDE=180-y, EDF=y.
As EDF=y, DEF=180-y-z.
As DEF=180-y-z, DEO=y+z.
As triangle DEO is isosceles, DOE=180-2y-2z.
As DOE is at the center of a circle on the same chord as DBE, DOE=2DBE, so DBE=90-y-z.
But DBE=x, so 90-y-z=x, or x+y+z=90, or DBE+CAE+DFE=90.

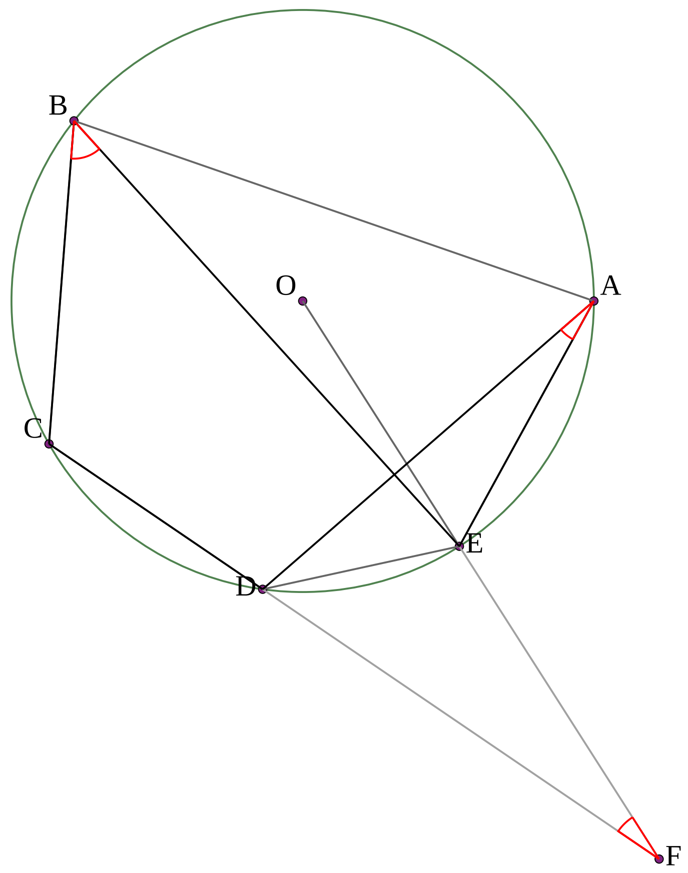
Solution to example 81



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BC and EO.
Prove that BDE+BFE = CAE+90

Draw line BE.
Let BDE=x. Let CAE=y. Let BFE=z.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=180-x.
As CAE and CBE stand on the same chord, CBE=CAE, so CBE=y.
As CBE=y, EBF=180-y.
As EBF=180-y, BEF=y-z.
As triangle BEO is isosceles, BOE=2z-2y+180.
As BOE is at the center of a circle on the same chord as BAE, BOE=2BAE, so BAE=z-y+90.
But BAE=180-x, so z-y+90=180-x, or x+z=y+90, or BDE+BFE=CAE+90.

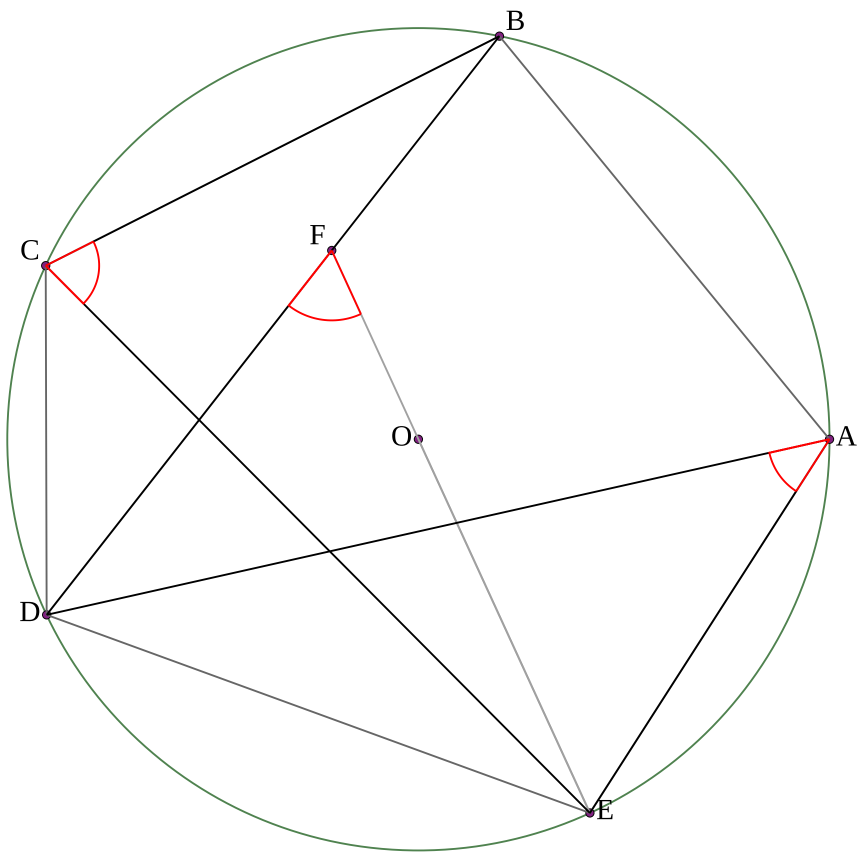
Solution to example 82



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of CD and EO.
Prove that CBE+DAE+DFE = 90

Let CBE=x. Let DAE=y. Let DFE=z.
As CBED is a cyclic quadrilateral, CDE=180-CBE, so CDE=180-x.
As CDE=180-x, EDF=x.
As EDF=x, DEF=180-x-z.
As DEF=180-x-z, DEO=x+z.
As triangle DEO is isosceles, DOE=180-2x-2z.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DAE=90-x-z.
But DAE=y, so 90-x-z=y, or x+y+z=90, or CBE+DAE+DFE=90.

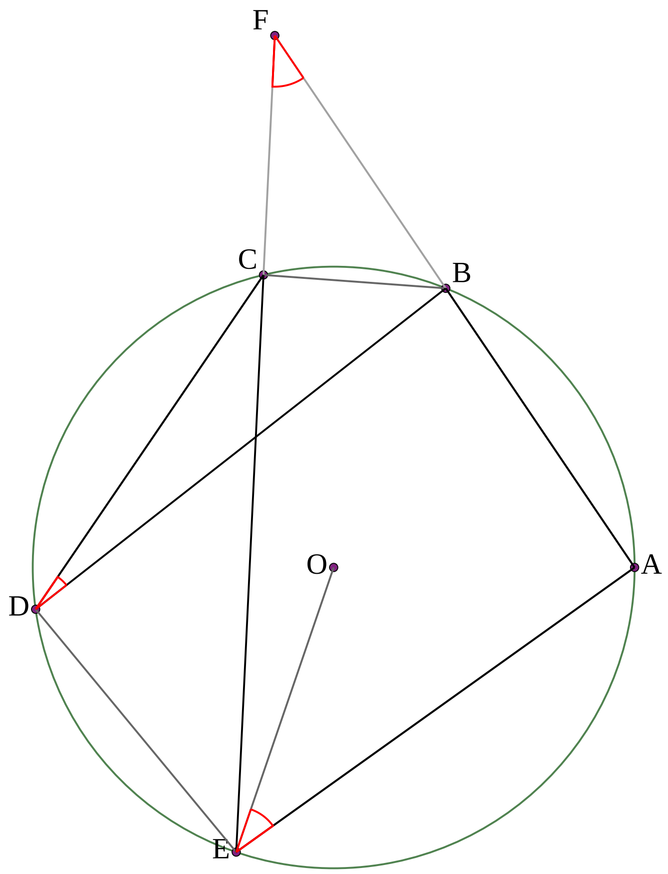
Solution to example 83



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of BD and EO.
Prove that BCE+DFE = DAE+90

Let BCE=x. Let DAE=y. Let DFE=z.
As DAE and DCE stand on the same chord, DCE=DAE, so DCE=y.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=x.
As EDF=x, DEF=180-x-z.
As triangle DEO is isosceles, DOE=2x+2z-180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=x+z-90.
But DCE=y, so x+z-90=y, or x+z=y+90, or BCE+DFE=DAE+90.

