A Set of 200 Automatically Generated Cyclic Polygon Angle Problems

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A random selection of automatically generated problems involving cyclic hexagons, heptagons and octagons is presented. The hexagon problems relate four, rather than three angles. Each problem can take one of three forms. In one form, certain angles are specified numerically, and teh numeric value of an unknown angle is required. In a second form, unknown angles are specified as indeterminates, and the unknown angle is required in terms of these indeterminates. The third form states a theorem relating the angles and requires a proof.

The problems are presented in approximate order of difficulty, as measured by the complexity of the machine generated human-readable proofs.

An introduction describes techniques which can be used in the solution of the problems. Answers for the non-proof problems are given, and step by step solutions, or proofs for odd numbered problems.

Introduction

Here is a set of geometry puzzles. In addition to the angles of a triangle adding up to 180 degrees, you'll need to use the facts that opposite angles of a cyclic quadrilateral add up to 180 degrees and that two angles at the circumference which sit on the same chord are equal.

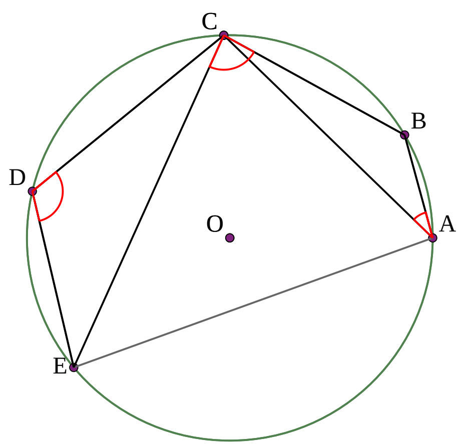


Fig.1: BAC=x, BCE=y, CDE=z

For example, in Fig. 1, ACDE is a cyclic quadrilateral, hence angles CAE=180-z. But ABCE is also a cyclic quadrilateral, hence BCE+BAE=180. As BAE=BAC+CAE, we have x+y+z=180.

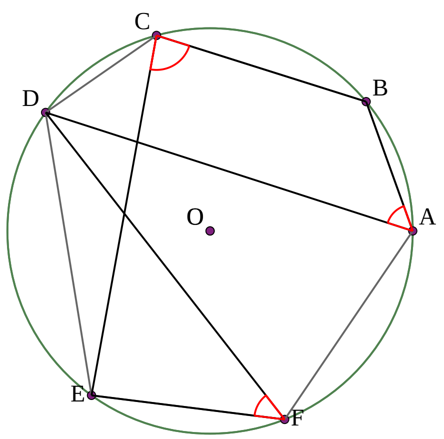


Fig.2: DFE=x, BCE=y, BAD=z

In Fig. 2, DFE and DCE both stand on the chord DE, and hence have the same value. So DCE=x. Hence DCB=x+y. But DCBA is a cyclic quadrilateral so x+y+z=180.

For problems which contain lines through the circle center, you may need the additional result that the angle on a chord at the center of the circle is twice the angle on that chord at the circumference.

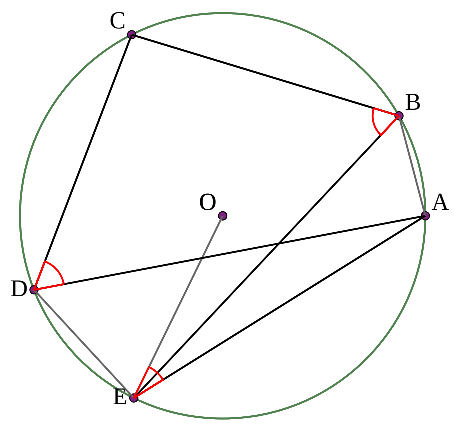
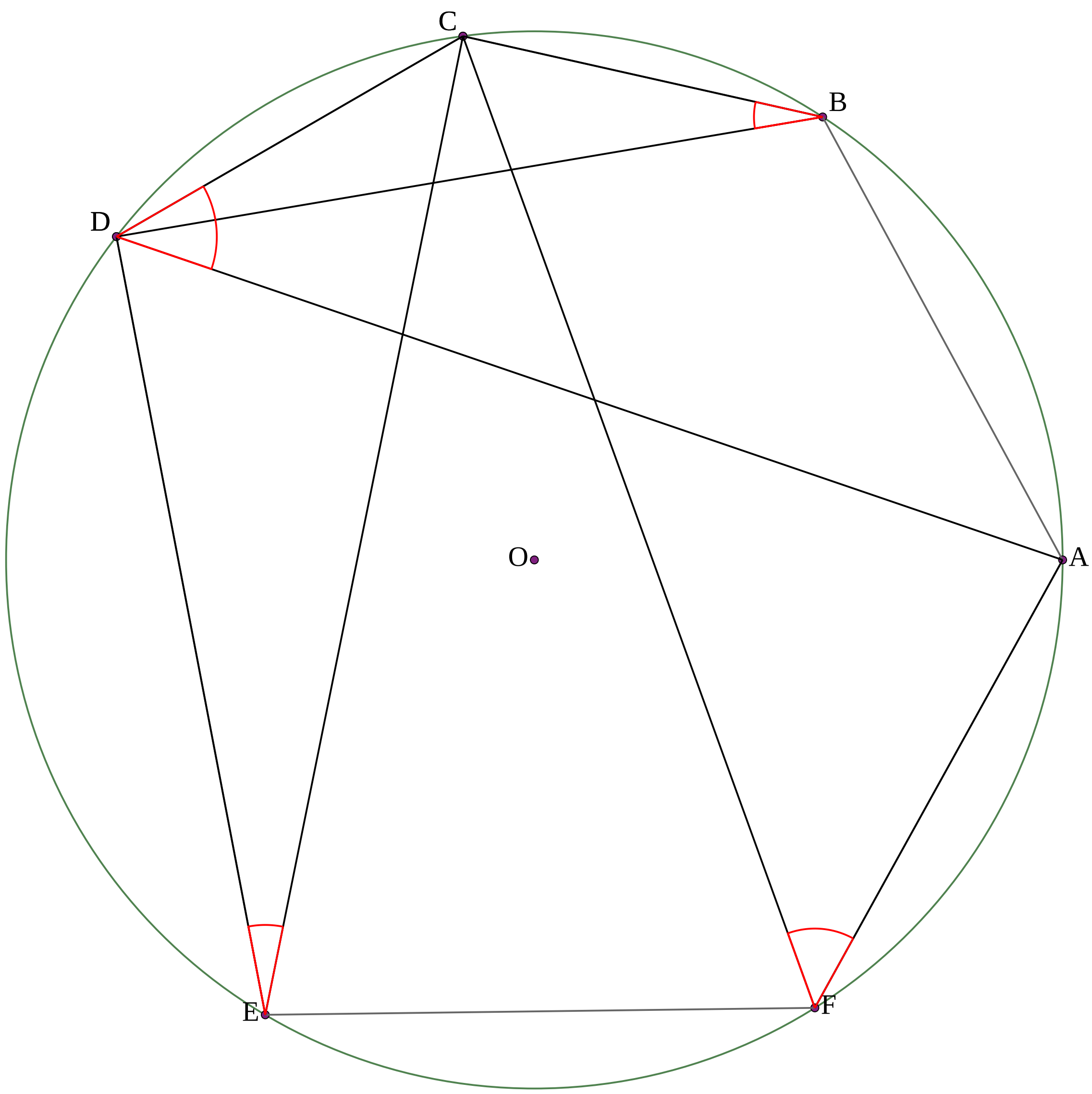


Fig.3: OEA=x, ADC=y, CBE=z

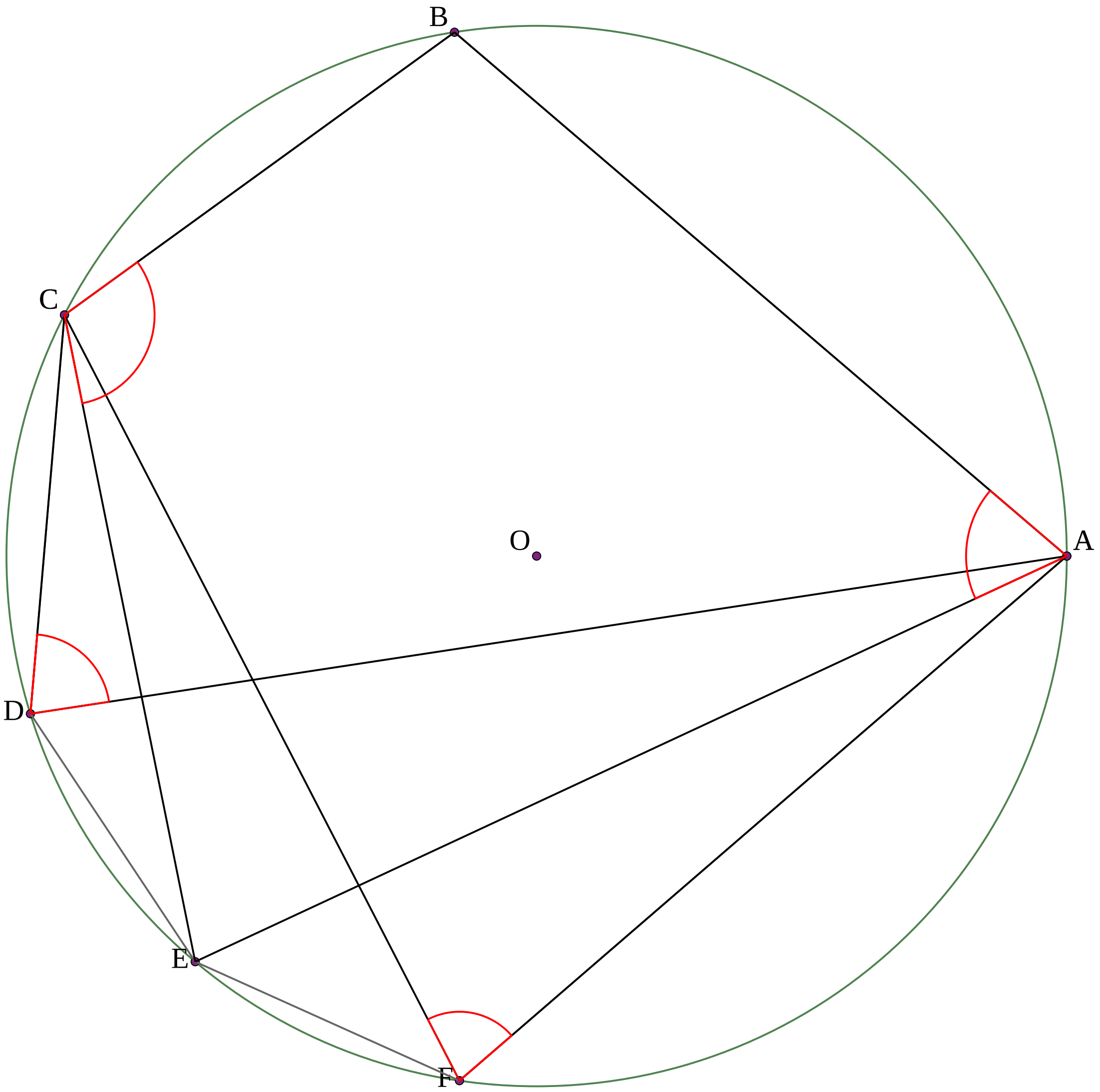
In Fig. 3, if angle OEA is x, then so is angle OAE, as the triangle is isosceles. So angle AOE = 180-2x. Hence ADE=90-x. If ADE=y, then CDE=90-x+y. But BCDE is a cyclic quadrilateral, so CDE+CBE=180. If CBE=z, then we have 90-x+y+z=180, or y+z=x+90.

Example 1



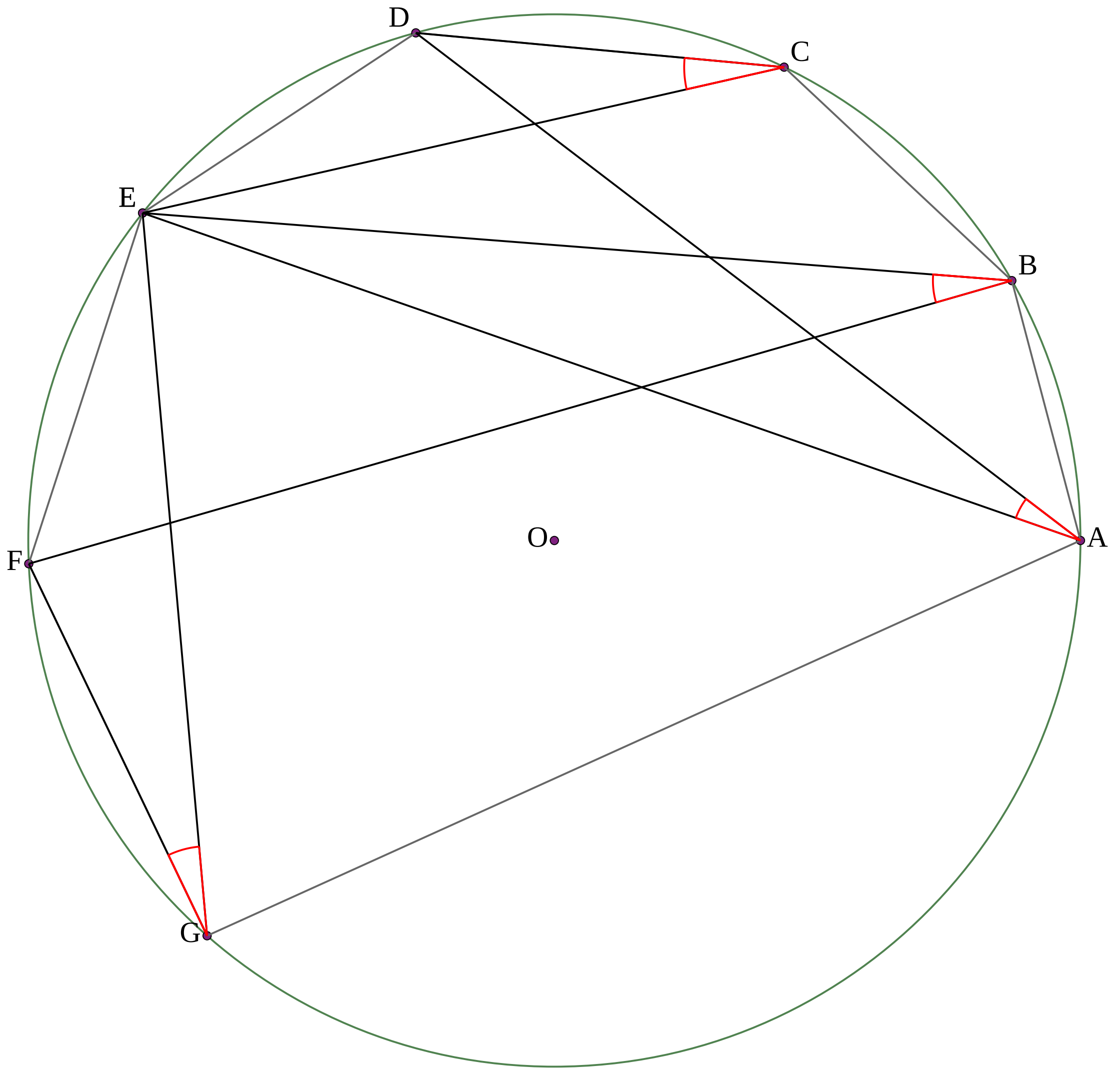
Let ABCDEF be a cyclic hexagon with center O.
Angle DEC = 22°. Angle ADC = 49°. Angle CFA = 49°.
Find angle DBC.

Example 2



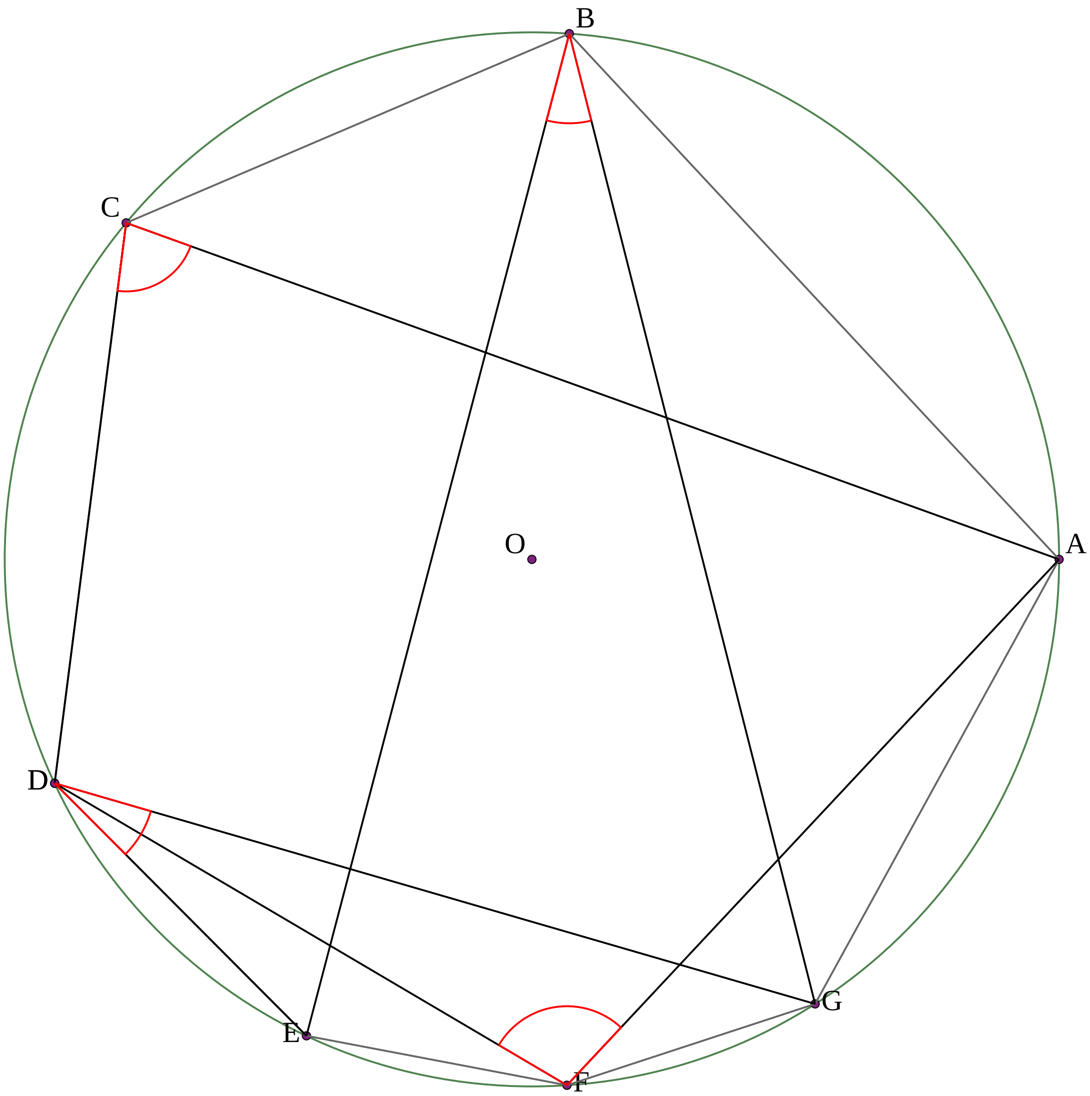
Let ABCDEF be a cyclic hexagon with center O.
Angle AFC = x. Angle CDA = y. Angle EAB = z.
Find angle ECB.