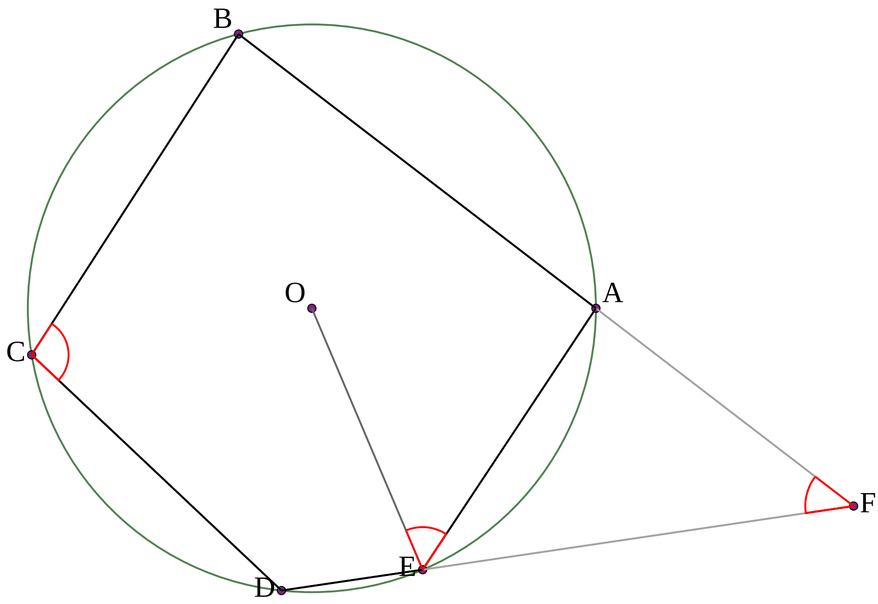
Solution to example 84



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and BA.
Prove that BDC+AEO+BFC = 90

Let BDC=x. Let AEO=y. Let BFC=z.
Let BCF=w.
As BFC=z, CBF=180-z-w.
As CBF=180-z-w, CBA=z+w.
As triangle AEO is isosceles, EAO=y.
As BCF=w, BCE=180-w.
As BCEA is a cyclic quadrilateral, BAE=180-BCE, so BAE=w.
As EAO=y, OAB=w-y.
As triangle BAO is isosceles, ABO=w-y.
As ABC=z+w, CBO=y+z.
As triangle CBO is isosceles, BOC=180-2y-2z.
As BOC is at the center of a circle on the same chord as BDC, BOC=2BDC, so BDC=90-y-z.
But BDC=x, so 90-y-z=x, or x+y+z=90, or BDC+AEO+BFC=90.

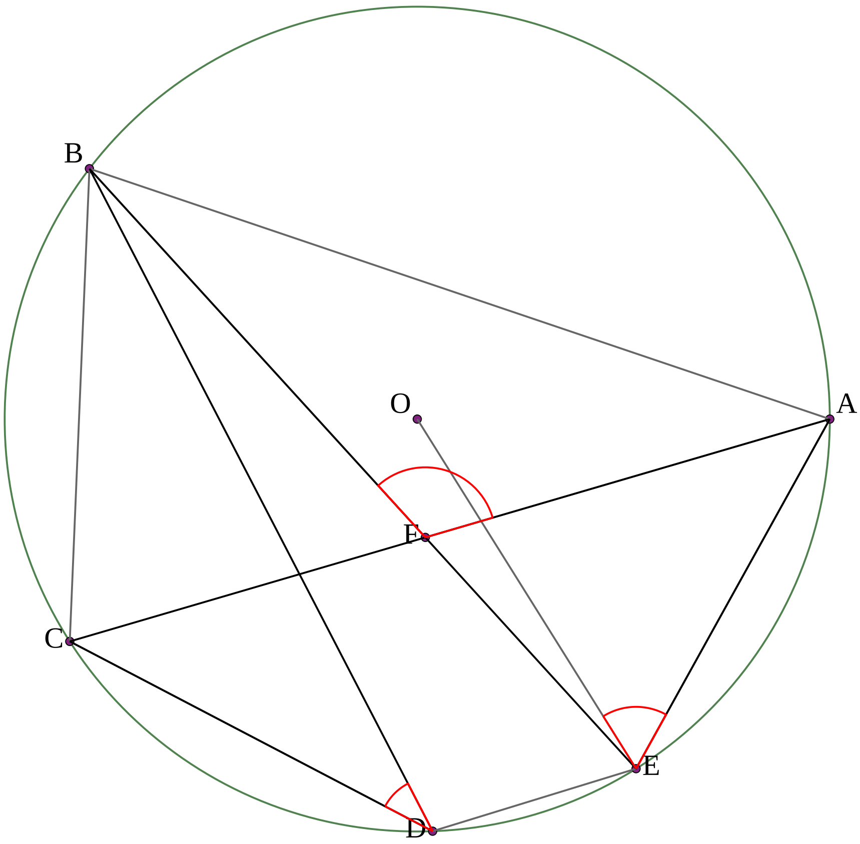
Solution to example 85



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and BA.
Prove that BCD+AFE = AEO+90

Let BCD=x. Let AEO=y. Let AFE=z.
Let AEF=w.
As AFE=z, EAF=180-z-w.
As EAF=180-z-w, EAB=z+w.
As triangle AEO is isosceles, EAO=y.
As BAE=z+w, BAO=z+w-y.
As triangle BAO is isosceles, AOB=2y-2z-2w+180.
As AEO=y, OEF=y+w.
As FEO=y+w, OED=180-y-w.
As triangle DEO is isosceles, DOE=2y+2w-180.
As triangle AEO is isosceles, AOE=180-2y.
As DOE=2y+2w-180, DOA=2w.
As AOB=2y-2z-2w+180, BOD=2z-2y+180.
As BOD is at the center of a circle on the same chord, but in the opposite direction to BCD, BOD=360-2BCD, so BCD=y-z+90.
But BCD=x, so y-z+90=x, or y+90=x+z, or AEO+90=BCD+AFE.

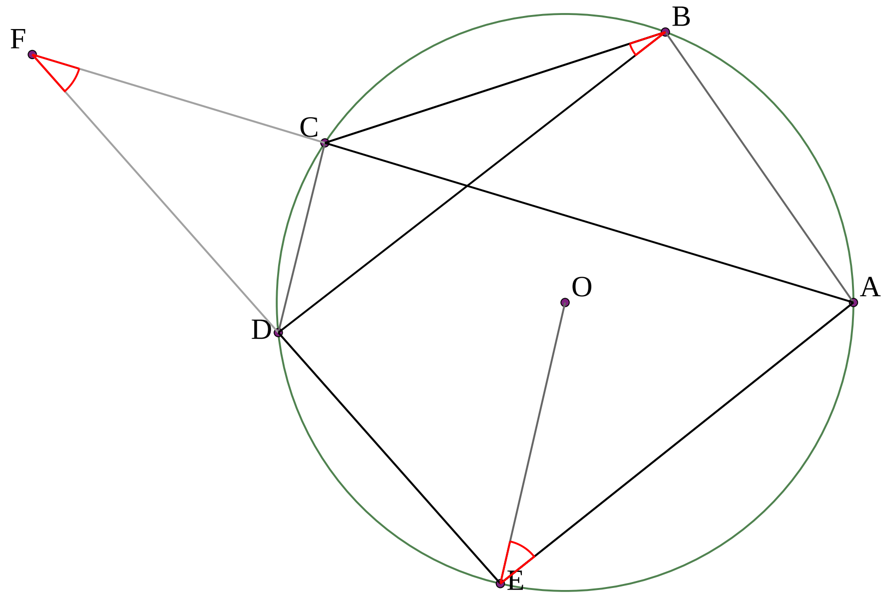
Solution to example 86



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and CA.
Prove that BDC+AFB = AEO+90

Let BDC=x. Let AEO=y. Let AFB=z.
As BDC and BAC stand on the same chord, BAC=BDC, so BAC=x.
As BAF=x, ABF=180-x-z.
As triangle AEO is isosceles, AOE=180-2y.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-y.
But ABF=180-x-z, so 90-y=180-x-z, or x+z=y+90, or BDC+AFB=AEO+90.

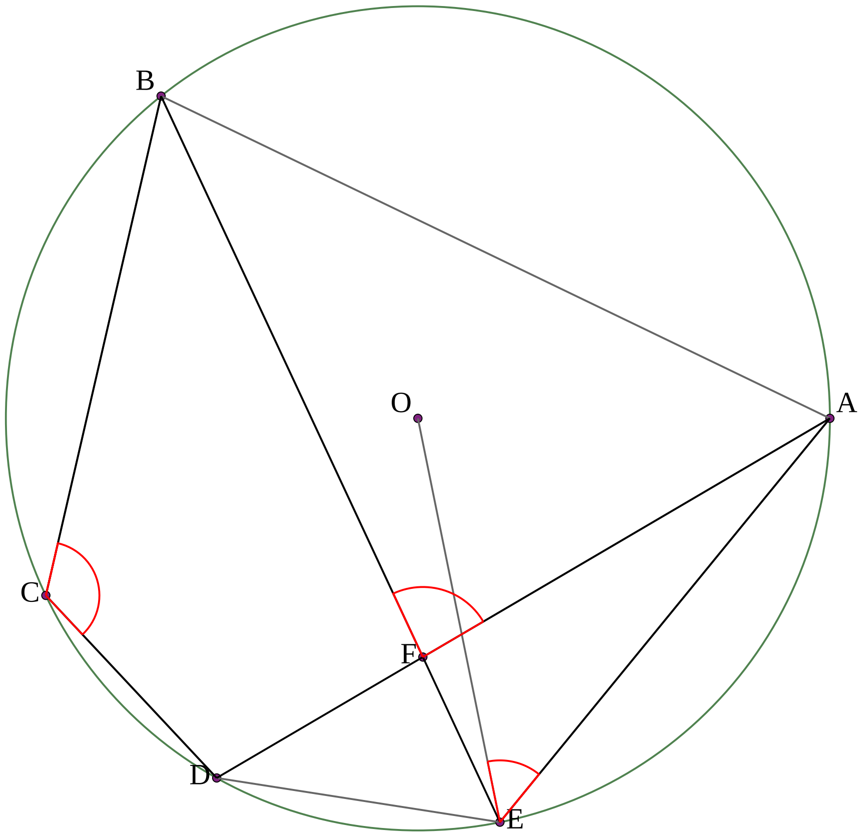
Solution to example 87



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and CA.
Prove that CBD+AEO+CFD = 90

Let CBD=x. Let AEO=y. Let CFD=z.
As triangle AEO is isosceles, EAO=y.
Let CDF=w.
As CDF=w, CDE=180-w.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=w.
As EAO=y, OAC=w-y.
As triangle CAO is isosceles, AOC=2y-2w+180.
As AOC is at the center of a circle on the same chord, but in the opposite direction to ABC, AOC=360-2ABC, so ABC=w-y+90.
As CFD=z, DCF=180-z-w.
As DCF=180-z-w, DCA=z+w.
As ACD and ABD stand on the same chord, ABD=ACD, so ABD=z+w.
As ABD=z+w, ABC=x+z+w.
But ABC=w-y+90, so x+z+w=w-y+90, or x+y+z=90, or CBD+AEO+CFD=90.

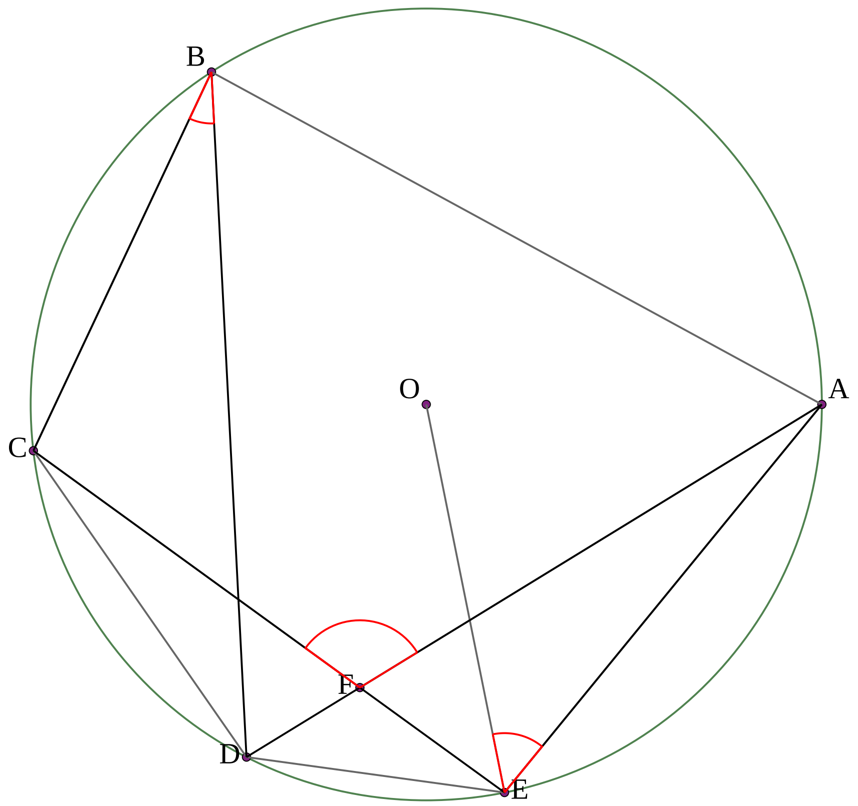
Solution to example 88



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA.
Prove that BCD+AEO = AFB+90

Let BCD=x. Let AEO=y. Let AFB=z.
As BCDA is a cyclic quadrilateral, BAD=180-BCD, so BAD=180-x.
As BAF=180-x, ABF=x-z.
As triangle AEO is isosceles, AOE=180-2y.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-y.
But ABF=x-z, so 90-y=x-z, or z+90=x+y, or AFB+90=BCD+AEO.