Example 3



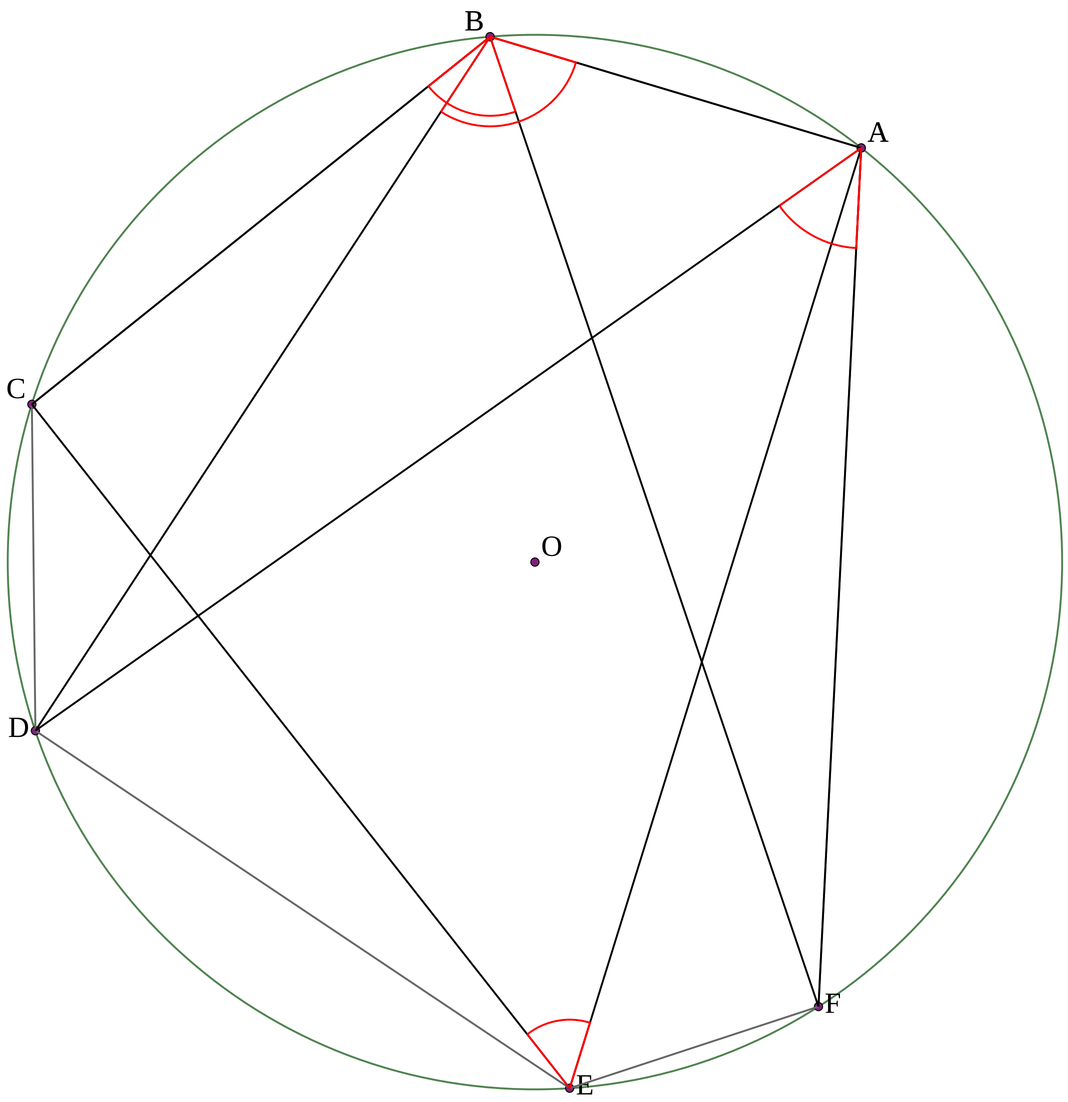
Let ABCDEFG be a cyclic heptagon with center O.
Angle FGE = 21°. Angle ECD = 18°. Angle DAE = 18°.
Find angle EBF.

Example 4



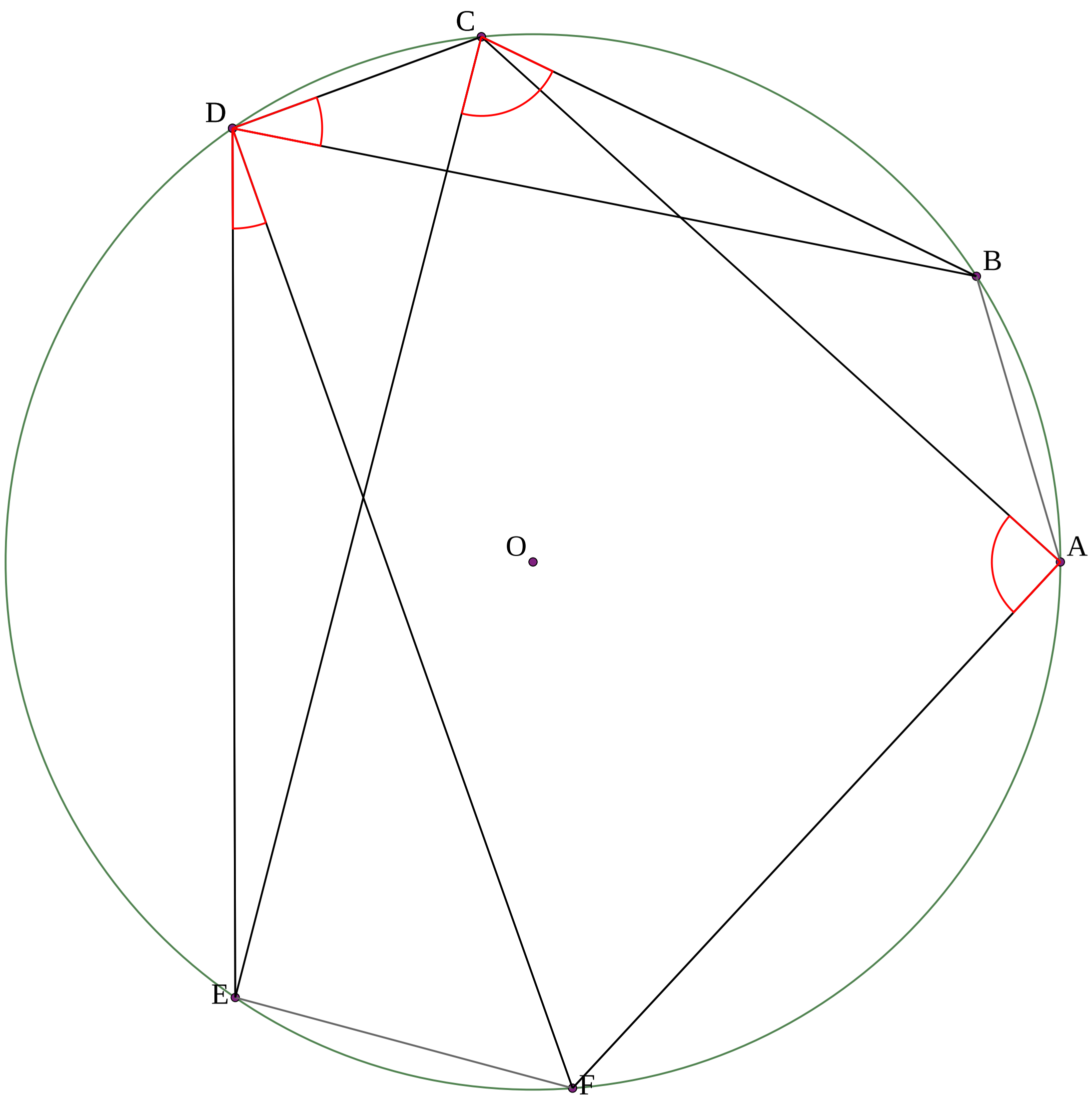
Let ABCDEFG be a cyclic heptagon with center O.
Angle EBG = x. Angle AFD = y. Angle EDG = z.
Find angle DCA.

Example 5



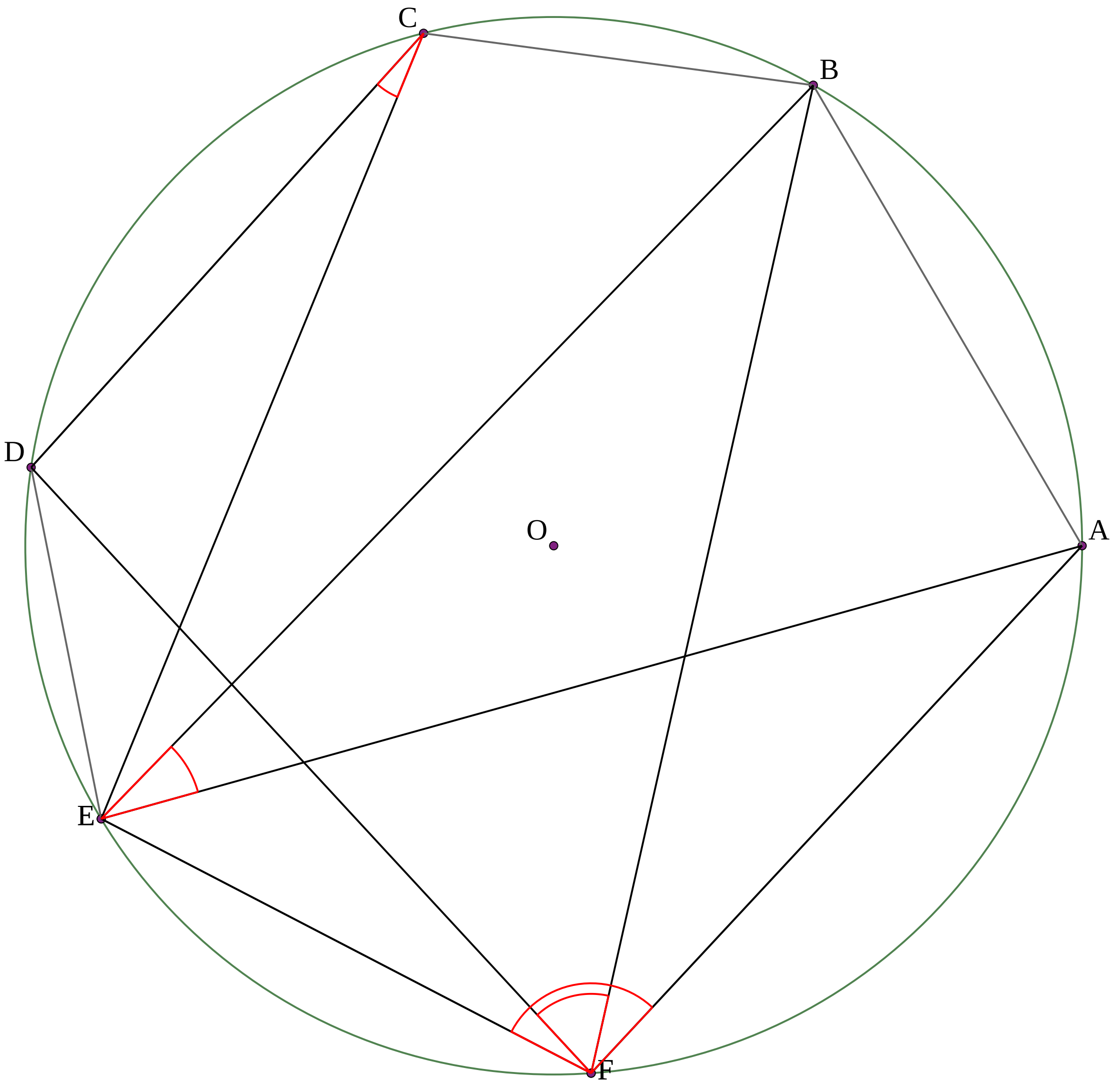
Let ABCDEF be a cyclic hexagon with center O.
Angle FAD = 52°. Angle AEC = 55°. Angle ABD = 107°.
Find angle CBF.

Example 6



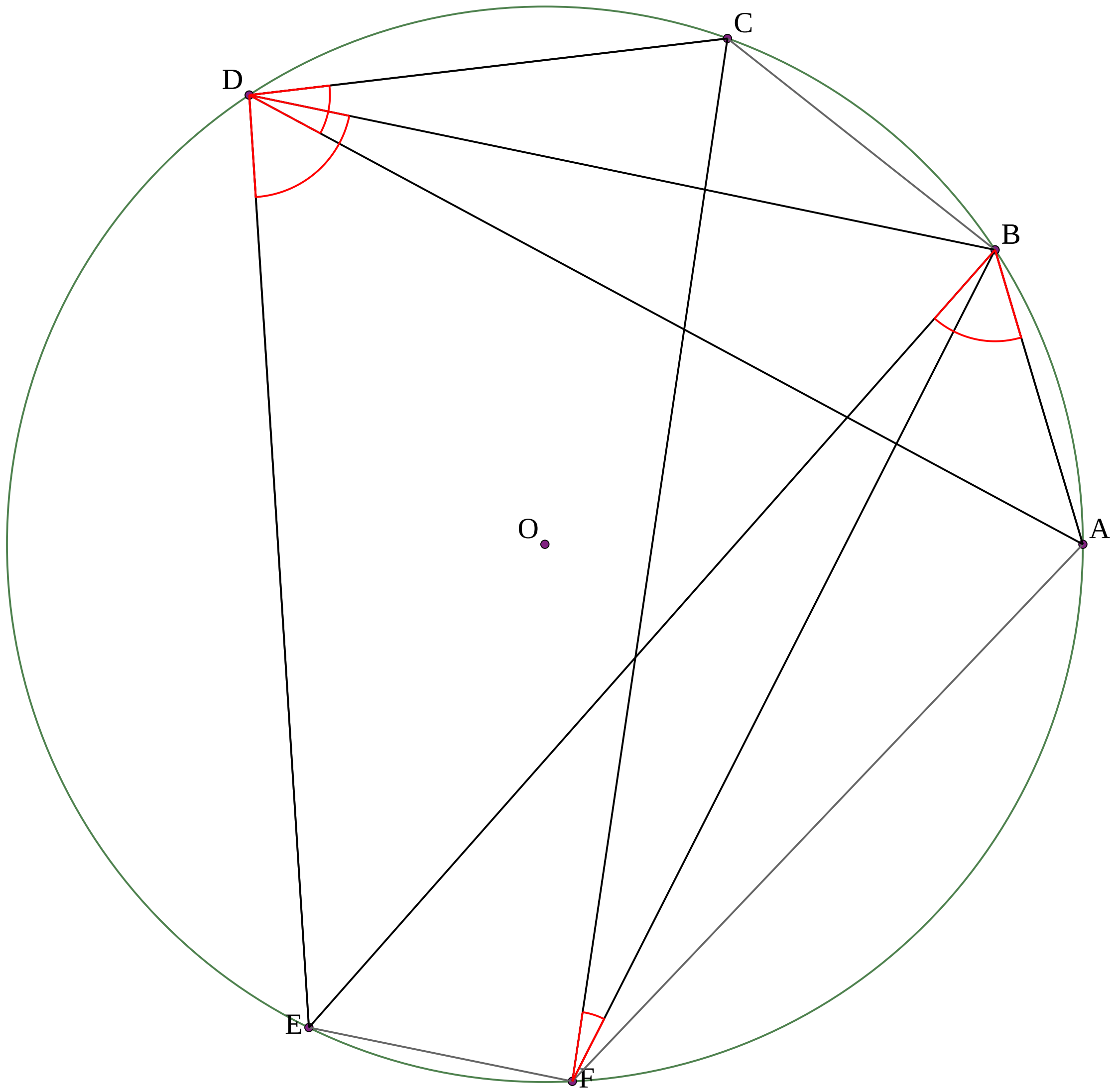
Let ABCDEF be a cyclic hexagon with center O.
Angle FAC = 89°. Angle BCE = 79°. Angle BDC = 31°.
Find angle EDF.

Example 7



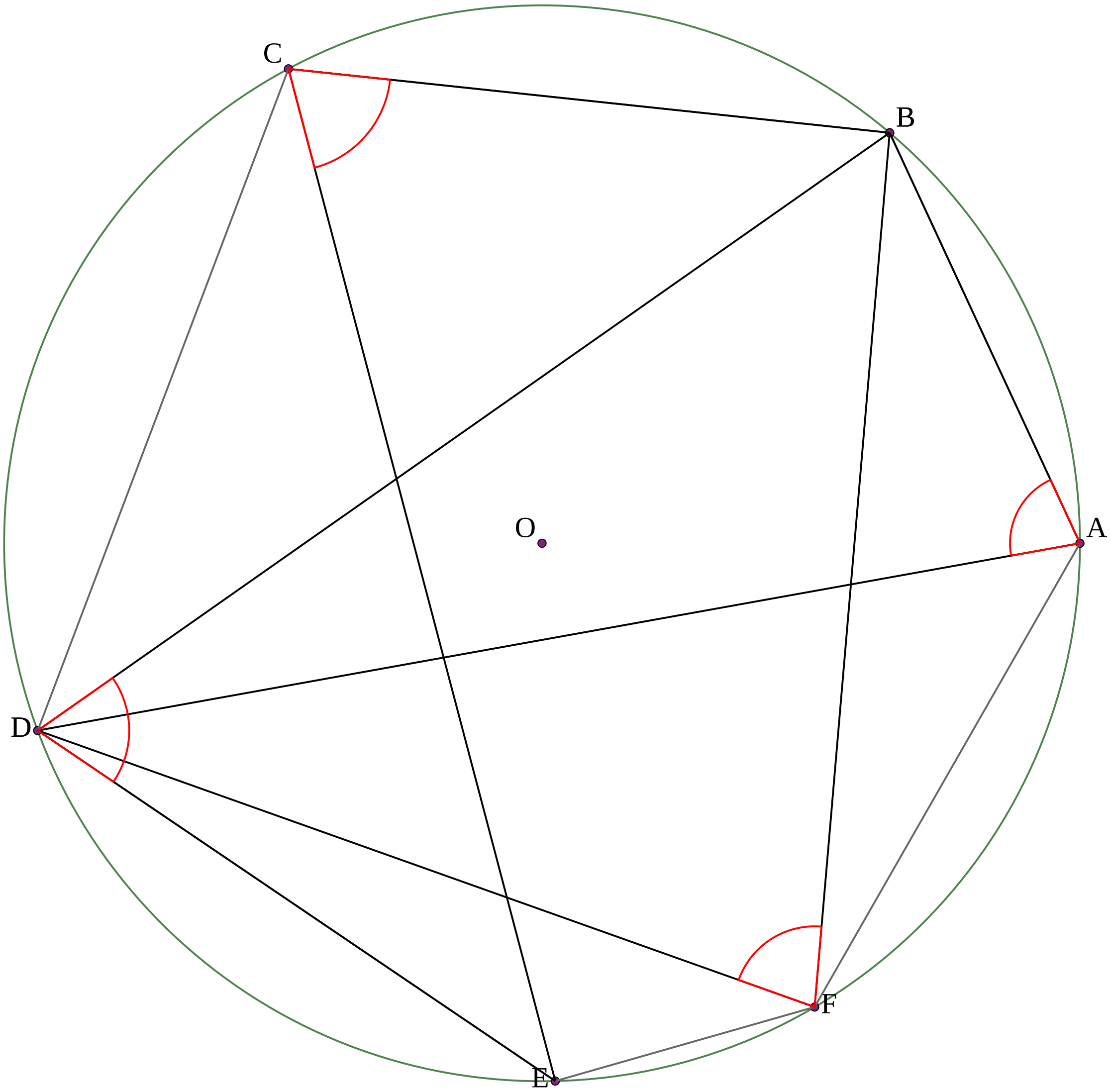
Let ABCDEF be a cyclic hexagon with center O.
Angle ECD = 20°. Angle DFB = 55°. Angle BEA = 30°.
Find angle EFA.