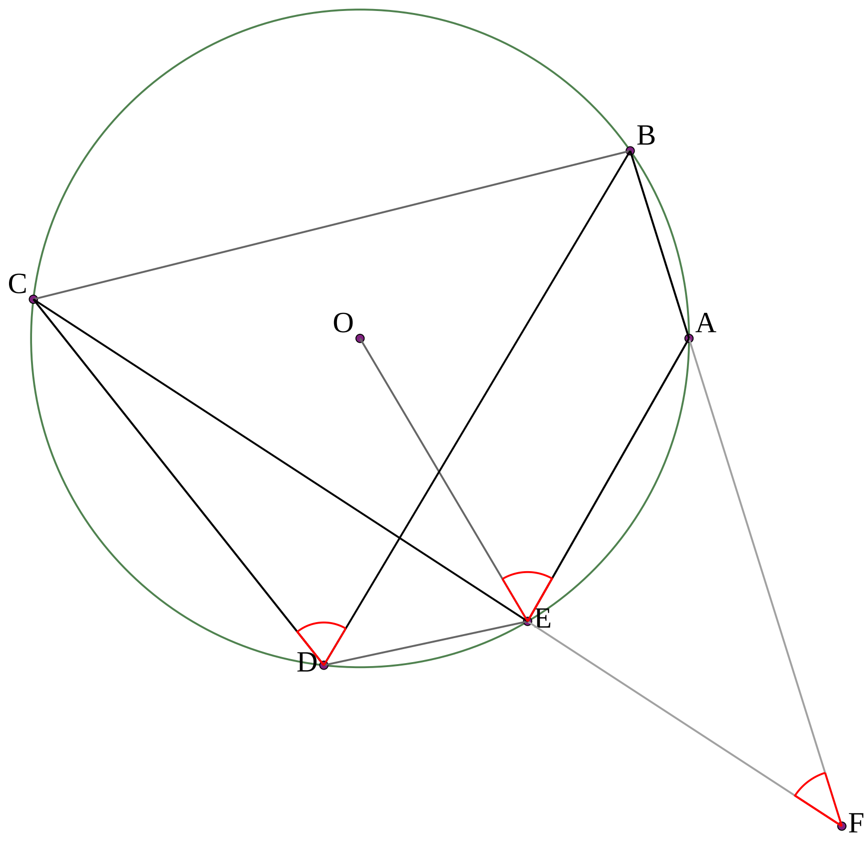
Solution to example 89



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA.
Prove that CBD+AFC = AEO+90

Let CBD=x. Let AEO=y. Let AFC=z.
As CBD and CED stand on the same chord, CED=CBD, so CED=x.
As AFC=z, AFE=180-z.
As AFE=180-z, EFD=z.
As DEF=x, EDF=180-x-z.
As triangle AEO is isosceles, AOE=180-2y.
As AOE is at the center of a circle on the same chord as ADE, AOE=2ADE, so ADE=90-y.
But EDF=180-x-z, so 90-y=180-x-z, or x+z=y+90, or CBD+AFC=AEO+90.

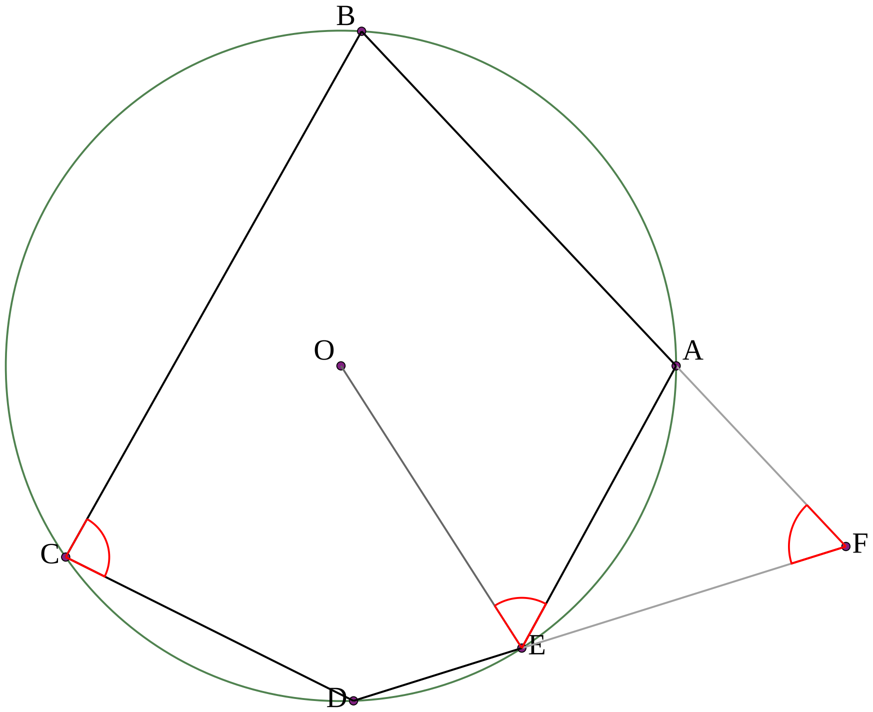
Solution to example 90



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and BA.
Prove that BDC+AEO = AFE+90

Let BDC=x. Let AEO=y. Let AFE=z.
Let AEF=w.
As AEO=y, OEF=y+w.
As FEO=y+w, OEC=180-y-w.
As triangle CEO is isosceles, COE=2y+2w-180.
As COE is at the center of a circle on the same chord, but in the opposite direction to CDE, COE=360-2CDE, so CDE=270-y-w.
As AFE=z, EAF=180-z-w.
As EAF=180-z-w, EAB=z+w.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-z-w.
As BDE=180-z-w, EDC=x-z-w+180.
But CDE=270-y-w, so x-z-w+180=270-y-w, or x+y=z+90, or BDC+AEO=AFE+90.

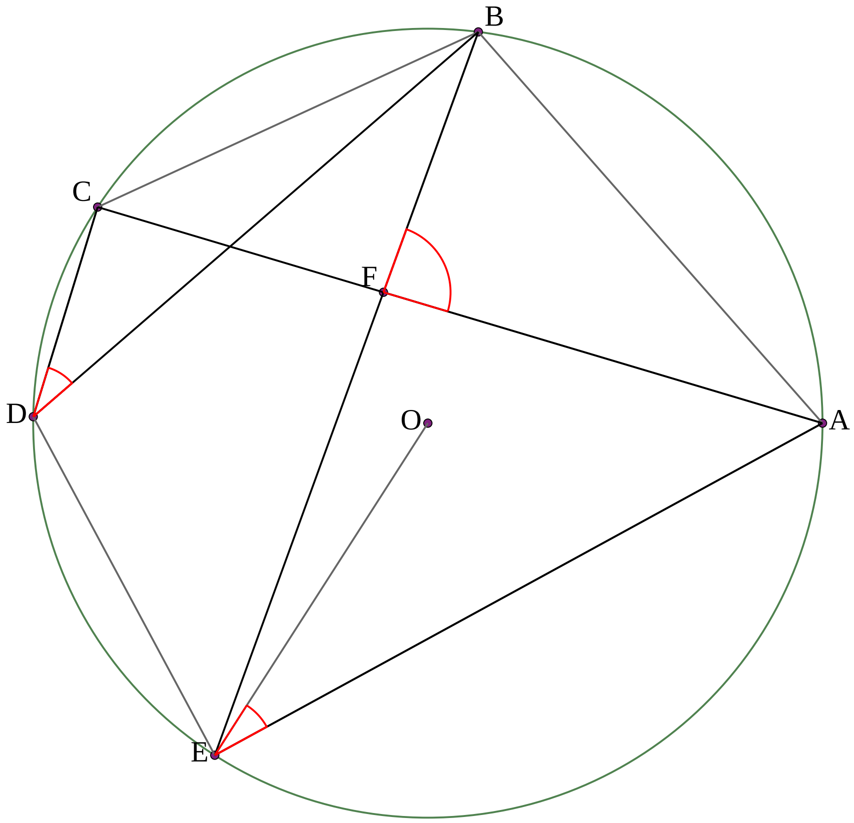
Solution to example 91



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and BA.
Prove that BCD+AFE = AEO+90

Let BCD=x. Let AEO=y. Let AFE=z.
Let AEF=w.
As AFE=z, EAF=180-z-w.
As EAF=180-z-w, EAB=z+w.
As triangle AEO is isosceles, EAO=y.
As BAE=z+w, BAO=z+w-y.
As triangle BAO is isosceles, AOB=2y-2z-2w+180.
As AEO=y, OEF=y+w.
As FEO=y+w, OED=180-y-w.
As triangle DEO is isosceles, DOE=2y+2w-180.
As triangle AEO is isosceles, AOE=180-2y.
As DOE=2y+2w-180, DOA=2w.
As AOB=2y-2z-2w+180, BOD=2y-2z+180.
As BOD is at the center of a circle on the same chord as BCD, BOD=2BCD, so BCD=y-z+90.
But BCD=x, so y-z+90=x, or y+90=x+z, or AEO+90=BCD+AFE.

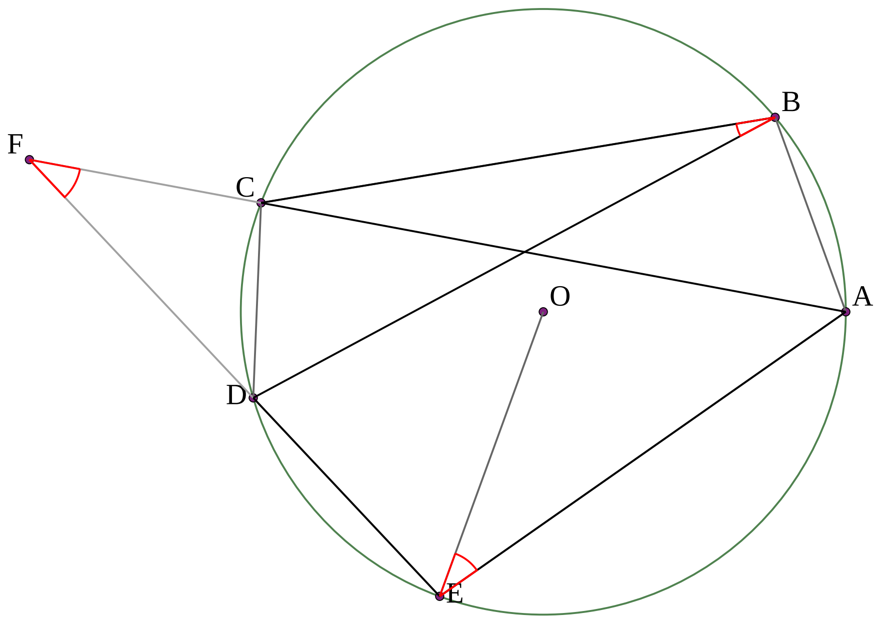
Solution to example 92



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and CA.
Prove that BDC+AFB = AEO+90

Let BDC=x. Let AEO=y. Let AFB=z.
As BDC and BAC stand on the same chord, BAC=BDC, so BAC=x.
As BAF=x, ABF=180-x-z.
As triangle AEO is isosceles, AOE=180-2y.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-y.
But ABF=180-x-z, so 90-y=180-x-z, or x+z=y+90, or BDC+AFB=AEO+90.

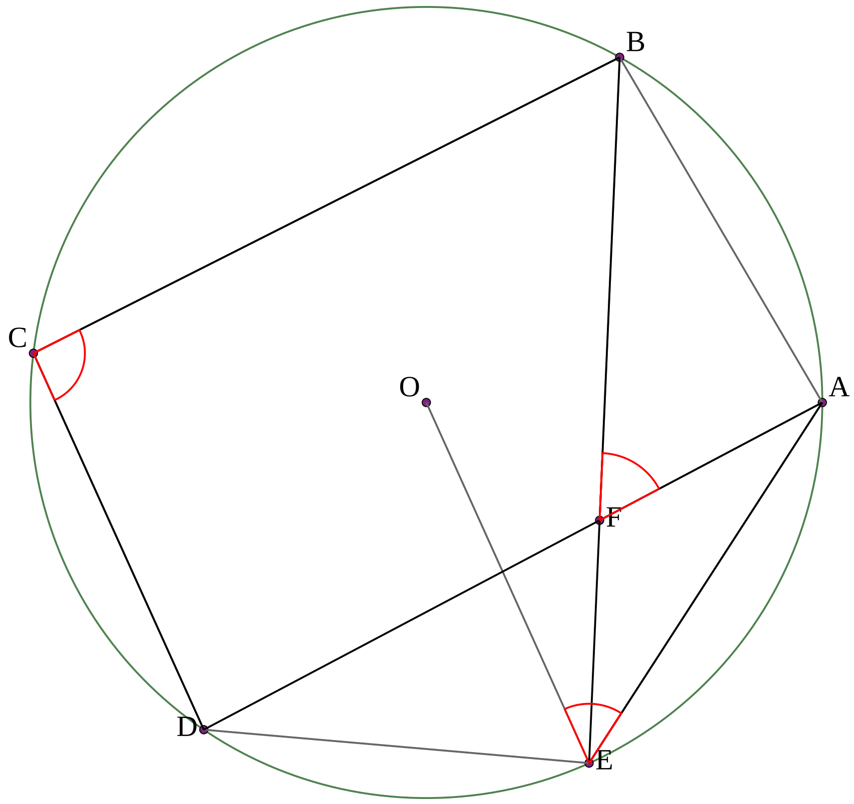
Solution to example 93



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and CA.
Prove that CBD+AEO+CFD = 90

Let CBD=x. Let AEO=y. Let CFD=z.
As triangle AEO is isosceles, EAO=y.
Let CDF=w.
As CDF=w, CDE=180-w.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=w.
As EAO=y, OAC=w-y.
As triangle CAO is isosceles, AOC=2y-2w+180.
As AOC is at the center of a circle on the same chord, but in the opposite direction to ABC, AOC=360-2ABC, so ABC=w-y+90.
As CFD=z, DCF=180-z-w.
As DCF=180-z-w, DCA=z+w.
As ACD and ABD stand on the same chord, ABD=ACD, so ABD=z+w.
As ABD=z+w, ABC=x+z+w.
But ABC=w-y+90, so x+z+w=w-y+90, or x+y+z=90, or CBD+AEO+CFD=90.

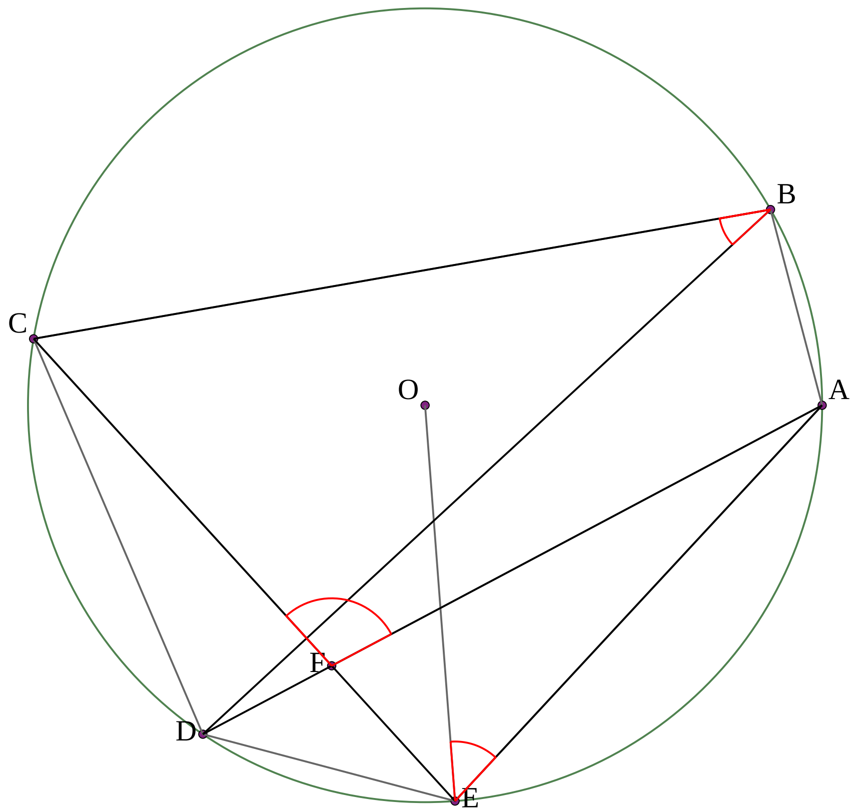
Solution to example 94



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA.
Prove that BCD+AEO = AFB+90

Let BCD=x. Let AEO=y. Let AFB=z.
As BCDA is a cyclic quadrilateral, BAD=180-BCD, so BAD=180-x.
As BAF=180-x, ABF=x-z.
As triangle AEO is isosceles, AOE=180-2y.
As AOE is at the center of a circle on the same chord as ABE, AOE=2ABE, so ABE=90-y.
But ABF=x-z, so 90-y=x-z, or z+90=x+y, or AFB+90=BCD+AEO.