Example 8



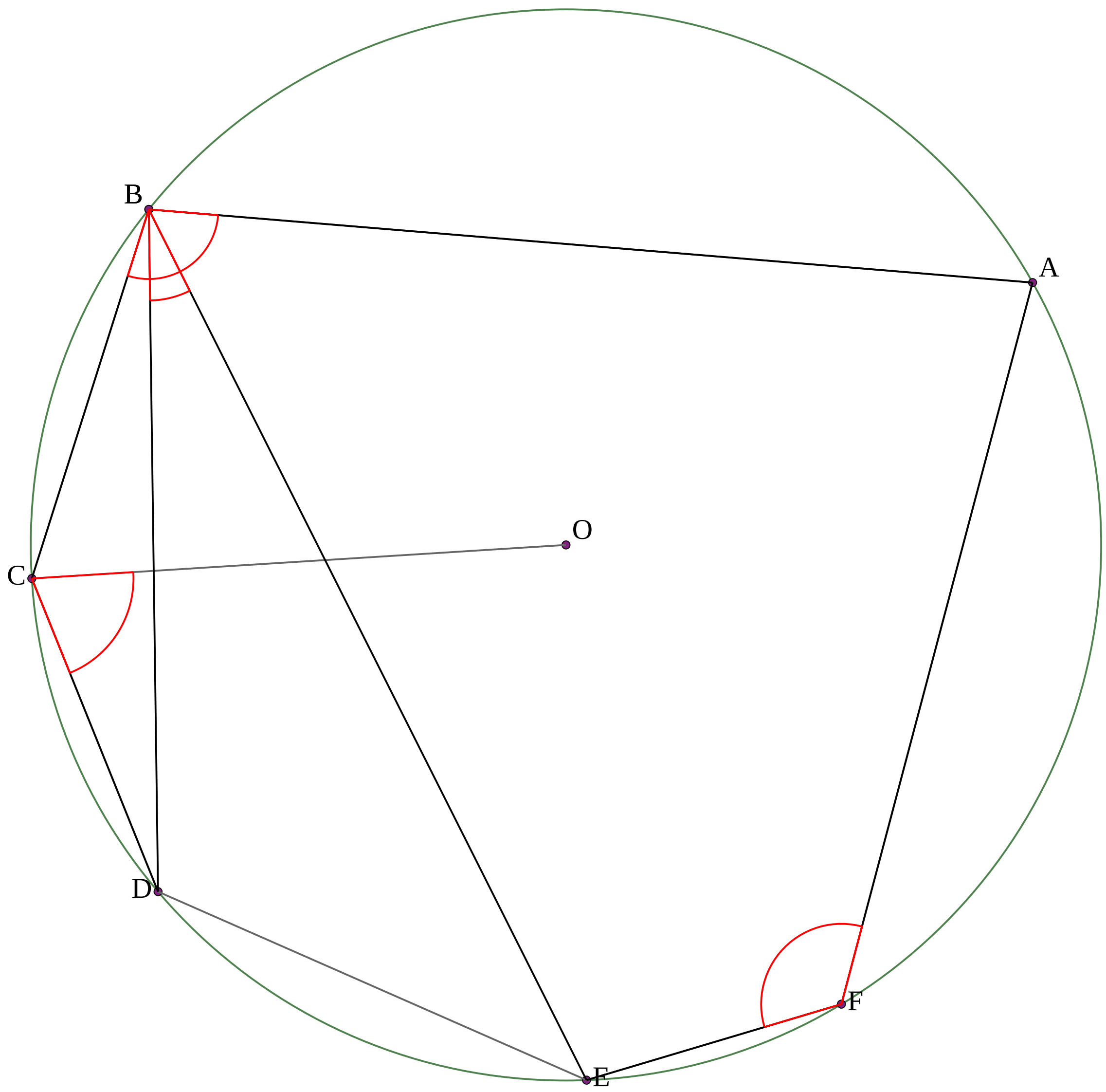
Let ABCDEF be a cyclic hexagon with center O.
Angle BFC = x. Angle EDB = y. Angle ABE = z.
Find angle CDA.

Example 9



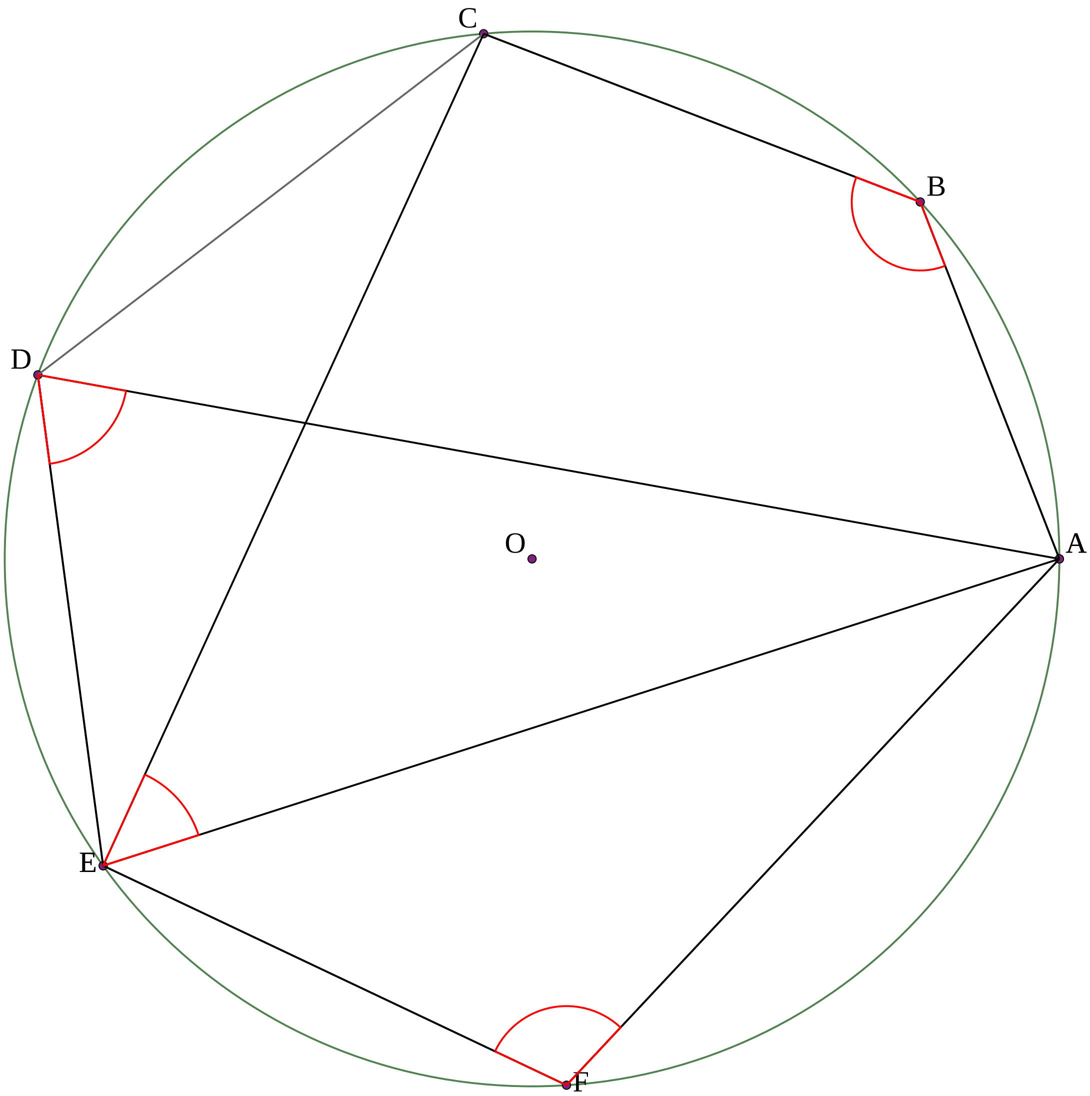
Let ABCDEF be a cyclic hexagon with center O.
Prove that BFD+BDE = BAD+BCE

Example 10



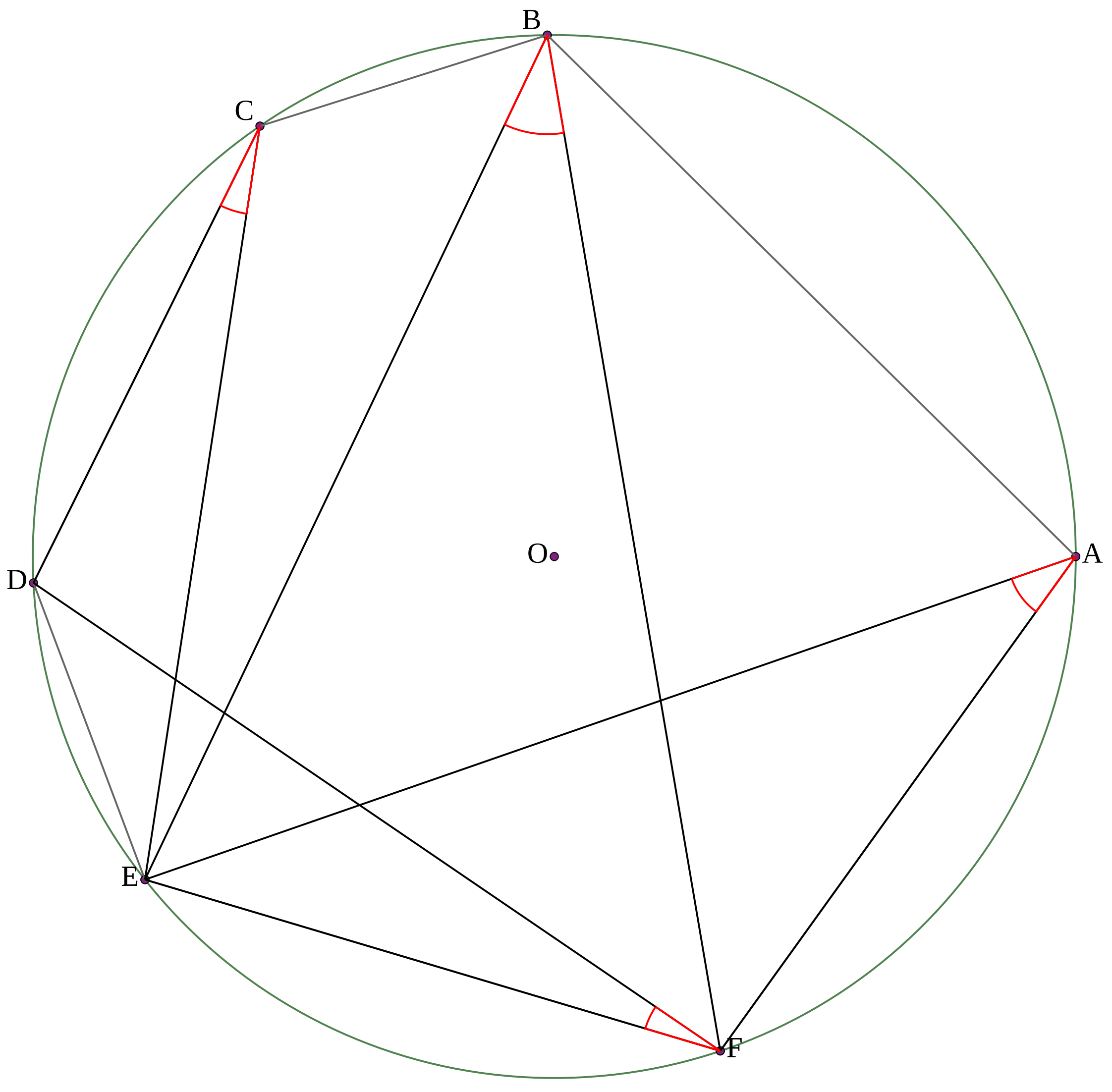
Let ABCDEF be a cyclic hexagon with center O.
Angle EBD = x. Angle AFE = y. Angle DCO = z.
Find angle CBA.

Example 11



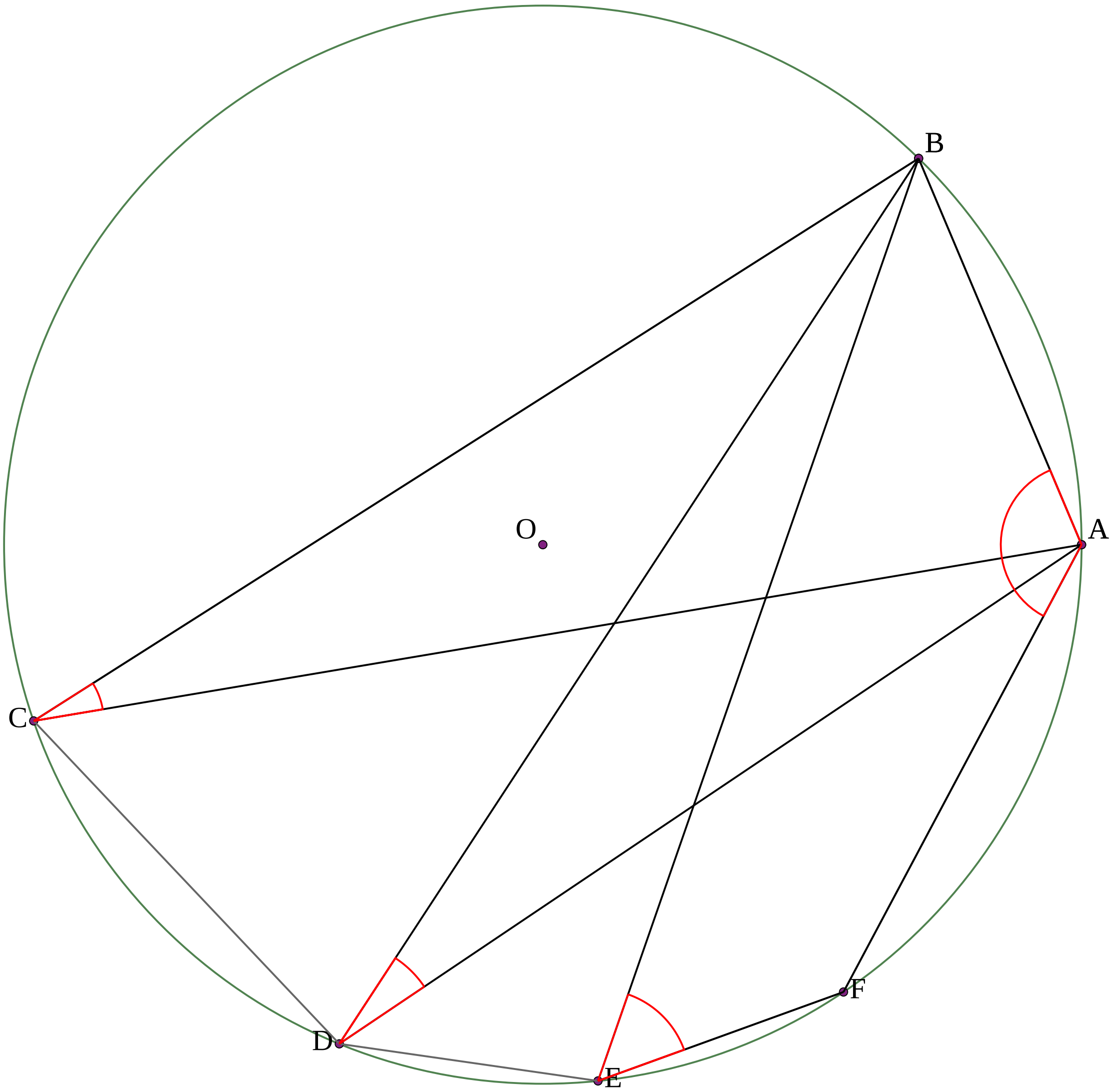
Let ABCDEF be a cyclic hexagon with center O.
Prove that ABC+AFE+ADE+AEC = 360

Example 12



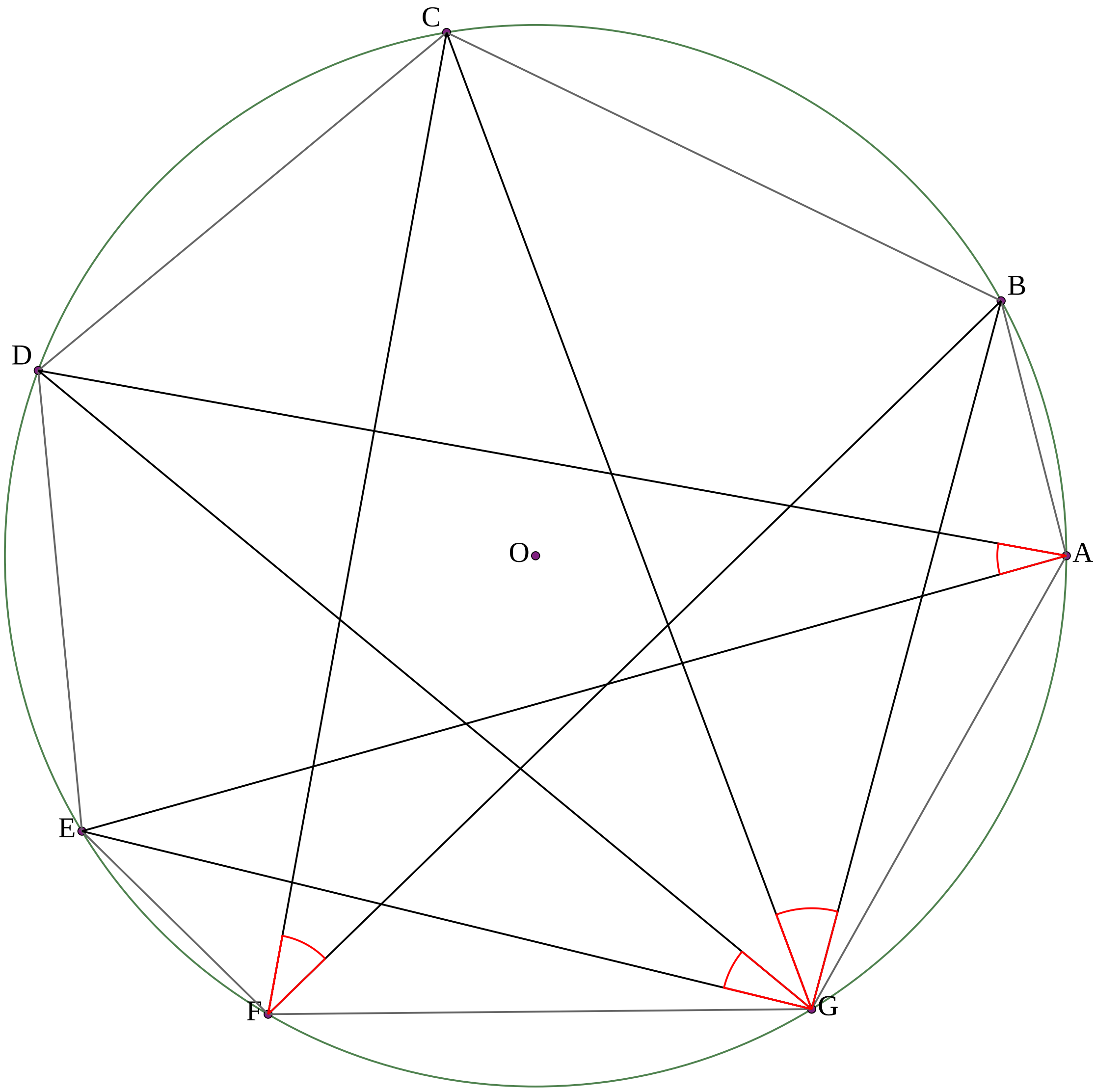
Let ABCDEF be a cyclic hexagon with center O.
Prove that EAF+DFE = DCE+EBF