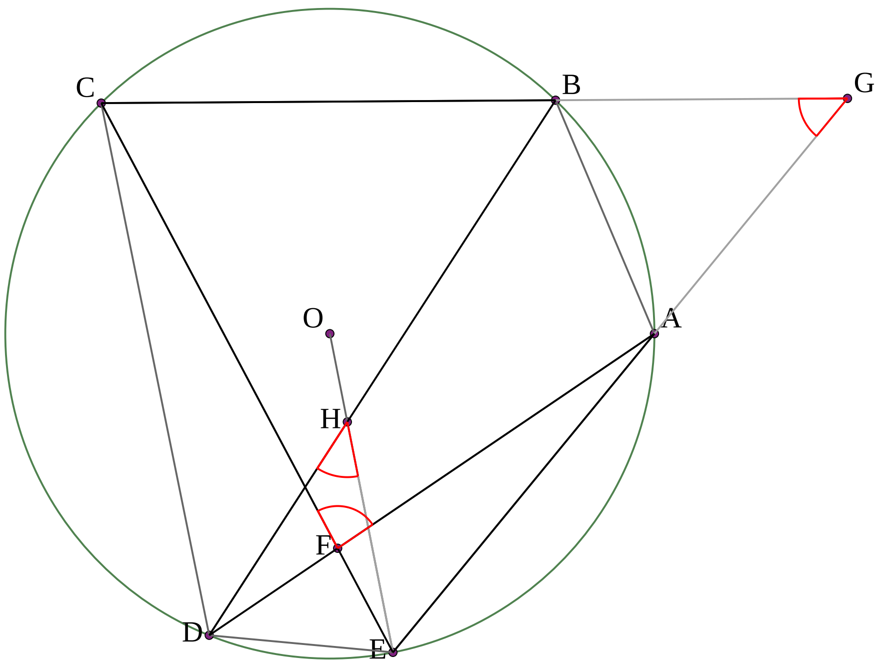
Solution to example 95



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA.
Prove that CBD+AFC = AEO+90

Let CBD=x. Let AEO=y. Let AFC=z.
As CBD and CED stand on the same chord, CED=CBD, so CED=x.
As AFC=z, AFE=180-z.
As AFE=180-z, EFD=z.
As DEF=x, EDF=180-x-z.
As triangle AEO is isosceles, AOE=180-2y.
As AOE is at the center of a circle on the same chord as ADE, AOE=2ADE, so ADE=90-y.
But EDF=180-x-z, so 90-y=180-x-z, or x+z=y+90, or CBD+AFC=AEO+90.

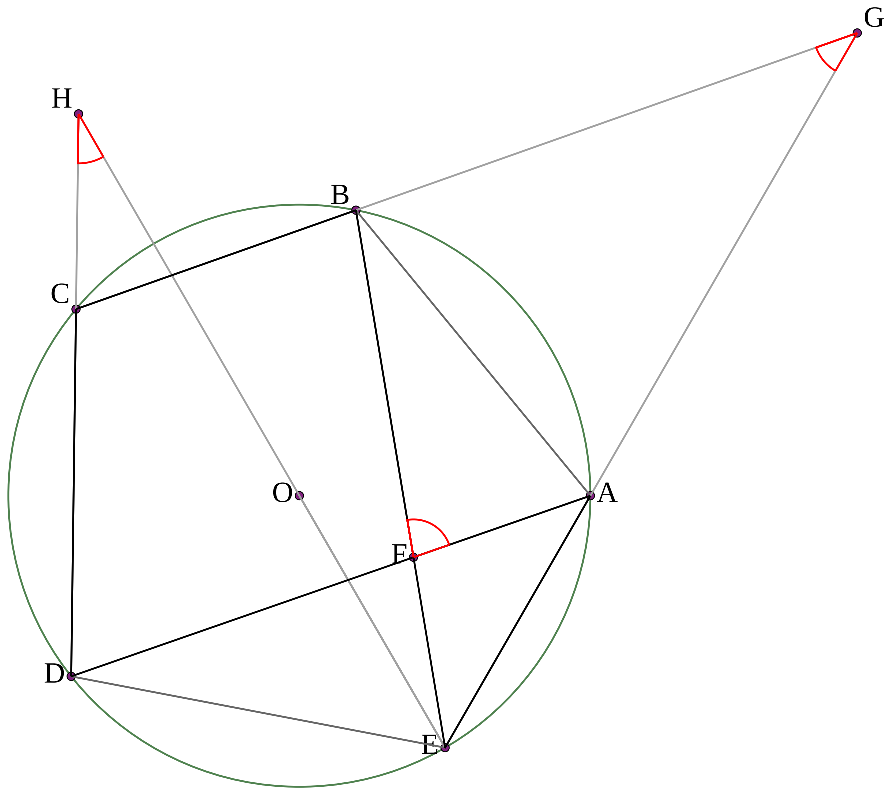
Solution to example 96



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA. Let G be the intersection of CB and AE. Let H be the intersection of BD and EO.
Prove that AFC+AGB = DHE+90

Let AFC=x. Let AGB=y. Let DHE=z.
Let ABG=w.
As ABG=w, ABC=180-w.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=w.
As AFC=x, CFD=180-x.
As CDF=w, DCF=x-w.
As AGB=y, BAG=180-y-w.
As BAG=180-y-w, BAE=y+w.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-y-w.
As EDH=180-y-w, DEH=y+w-z.
As triangle DEO is isosceles, DOE=2z-2y-2w+180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=z-y-w+90.
But DCE=x-w, so z-y-w+90=x-w, or z+90=x+y, or DHE+90=AFC+AGB.

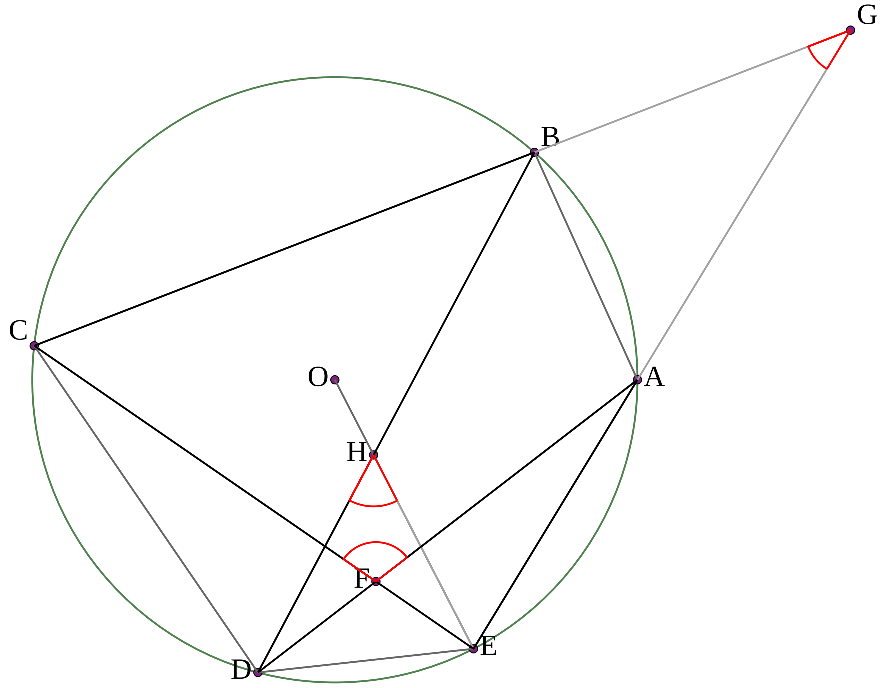
Solution to example 97



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA. Let G be the intersection of BC and AE. Let H be the intersection of CD and EO.
Prove that AFB+AGB = CHE+90

Let AFB=x. Let AGB=y. Let CHE=z.
As AFB=x, AFE=180-x.
Let EDH=w.
As DHE=z, DEH=180-z-w.
As triangle DEO is isosceles, DOE=2z+2w-180.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DAE=z+w-90.
As CDEB is a cyclic quadrilateral, CBE=180-CDE, so CBE=180-w.
As CBE=180-w, EBG=w.
As EBG=w, BEG=180-y-w.
As EAF=z+w-90, AFE=y-z+90.
But AFE=180-x, so y-z+90=180-x, or x+y=z+90, or AFB+AGB=CHE+90.

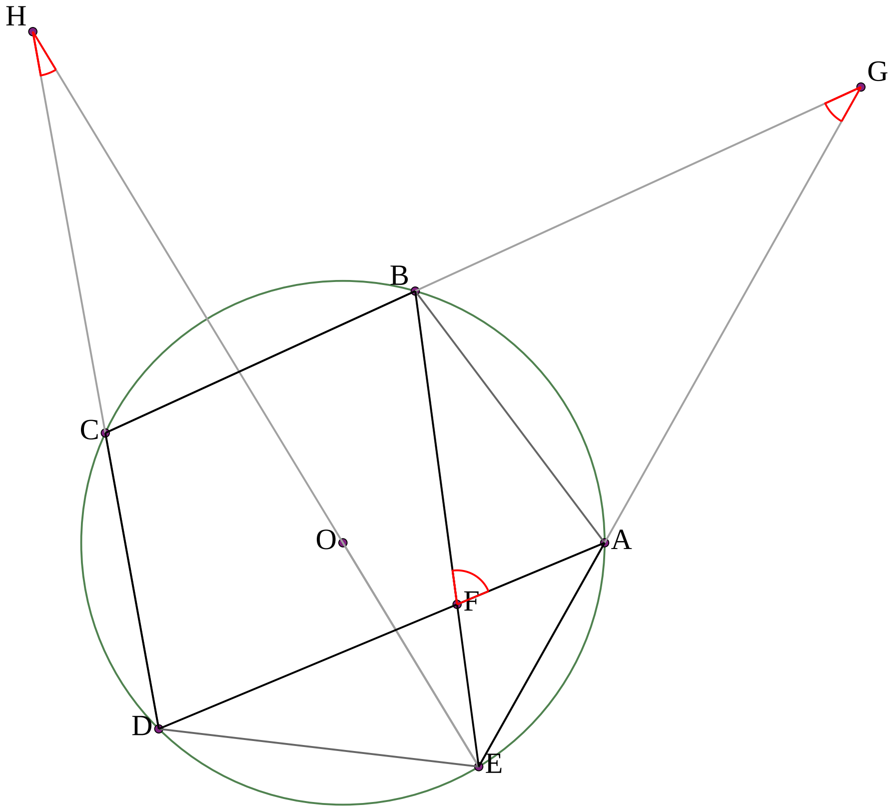
Solution to example 98



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA. Let G be the intersection of CB and AE. Let H be the intersection of BD and EO.
Prove that AFC+AGB = DHE+90

Let AFC=x. Let AGB=y. Let DHE=z.
Let ABG=w.
As ABG=w, ABC=180-w.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=w.
As AFC=x, CFD=180-x.
As CDF=w, DCF=x-w.
As AGB=y, BAG=180-y-w.
As BAG=180-y-w, BAE=y+w.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-y-w.
As EDH=180-y-w, DEH=y+w-z.
As triangle DEO is isosceles, DOE=2z-2y-2w+180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=z-y-w+90.
But DCE=x-w, so z-y-w+90=x-w, or z+90=x+y, or DHE+90=AFC+AGB.

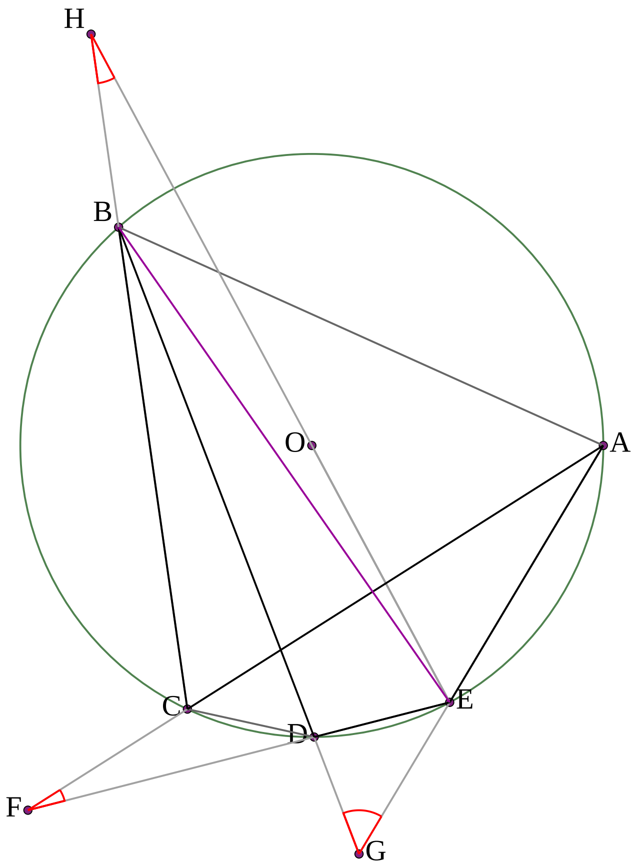
Solution to example 99



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA. Let G be the intersection of BC and AE. Let H be the intersection of CD and EO.
Prove that AFB+AGB = CHE+90

Let AFB=x. Let AGB=y. Let CHE=z.
As AFB=x, AFE=180-x.
Let EDH=w.
As DHE=z, DEH=180-z-w.
As triangle DEO is isosceles, DOE=2z+2w-180.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DAE=z+w-90.
As CDEB is a cyclic quadrilateral, CBE=180-CDE, so CBE=180-w.
As CBE=180-w, EBG=w.
As EBG=w, BEG=180-y-w.
As EAF=z+w-90, AFE=y-z+90.
But AFE=180-x, so y-z+90=180-x, or x+y=z+90, or AFB+AGB=CHE+90.

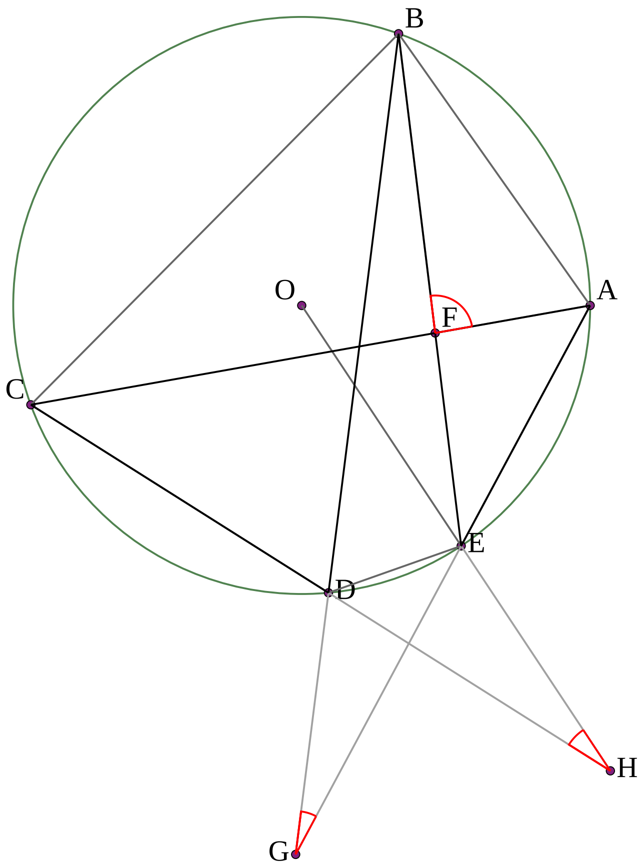
Solution to example 100



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of ED and CA. Let G be the intersection of DB and AE. Let H be the intersection of BC and EO.
Prove that CFD+DGE+BHE = 90

Draw line BE.
Let CFD=x. Let DGE=y. Let BHE=z.
Let EBH=w.
As BHE=z, BEH=180-z-w.
As triangle BEO is isosceles, BOE=2z+2w-180.
As BOE is at the center of a circle on the same chord, but in the opposite direction to BDE, BOE=360-2BDE, so BDE=270-z-w.
As BDE=270-z-w, EDG=z+w-90.
As EDG=z+w-90, DEG=270-y-z-w.
As EBH=w, EBC=180-w.
As CBE and CAE stand on the same chord, CAE=CBE, so CAE=180-w.
As EAF=180-w, AEF=w-x.
As AED=w-x, DEG=x-w+180.
But DEG=270-y-z-w, so x-w+180=270-y-z-w, or x+y+z=90, or CFD+DGE+BHE=90.