Example 13



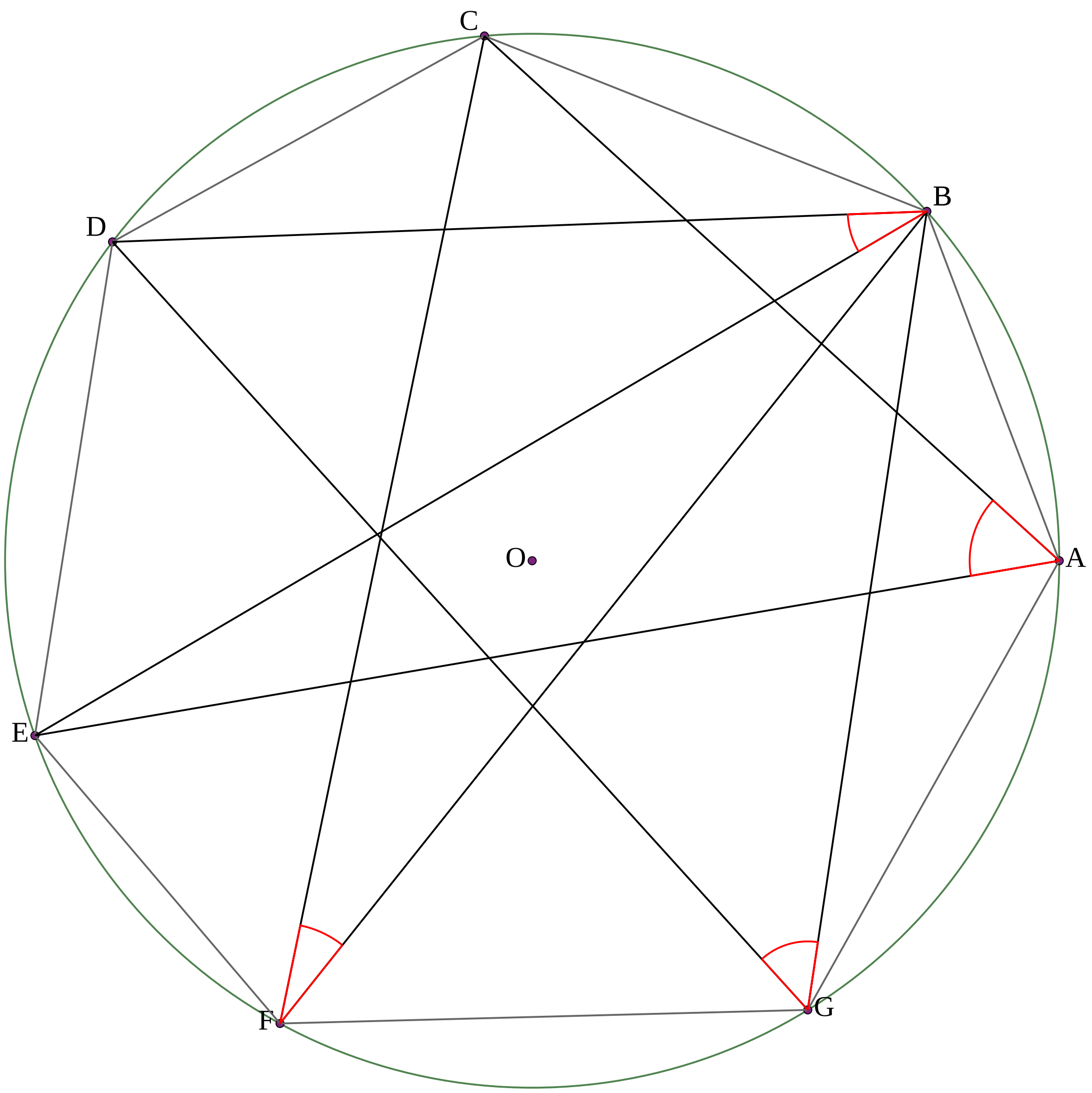
Let ABCDEF be a cyclic hexagon with center O.
Prove that ACB+BAF+BEF = ADB+180

Example 14



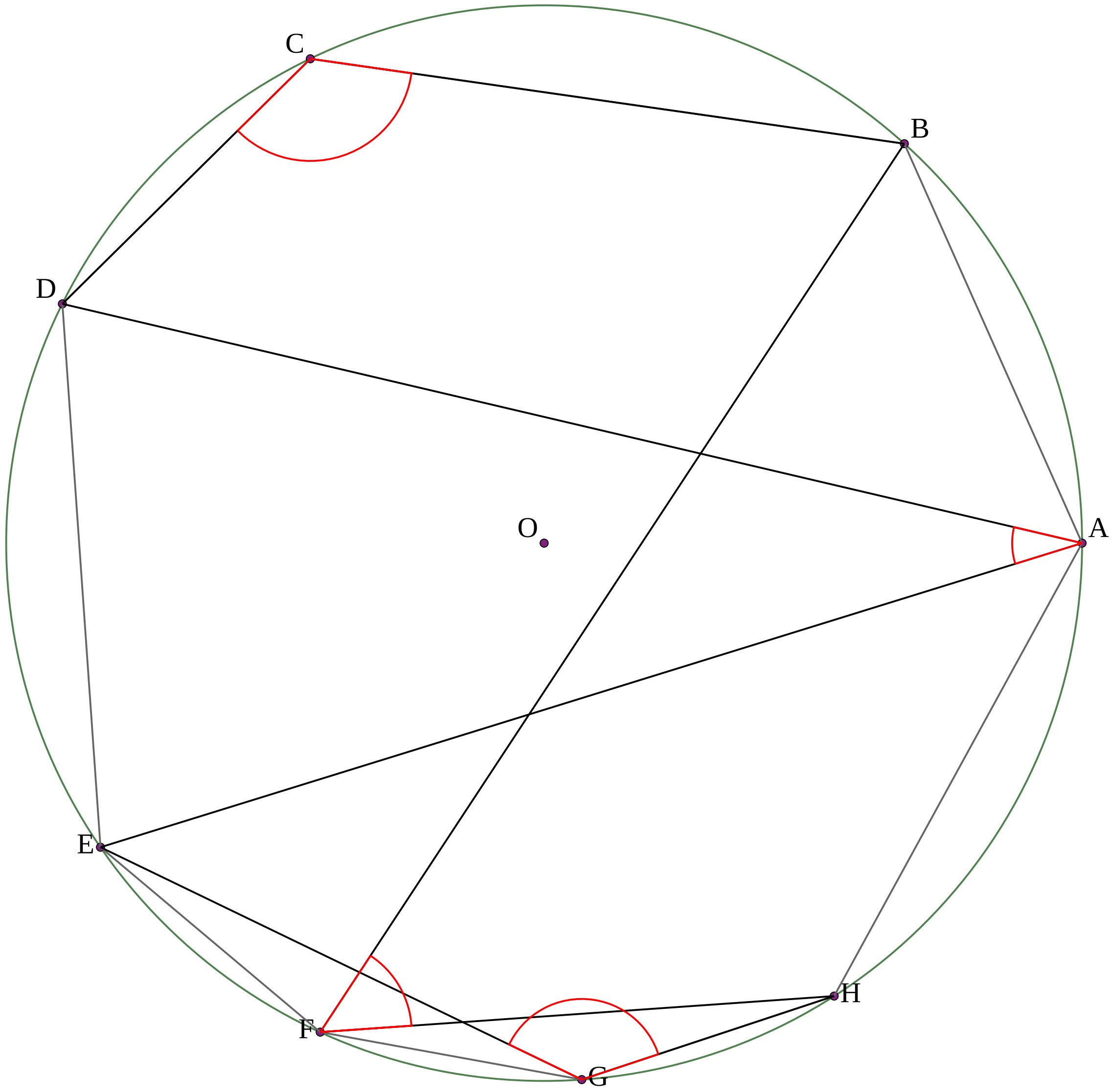
Let ABCDEFG be a cyclic heptagon with center O.
Prove that BFC+DGE = DAE+BGC

Example 15



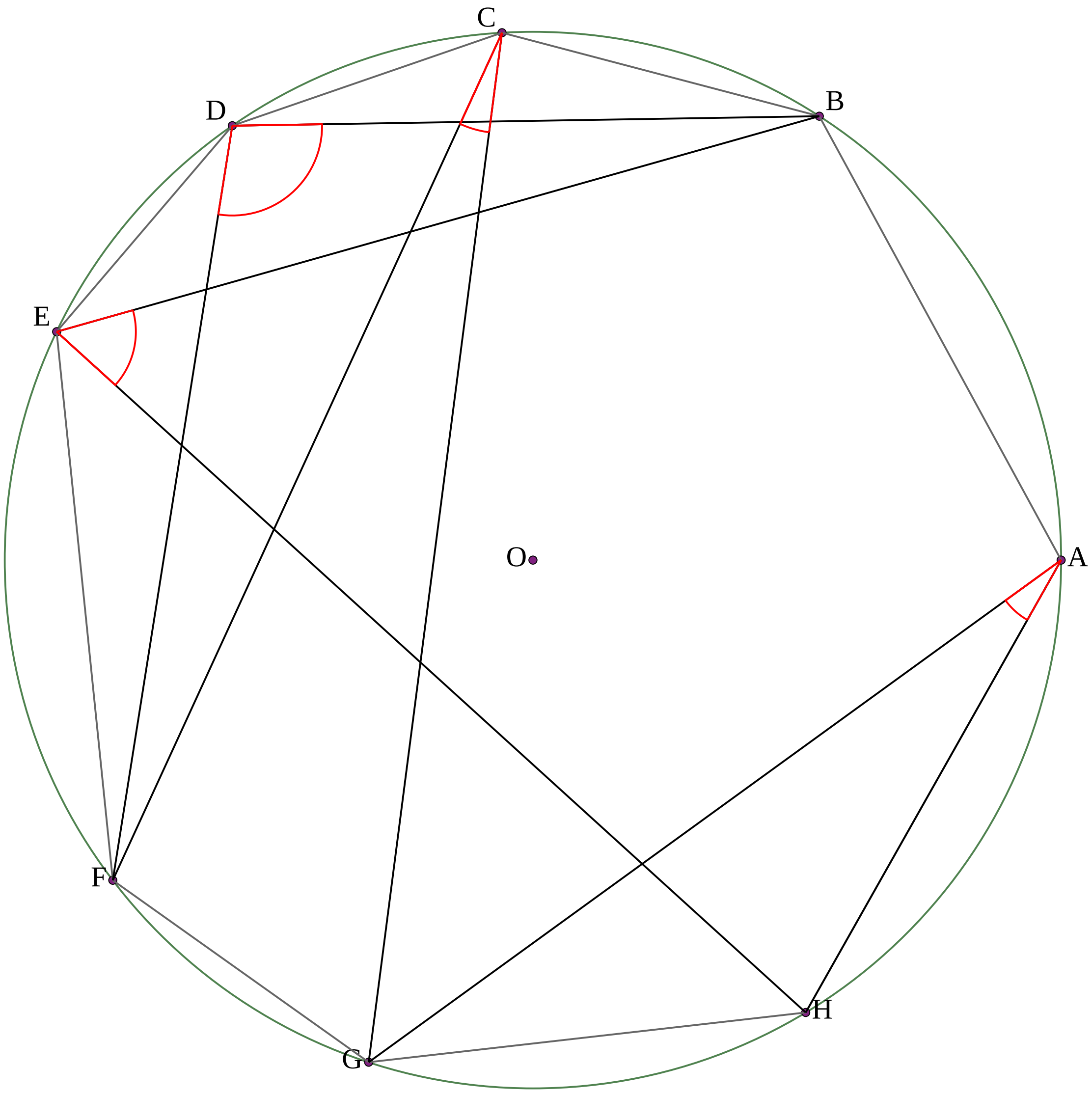
Let ABCDEFG be a cyclic heptagon with center O.
Angle BGD = x. Angle DBE = y. Angle CFB = z.
Find angle EAC.

Example 16



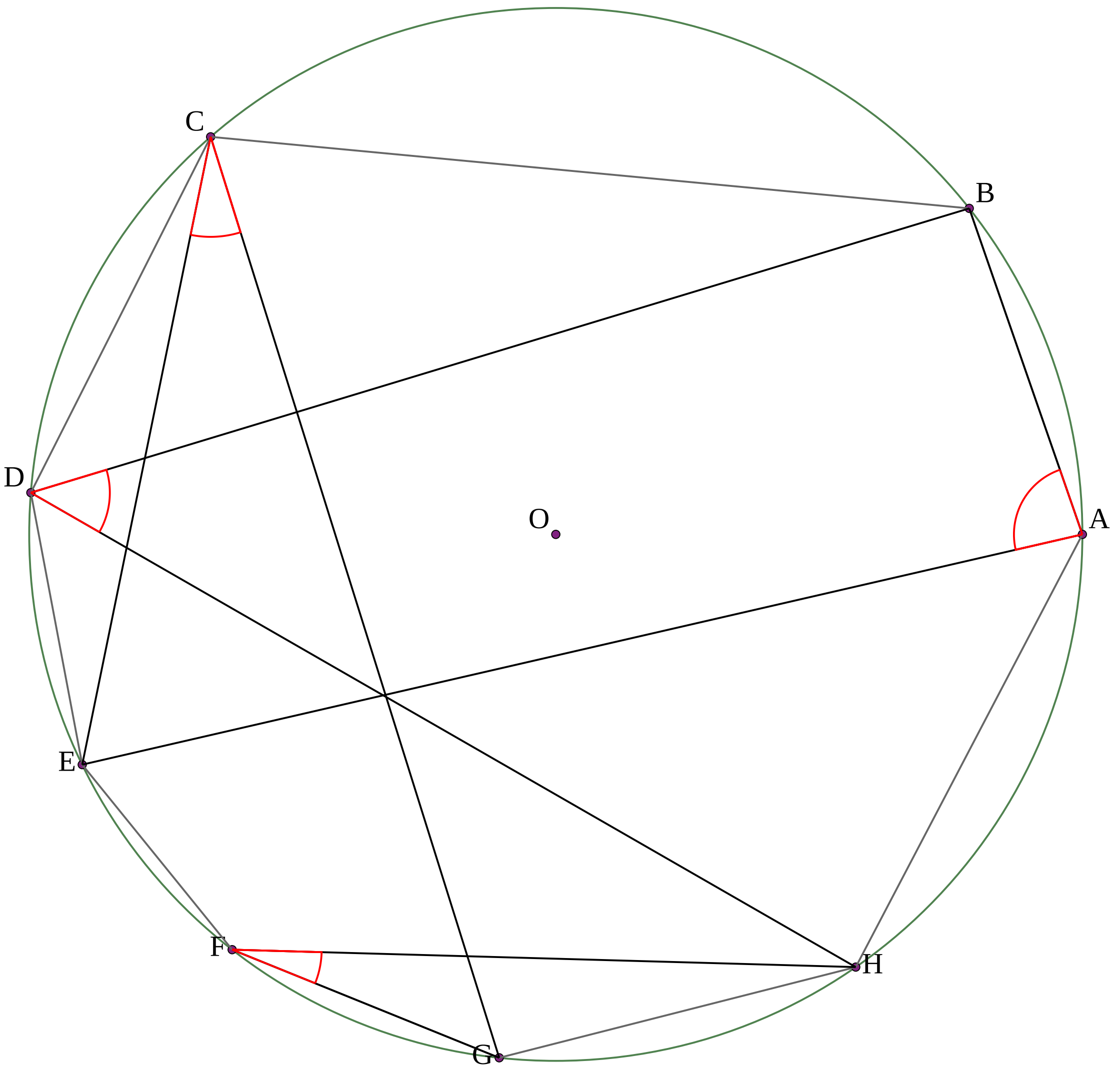
Let ABCDEFGH be a cyclic octagon with center O.
Angle DAE = 30°. Angle HFB = 53°. Angle EGH = 136°.
Find angle BCD.

Example 17



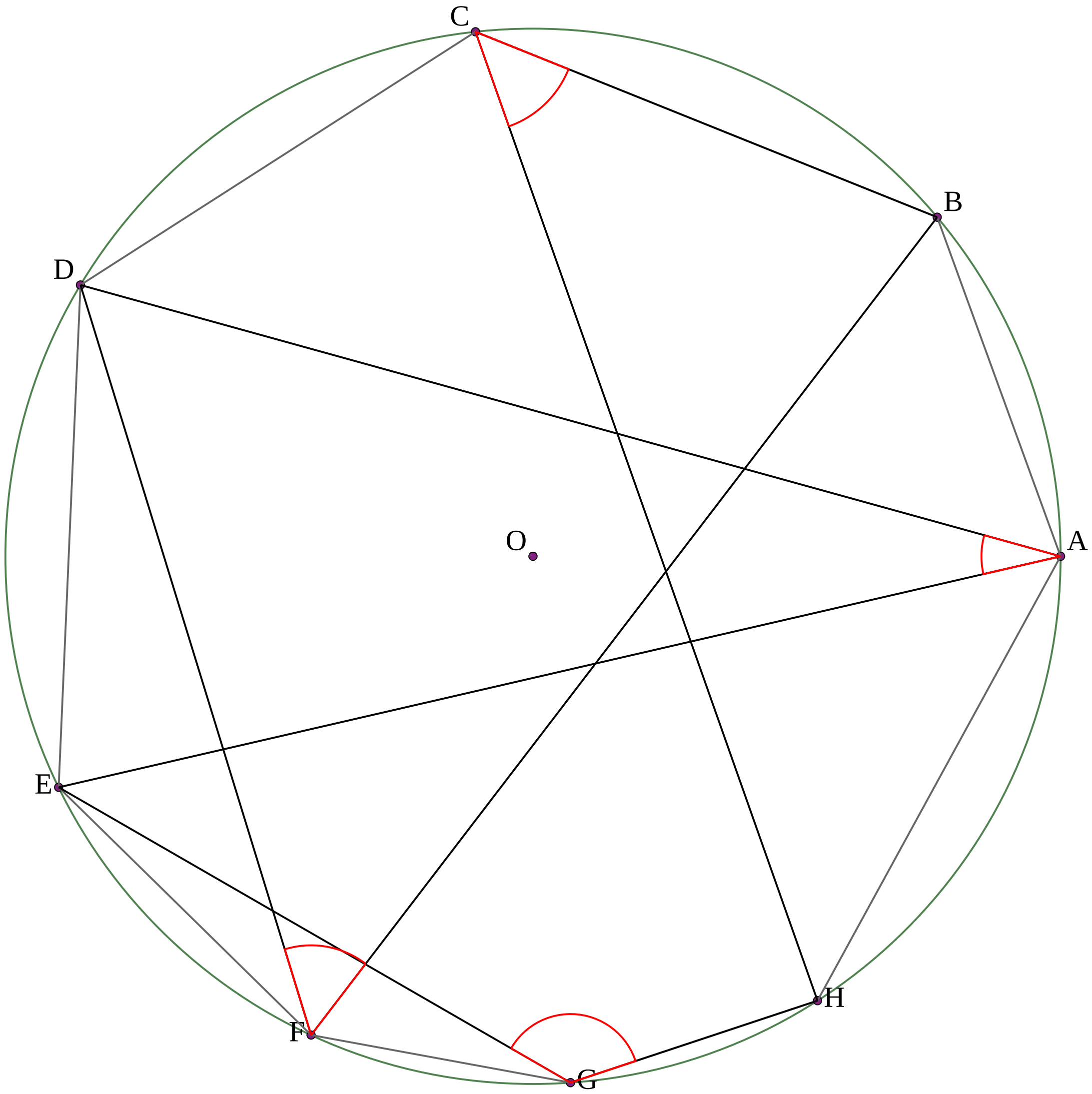
Let ABCDEFGH be a cyclic octagon with center O.
Angle BDF = x. Angle HEB = y. Angle FCG = z.
Find angle GAH.

Example 18



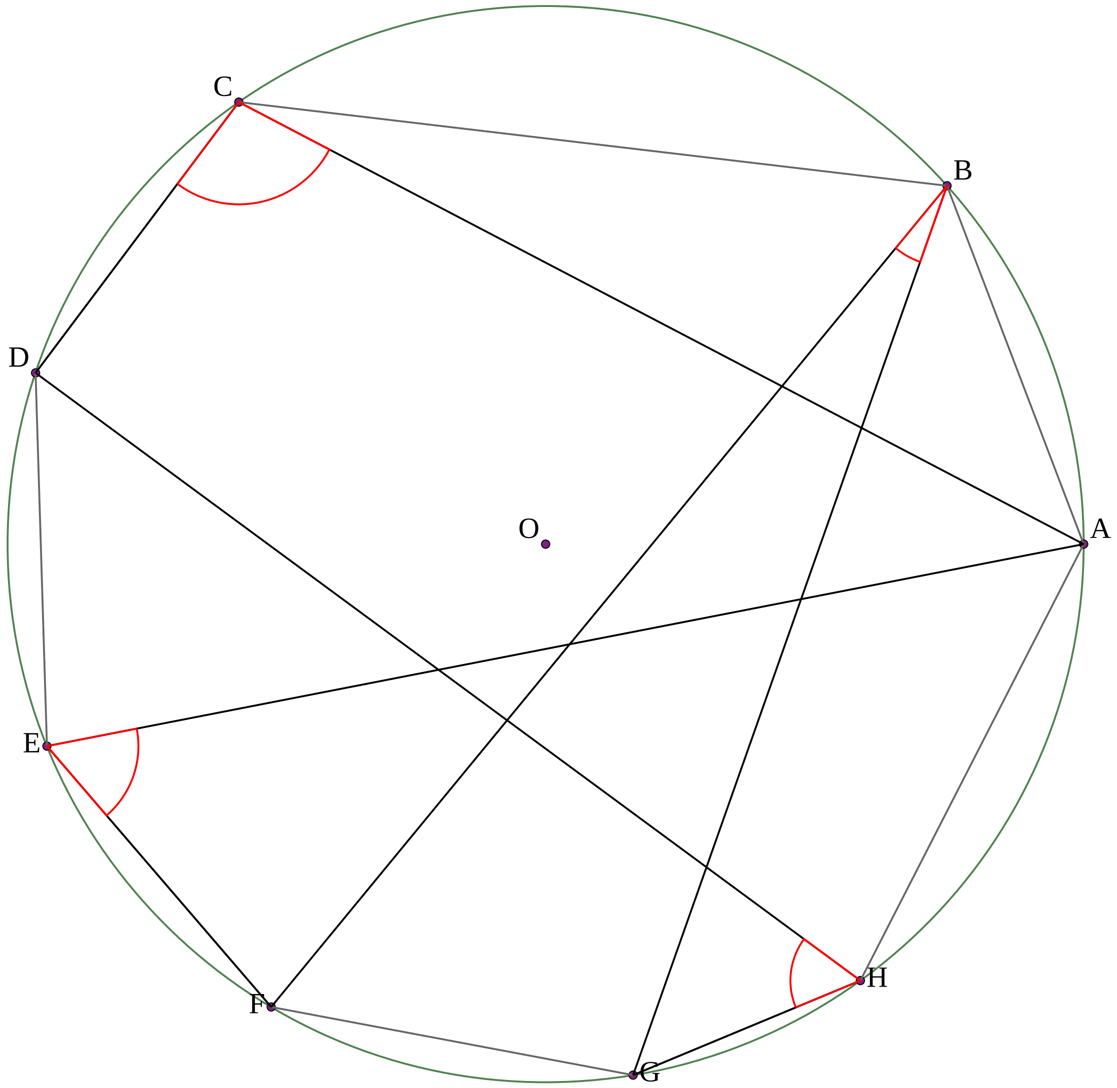
Let ABCDEFGH be a cyclic octagon with center O.
Angle EAB = x. Angle BDH = y. Angle GCE = z.
Find angle HFG.

Example 19



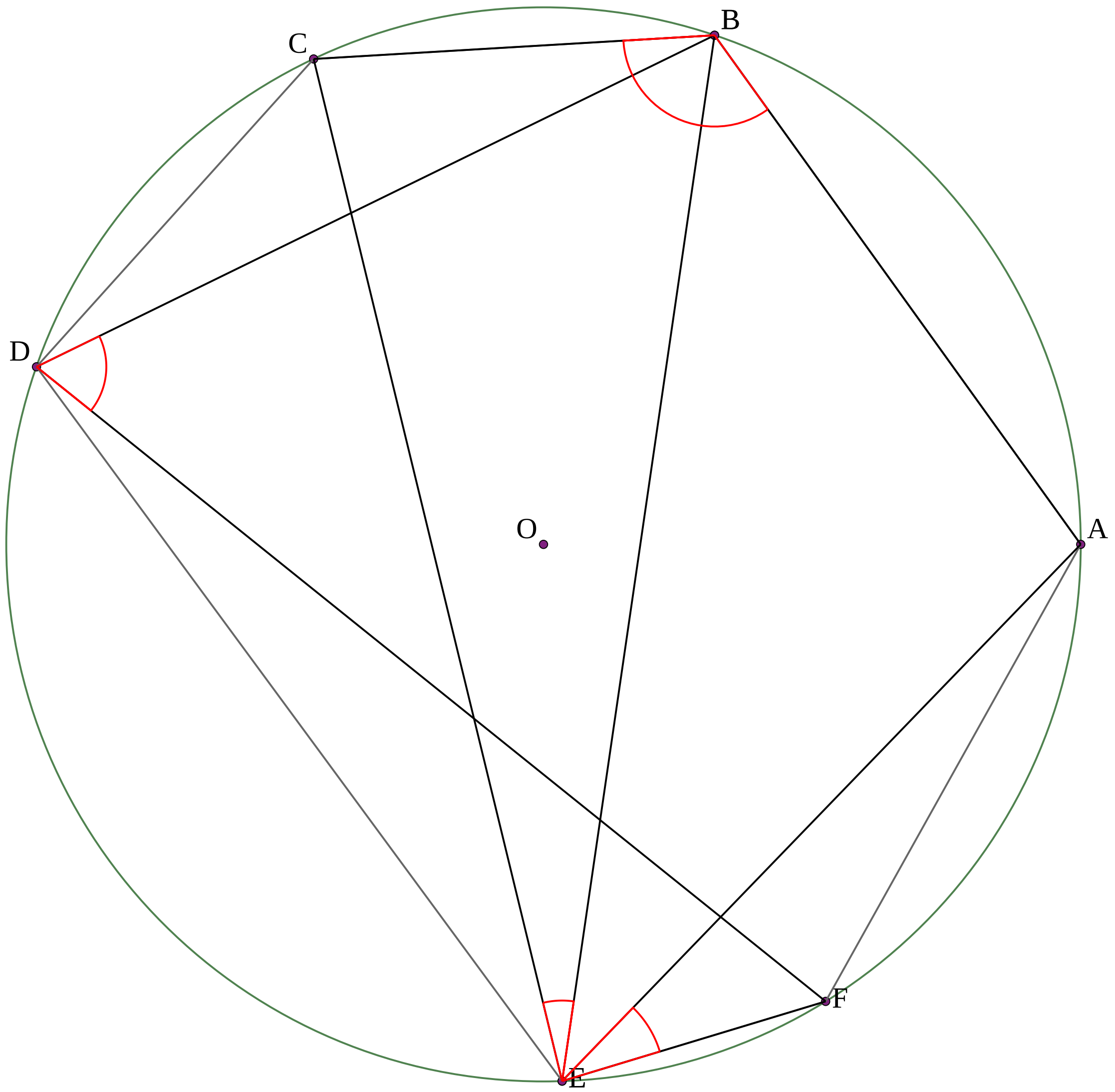
Let ABCDEFGH be a cyclic octagon with center O.
Angle EAD = 28°. Angle DFB = 55°. Angle BCH = 49°.
Find angle HGE.

Example 20



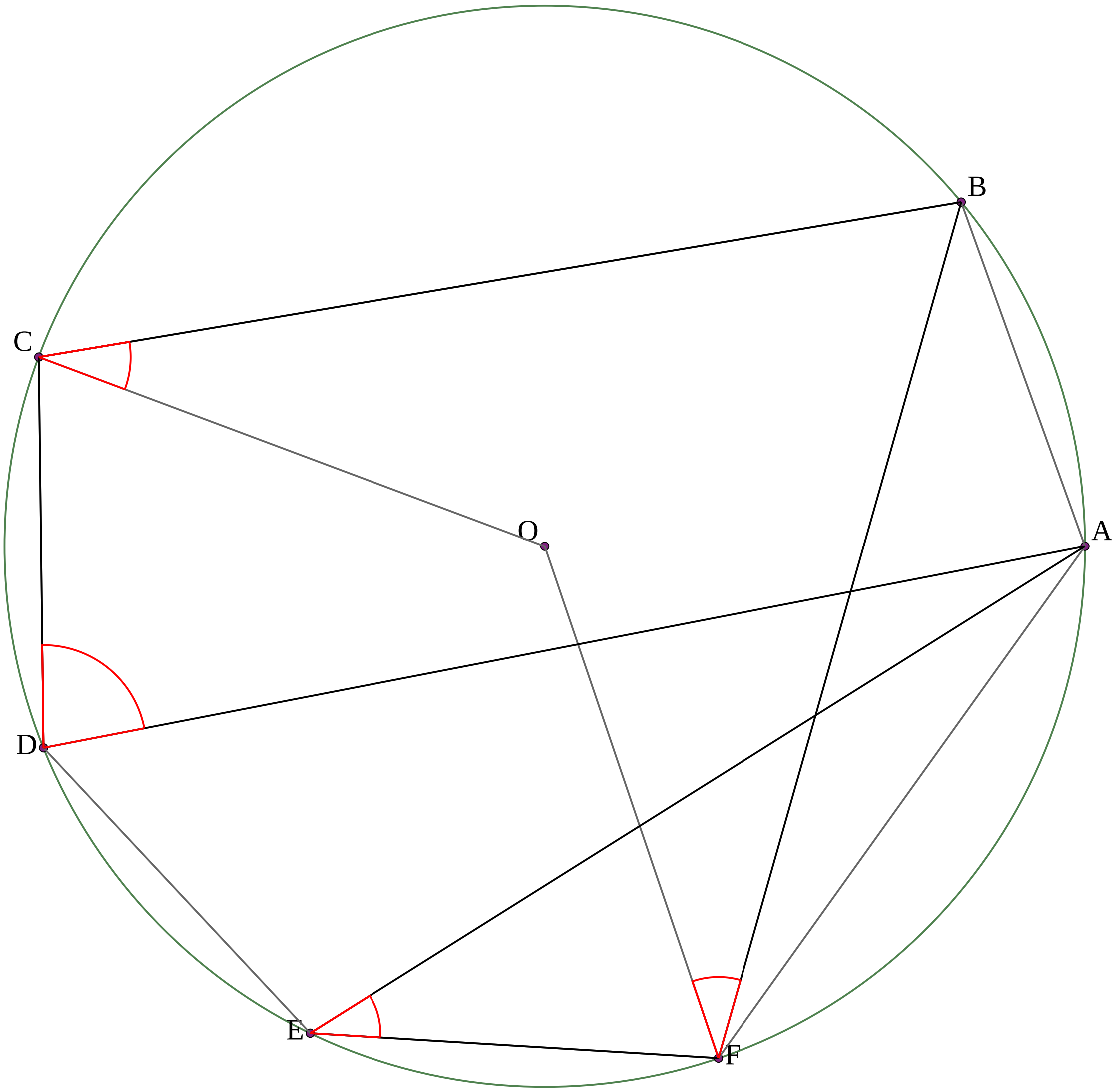
Let ABCDEFGH be a cyclic octagon with center O.
Angle DHG = 59°. Angle FEA = 60°. Angle GBF = 20°.
Find angle ACD.

Example 21



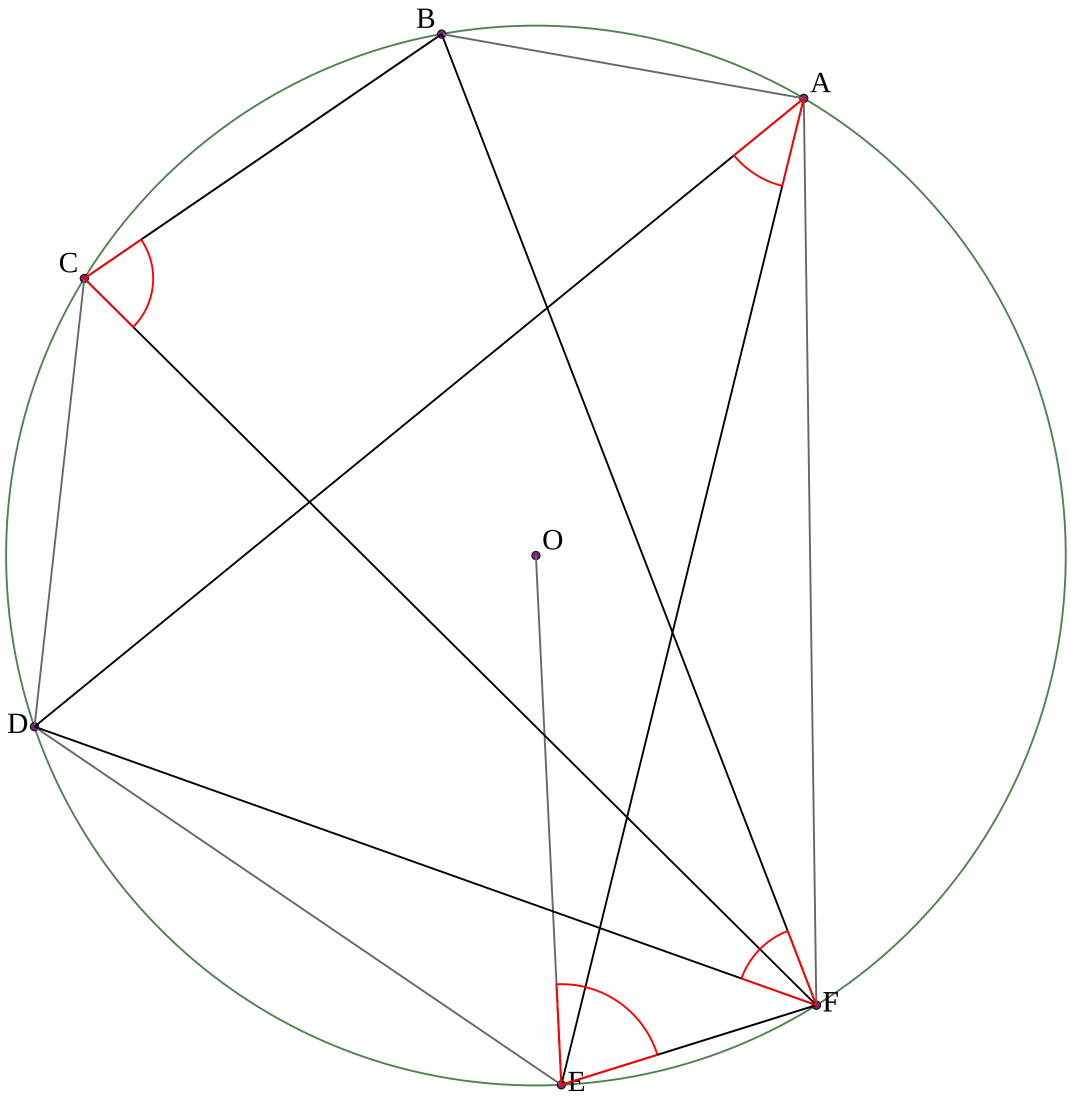
Let ABCDEF be a cyclic hexagon with center O.
Prove that BDF+BEC+ABC = AEF+180

Example 22



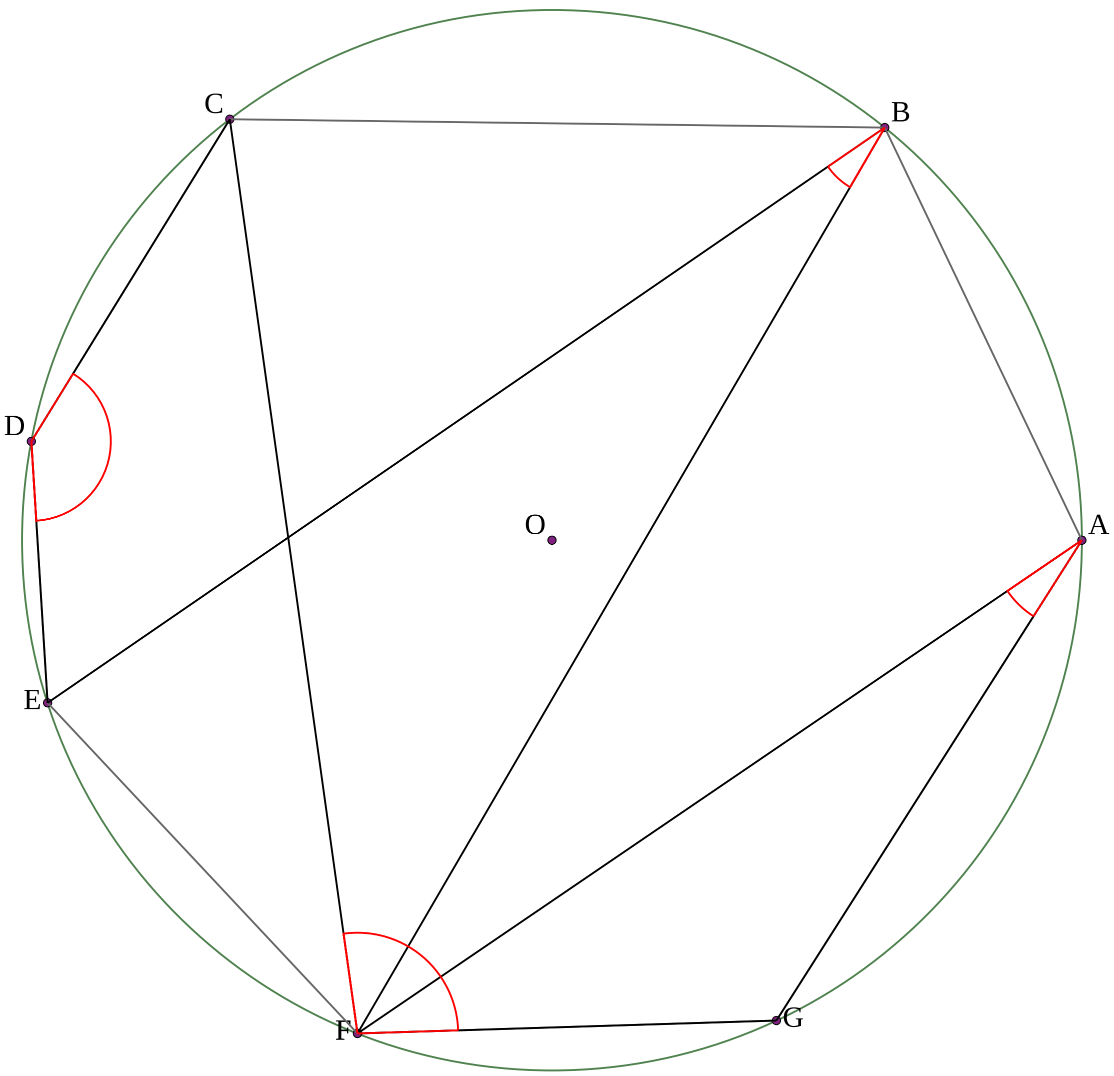
Let ABCDEF be a cyclic hexagon with center O.
Angle AEF = 36°. Angle BCO = 30°. Angle CDA = 80°.
Find angle OFB.