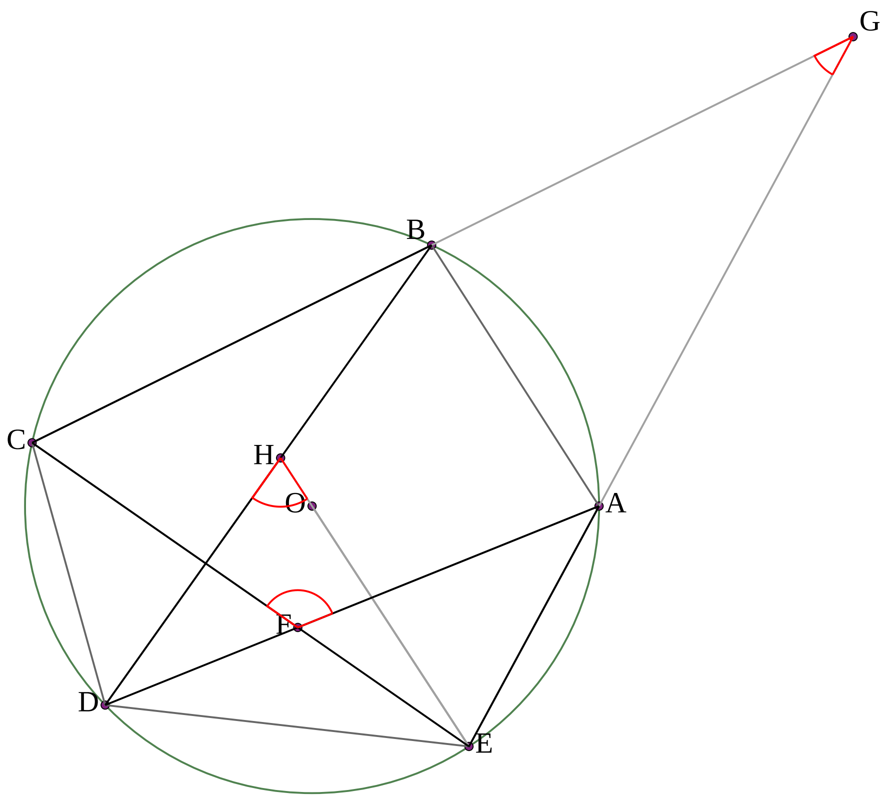
Solution to example 101



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and CA. Let G be the intersection of BD and AE. Let H be the intersection of DC and EO.
Prove that AFB+DHE = DGE+90

Let AFB=x. Let DGE=y. Let DHE=z.
Let EDH=w.
As DHE=z, DEH=180-z-w.
As DEH=180-z-w, DEO=z+w.
As triangle DEO is isosceles, DOE=180-2z-2w.
As DOE is at the center of a circle on the same chord as DBE, DOE=2DBE, so DBE=90-z-w.
As EBG=90-z-w, BEG=z+w-y+90.
As EDH=w, EDC=180-w.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=w.
As AFB=x, AFE=180-x.
As EAF=w, AEF=x-w.
As AEB=x-w, BEG=w-x+180.
But BEG=z+w-y+90, so w-x+180=z+w-y+90, or y+90=x+z, or DGE+90=AFB+DHE.

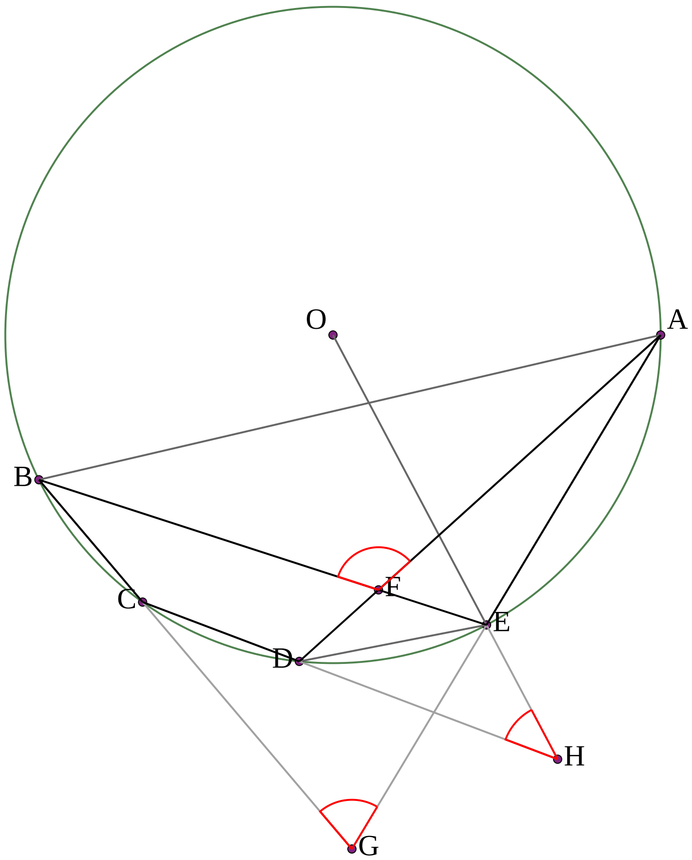
Solution to example 102



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EC and DA. Let G be the intersection of CB and AE. Let H be the intersection of BD and EO.
Prove that AFC+AGB = DHE+90

Let AFC=x. Let AGB=y. Let DHE=z.
Let ABG=w.
As ABG=w, ABC=180-w.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=w.
As AFC=x, CFD=180-x.
As CDF=w, DCF=x-w.
As AGB=y, BAG=180-y-w.
As BAG=180-y-w, BAE=y+w.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-y-w.
As EDH=180-y-w, DEH=y+w-z.
As triangle DEO is isosceles, DOE=2z-2y-2w+180.
As DOE is at the center of a circle on the same chord as DCE, DOE=2DCE, so DCE=z-y-w+90.
But DCE=x-w, so z-y-w+90=x-w, or z+90=x+y, or DHE+90=AFC+AGB.

Solution to example 103



Let ABCDE be a cyclic pentagon with center O. Let F be the intersection of EB and DA. Let G be the intersection of BC and AE. Let H be the intersection of CD and EO.
Prove that AFB+DHE = CGE+90

Let AFB=x. Let CGE=y. Let DHE=z.
Let EDH=w.
As DHE=z, DEH=180-z-w.
As DEH=180-z-w, DEO=z+w.
As triangle DEO is isosceles, DOE=180-2z-2w.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DAE=90-z-w.
As AFB=x, AFE=180-x.
As EAF=90-z-w, AEF=x+z+w-90.
As EDH=w, EDC=180-w.
As CDEB is a cyclic quadrilateral, CBE=180-CDE, so CBE=w.
As EBG=w, BEG=180-y-w.
As FEG=180-y-w, FEA=y+w.
But AEF=x+z+w-90, so y+w=x+z+w-90, or y+90=x+z, or CGE+90=AFB+DHE.