Example 23



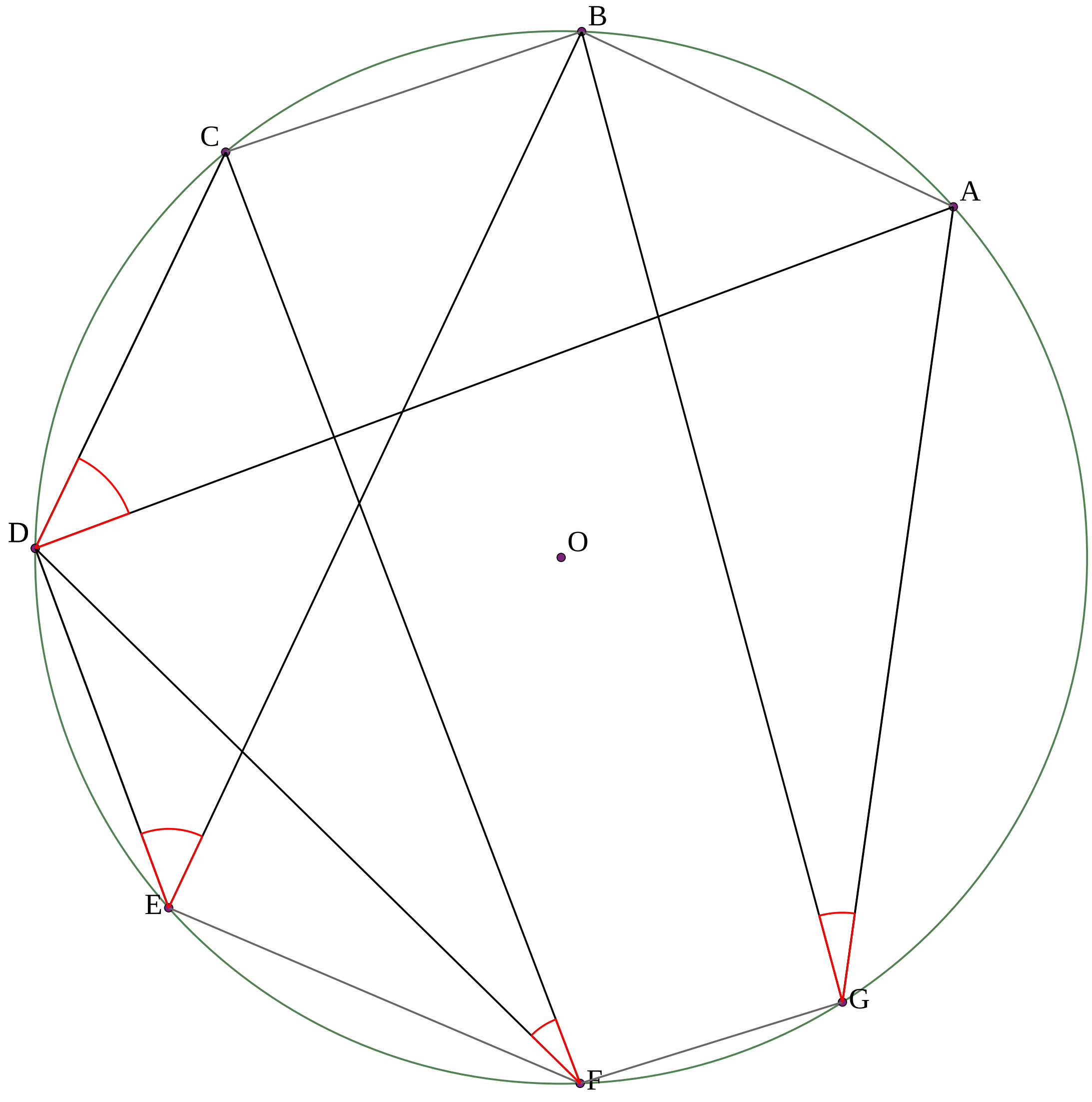
Let ABCDEF be a cyclic hexagon with center O.
Angle OEF = x. Angle FCB = y. Angle BFD = z.
Find angle DAE.

Example 24



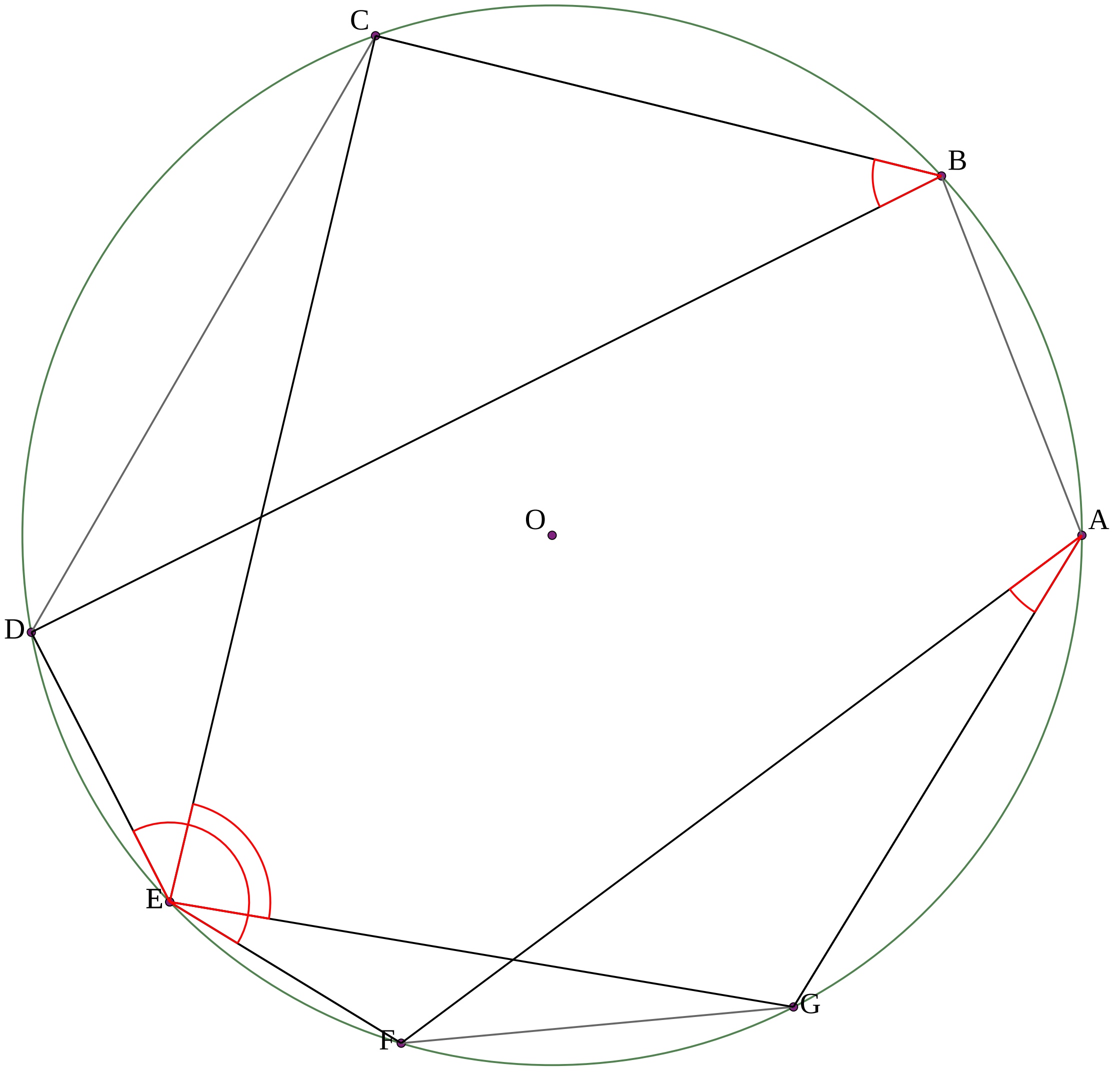
Let ABCDEFG be a cyclic heptagon with center O.
Angle EDC = x. Angle FBE = y. Angle FAG = z.
Find angle CFG.

Example 25



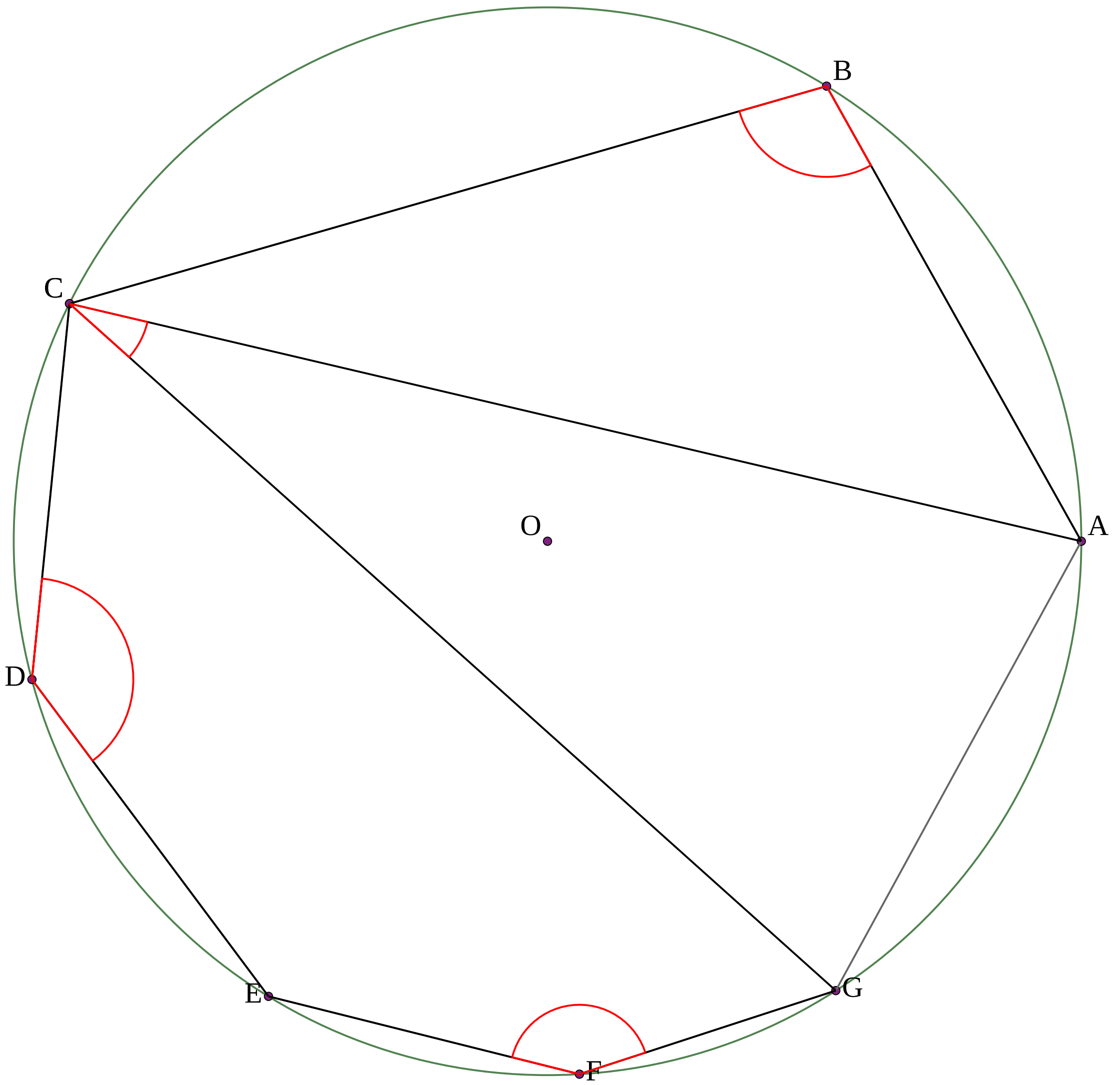
Let ABCDEFG be a cyclic heptagon with center O.
Angle DEB = x. Angle BGA = y. Angle ADC = z.
Find angle CFD.

Example 26



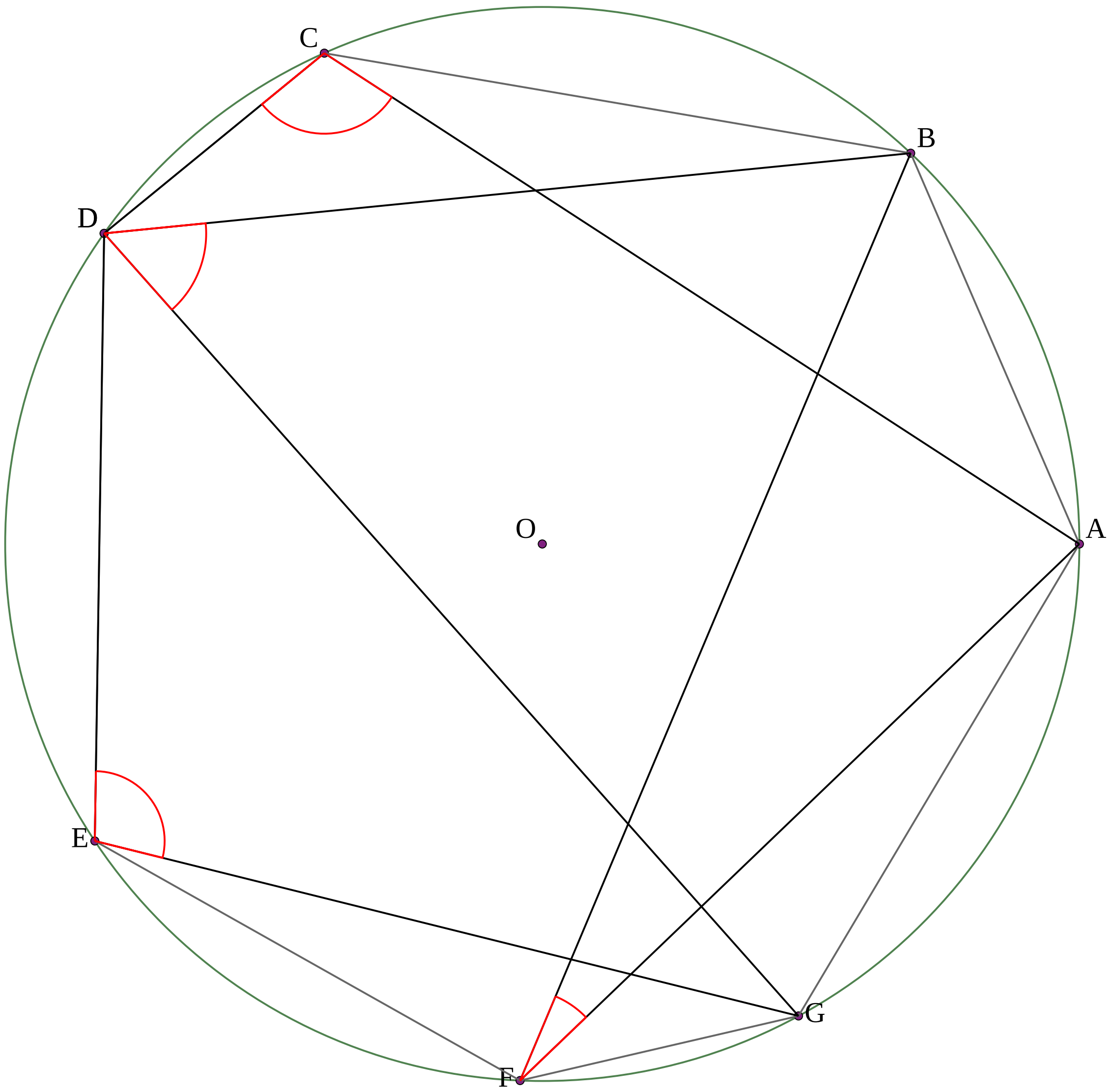
Let ABCDEFG be a cyclic heptagon with center O.
Prove that DEF = CBD+FAG+CEG

Example 27



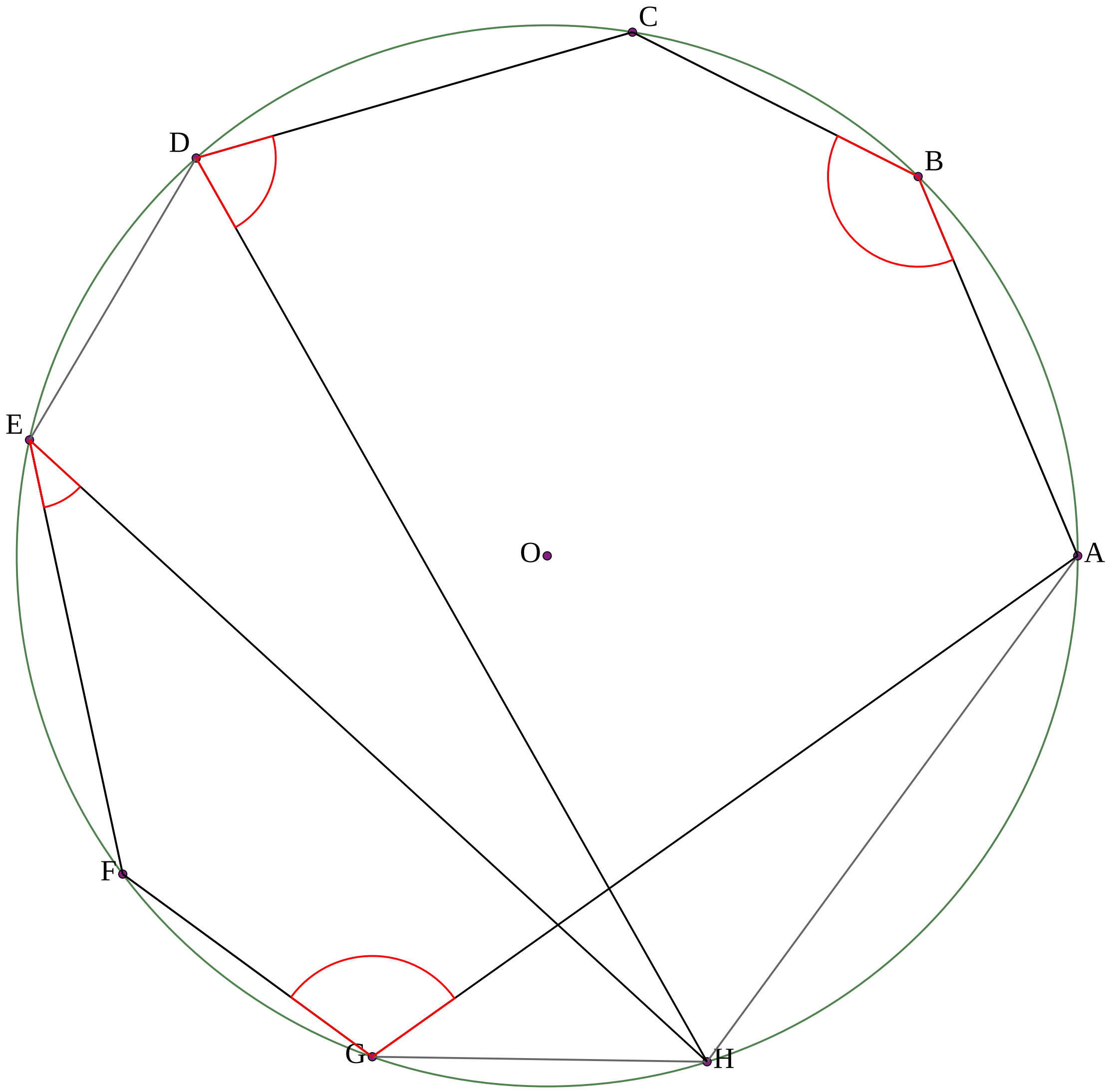
Let ABCDEFG be a cyclic heptagon with center O.
Angle EFG = x. Angle ABC = y. Angle CDE = z.
Find angle GCA.

Example 28



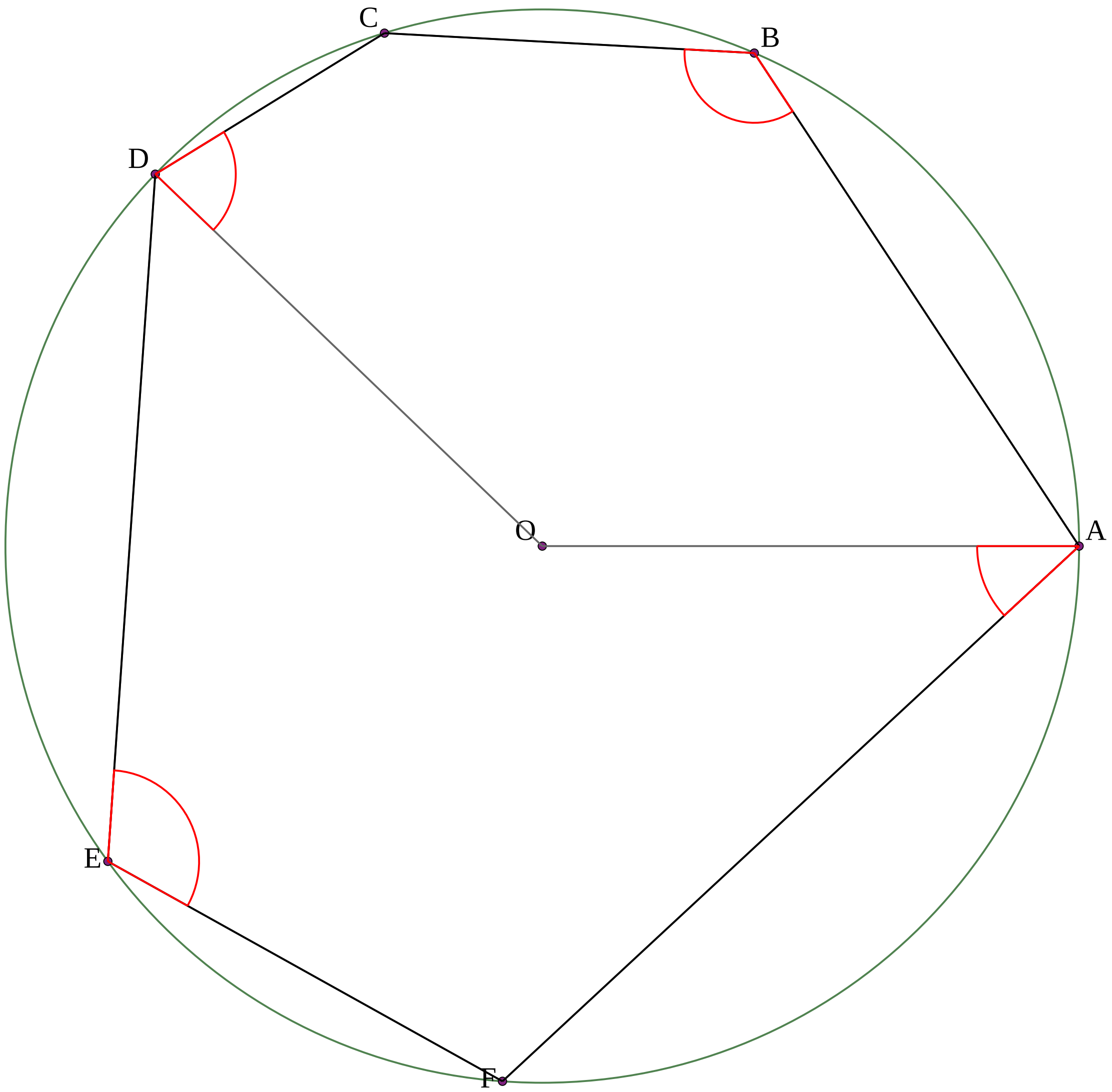
Let ABCDEFG be a cyclic heptagon with center O.
Angle BDG = 54°. Angle DCA = 108°. Angle AFB = 23°.
Find angle GED.

Example 29



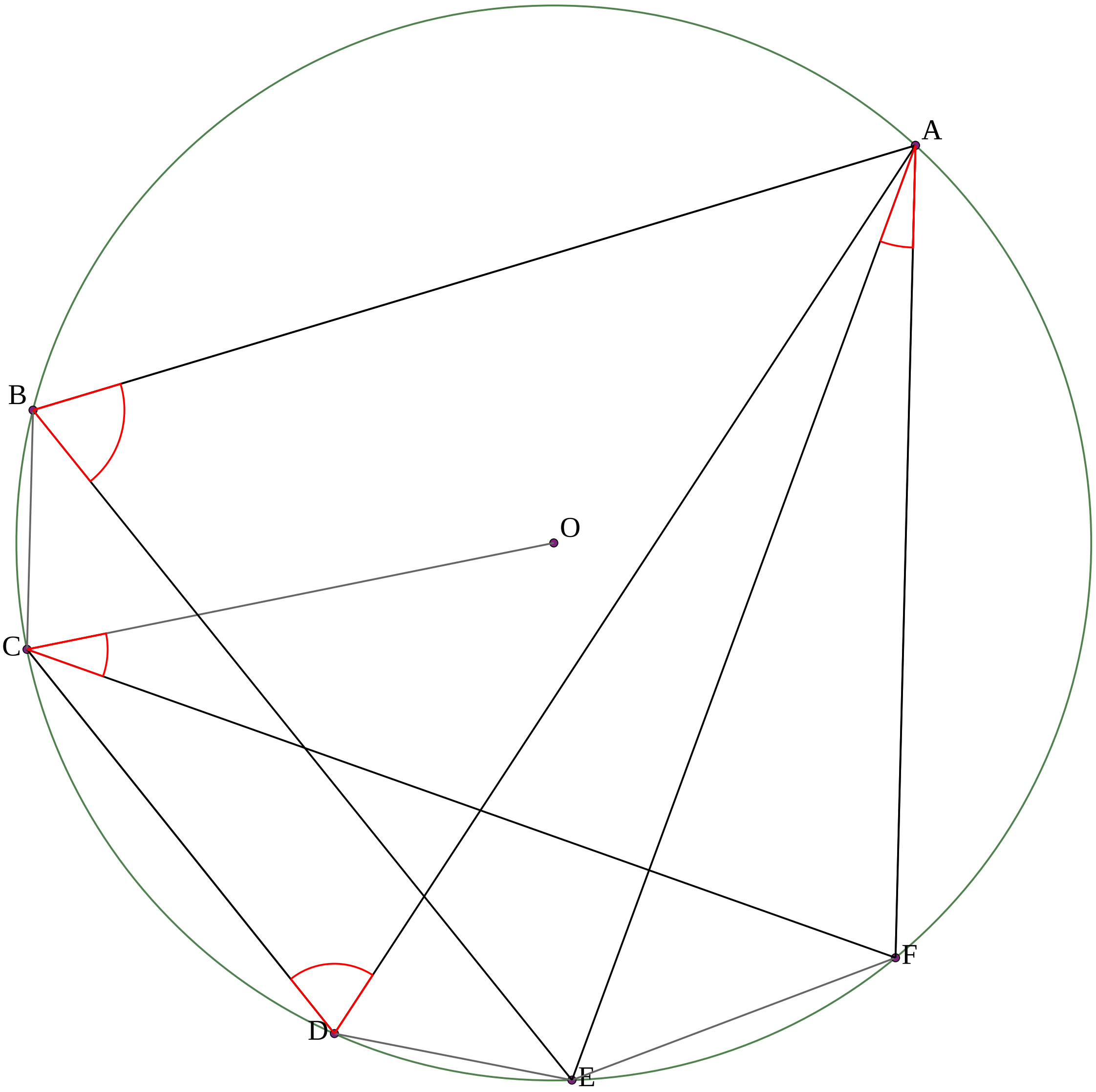
Let ABCDEFGH be a cyclic octagon with center O.
Angle HDC = 77°. Angle CBA = 140°. Angle AGF = 108°.
Find angle FEH.

Example 30



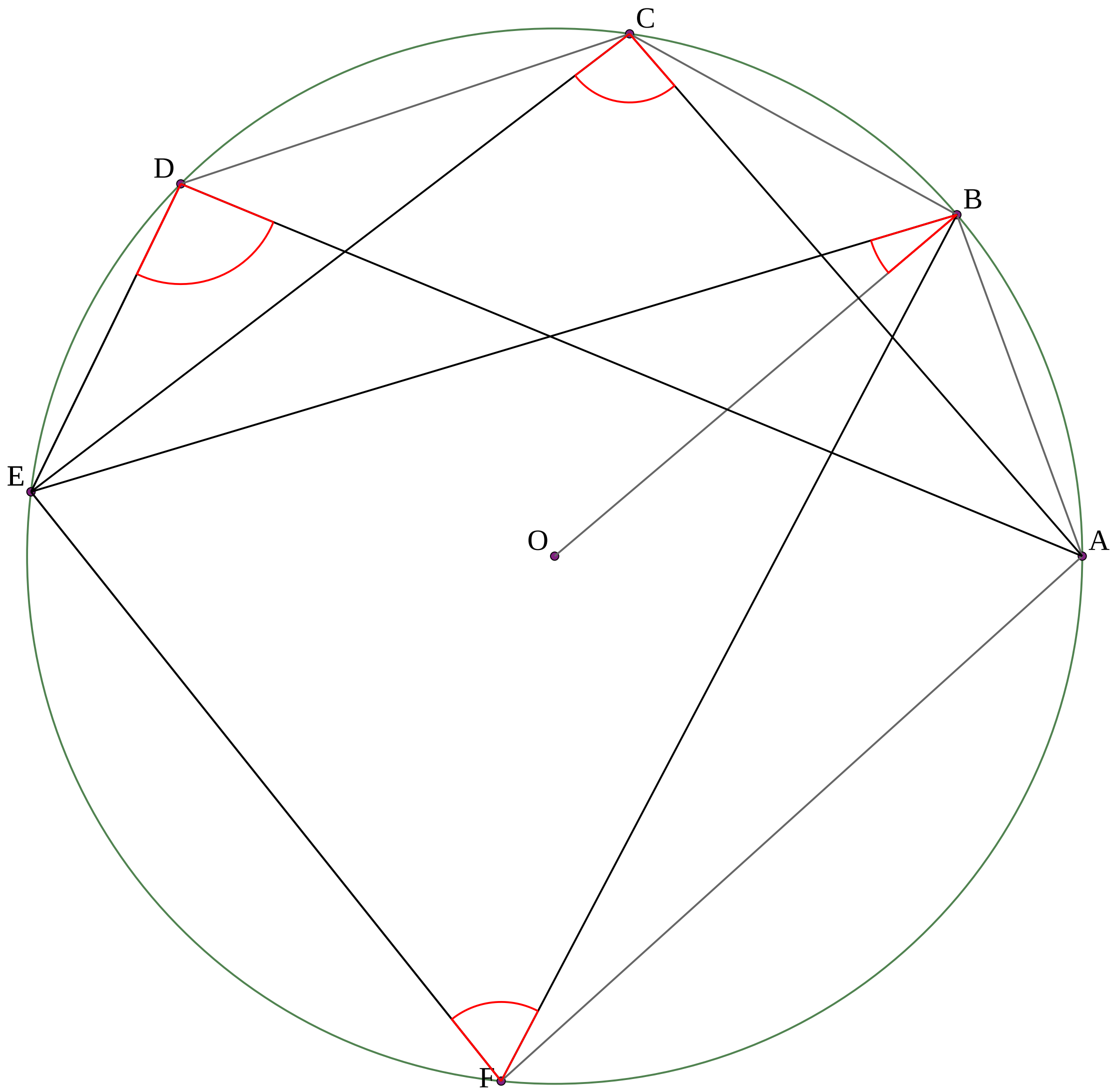
Let ABCDEF be a cyclic hexagon with center O.
Angle ABC = x. Angle CDO = y. Angle FAO = z.
Find angle DEF.

Example 31



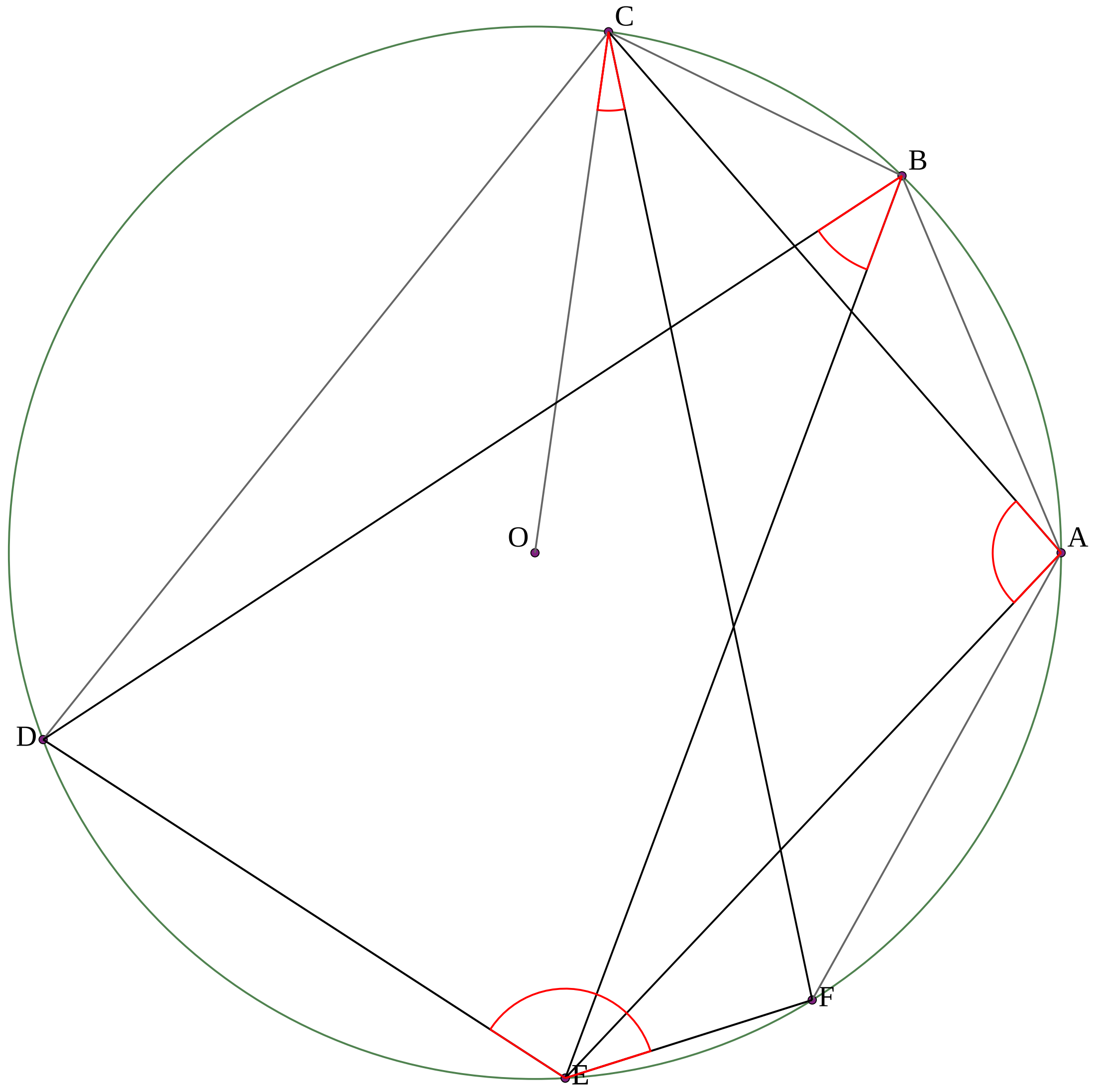
Let ABCDEF be a cyclic hexagon with center O.
Angle ADC = 72°. Angle ABE = 68°. Angle OCF = 31°.
Find angle FAE.

Example 32



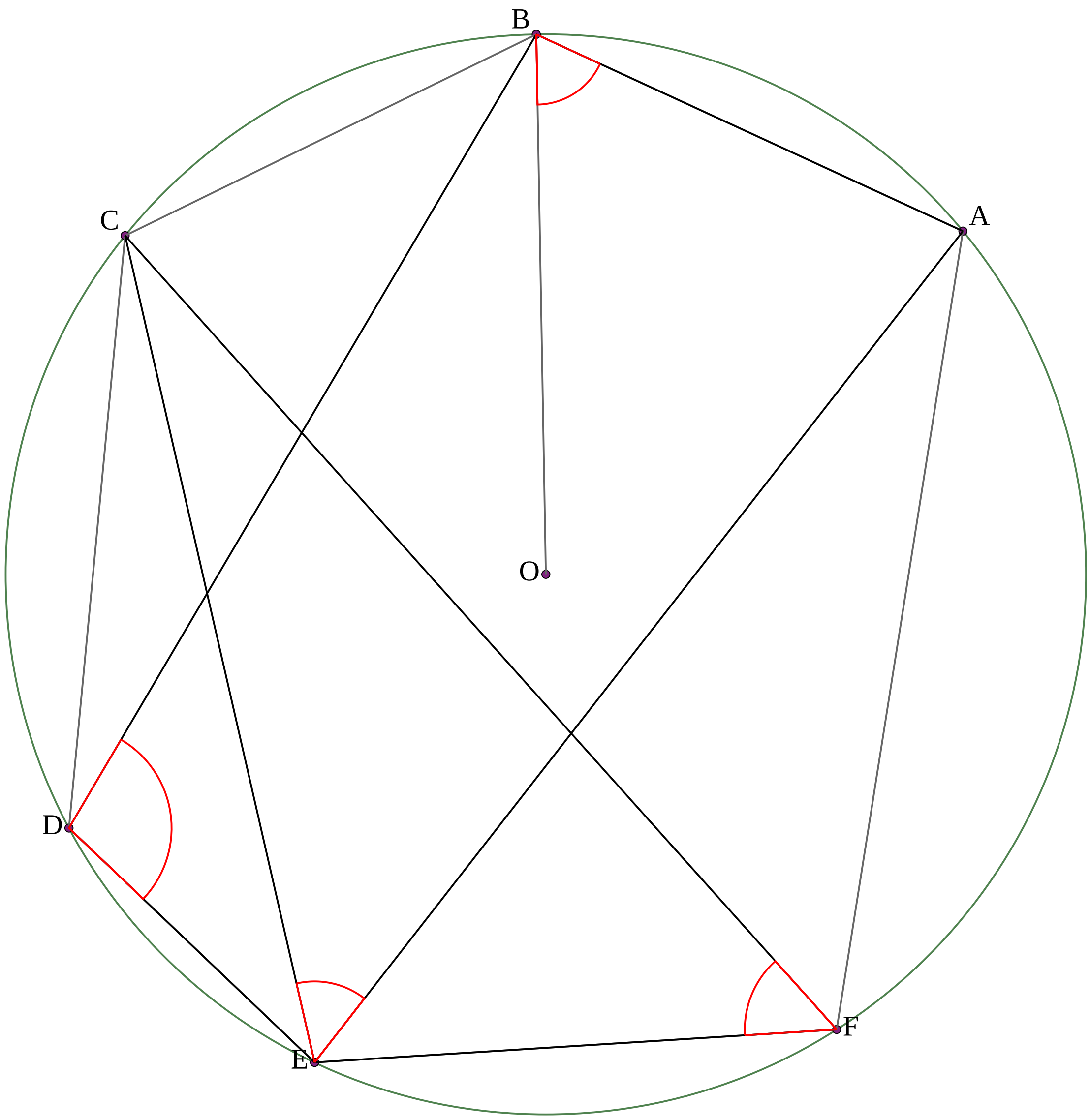
Let ABCDEF be a cyclic hexagon with center O.
Prove that ACE+BFE+EBO = ADE+90

Example 33



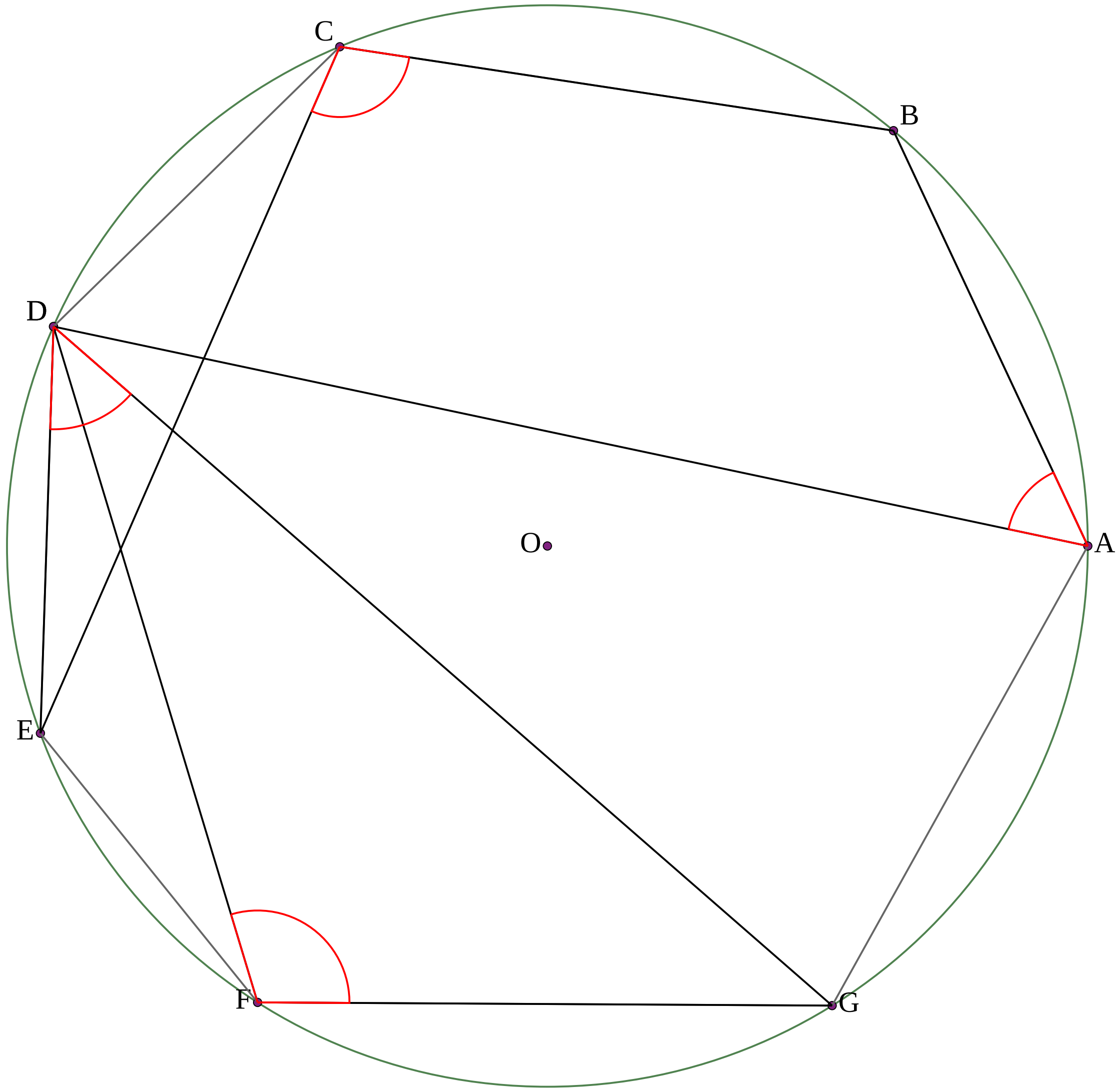
Let ABCDEF be a cyclic hexagon with center O.
Angle EAC = x. Angle OCF = y. Angle FED = z.
Find angle DBE.

Example 34



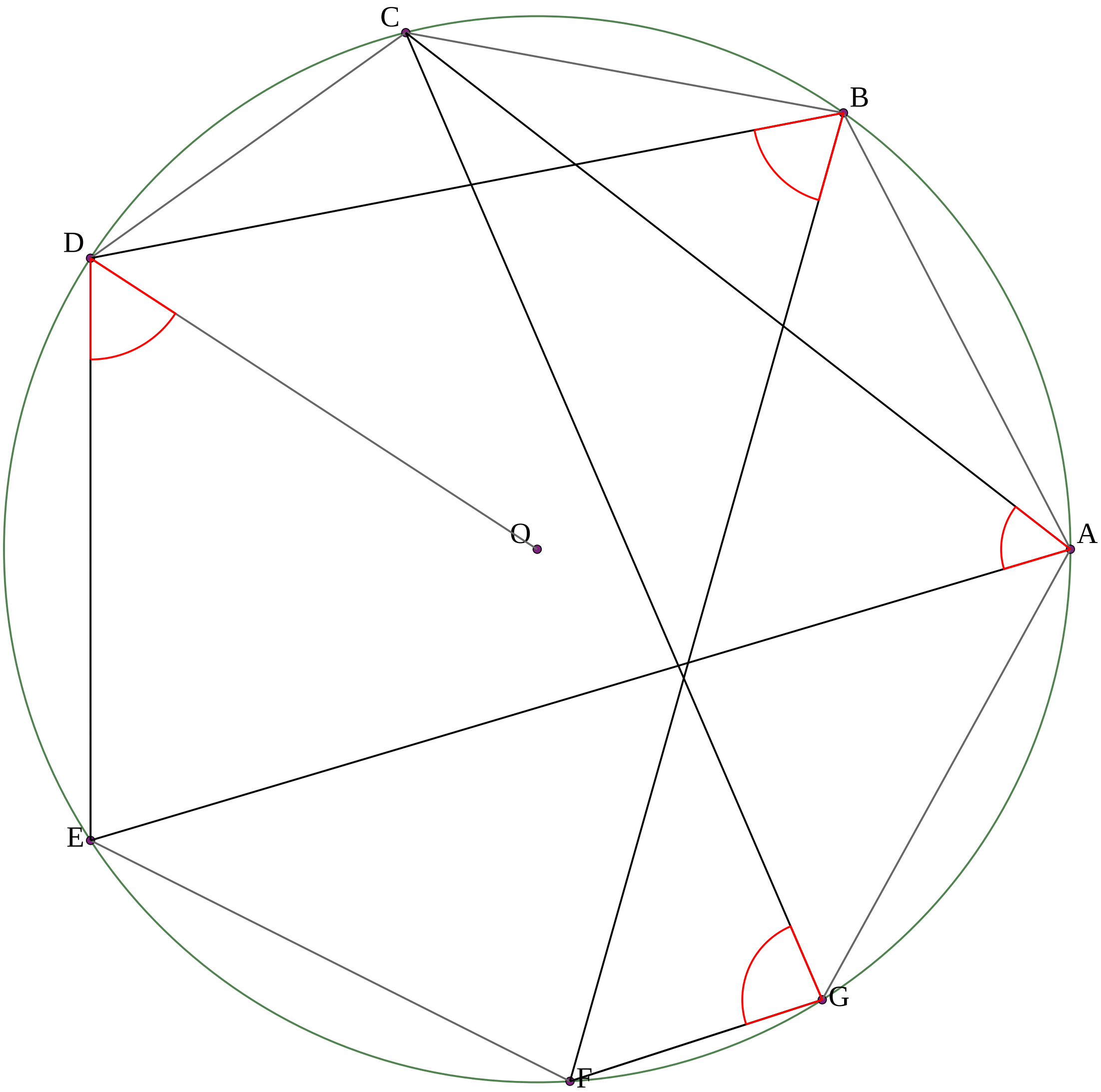
Let ABCDEF be a cyclic hexagon with center O.
Angle OBA = 64°. Angle AEC = 51°. Angle EDB = 103°.
Find angle CFE.

Example 35



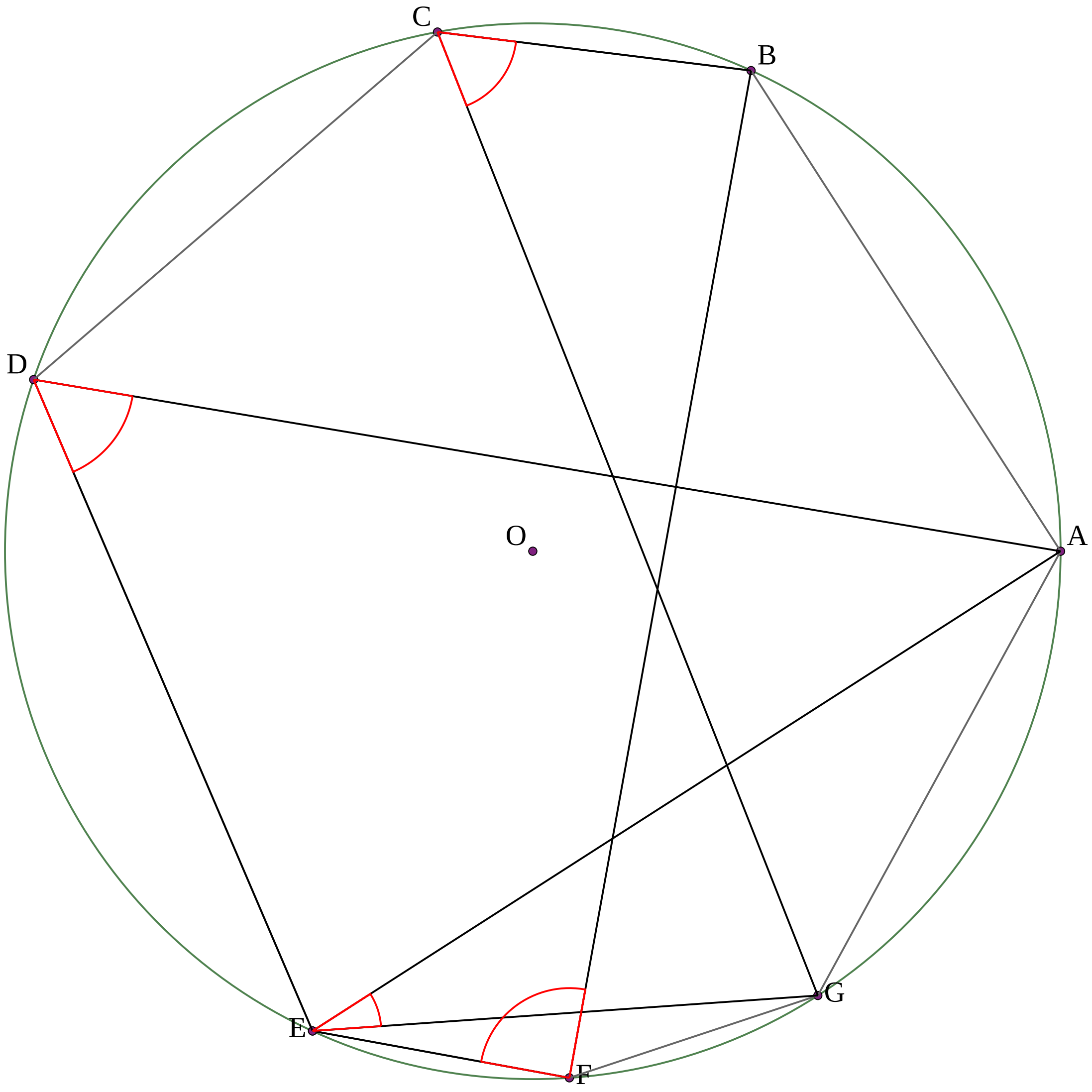
Let ABCDEFG be a cyclic heptagon with center O.
Prove that DFG+EDG = BCE+BAD

Example 36



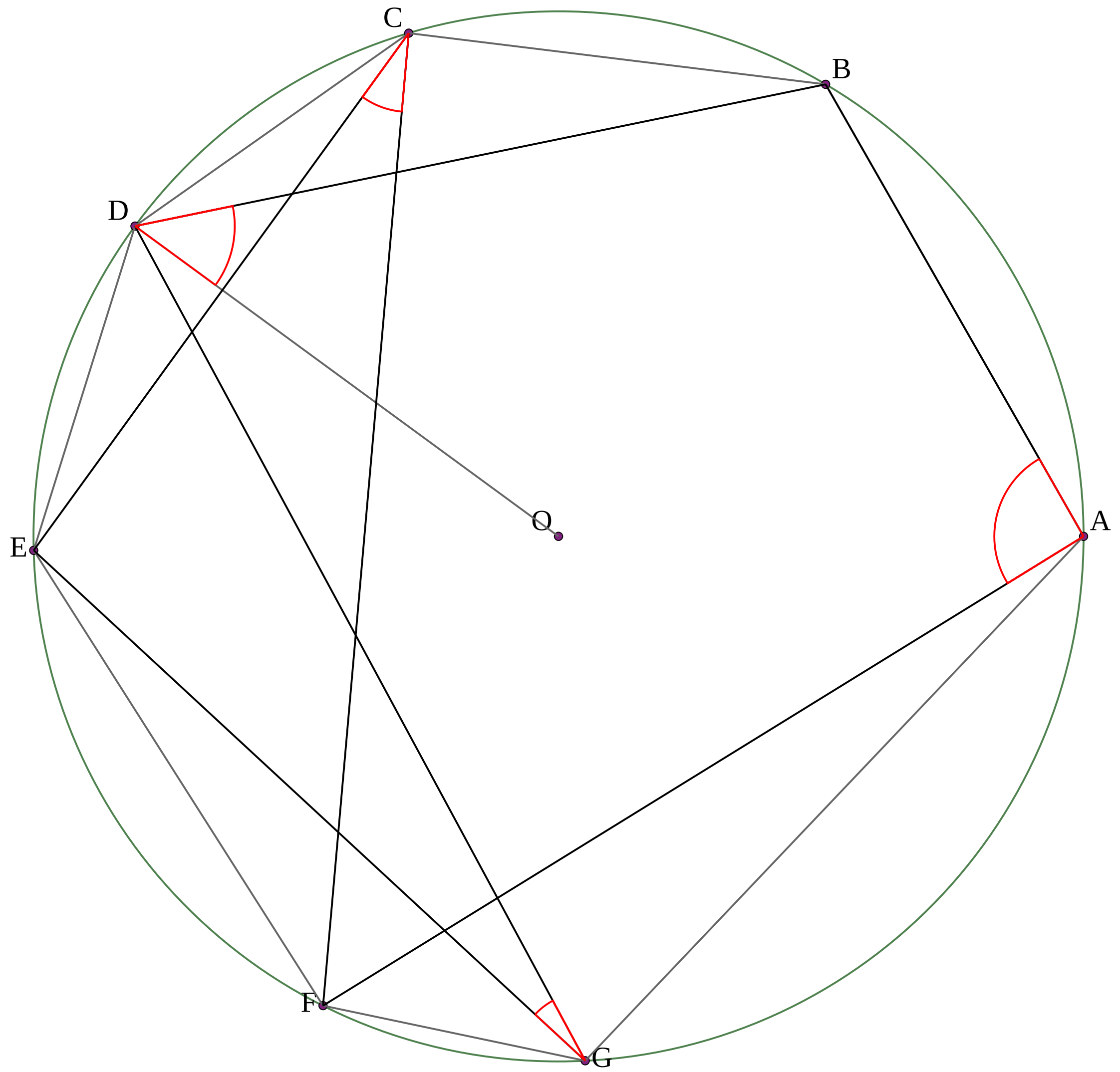
Let ABCDEFG be a cyclic heptagon with center O.
Angle CGF = 85°. Angle FBD = 63°. Angle ODE = 57°.
Find angle EAC.

Example 37



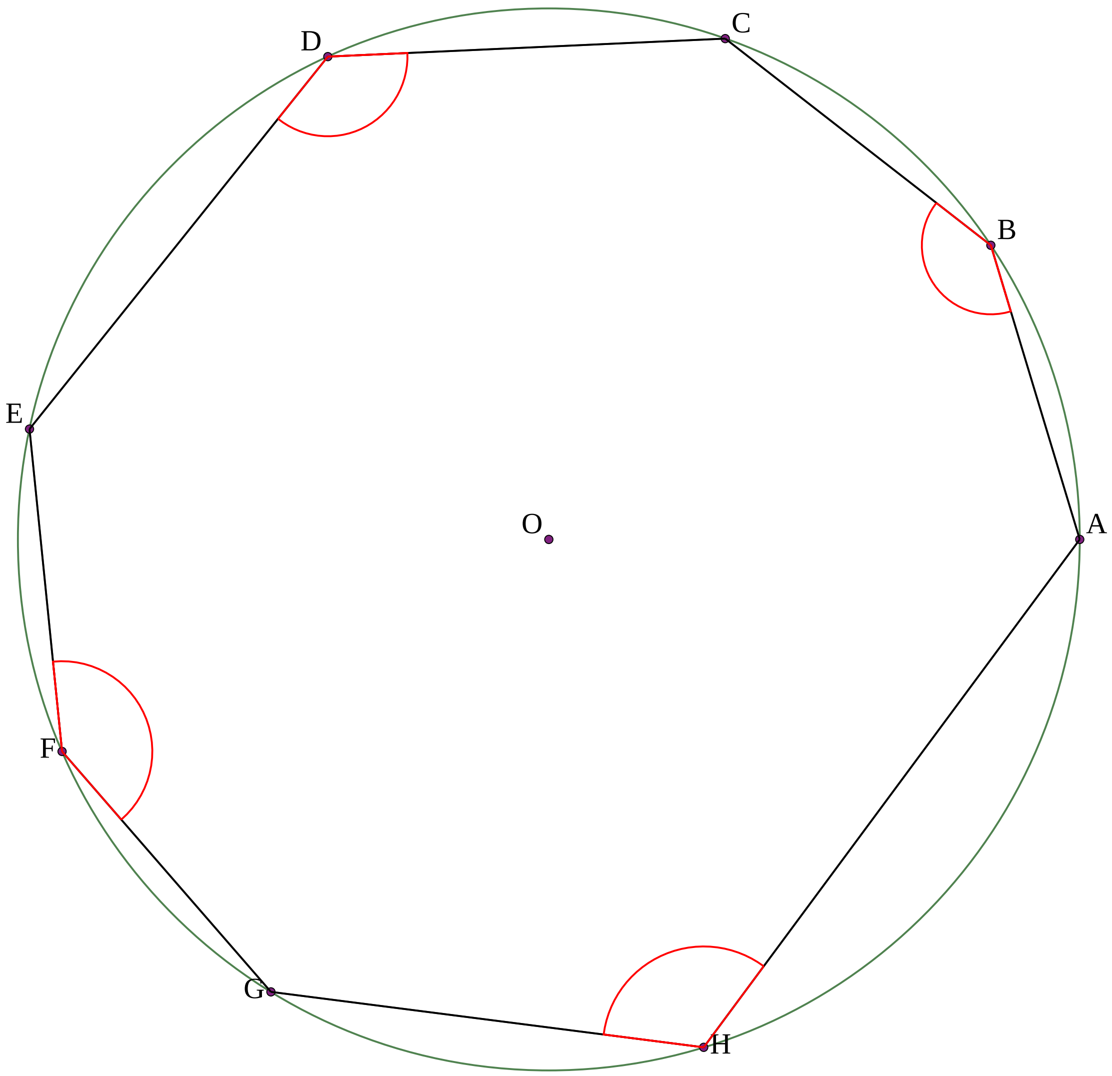
Let ABCDEFG be a cyclic heptagon with center O.
Prove that BCG+BFE+ADE = AEG+180

Example 38



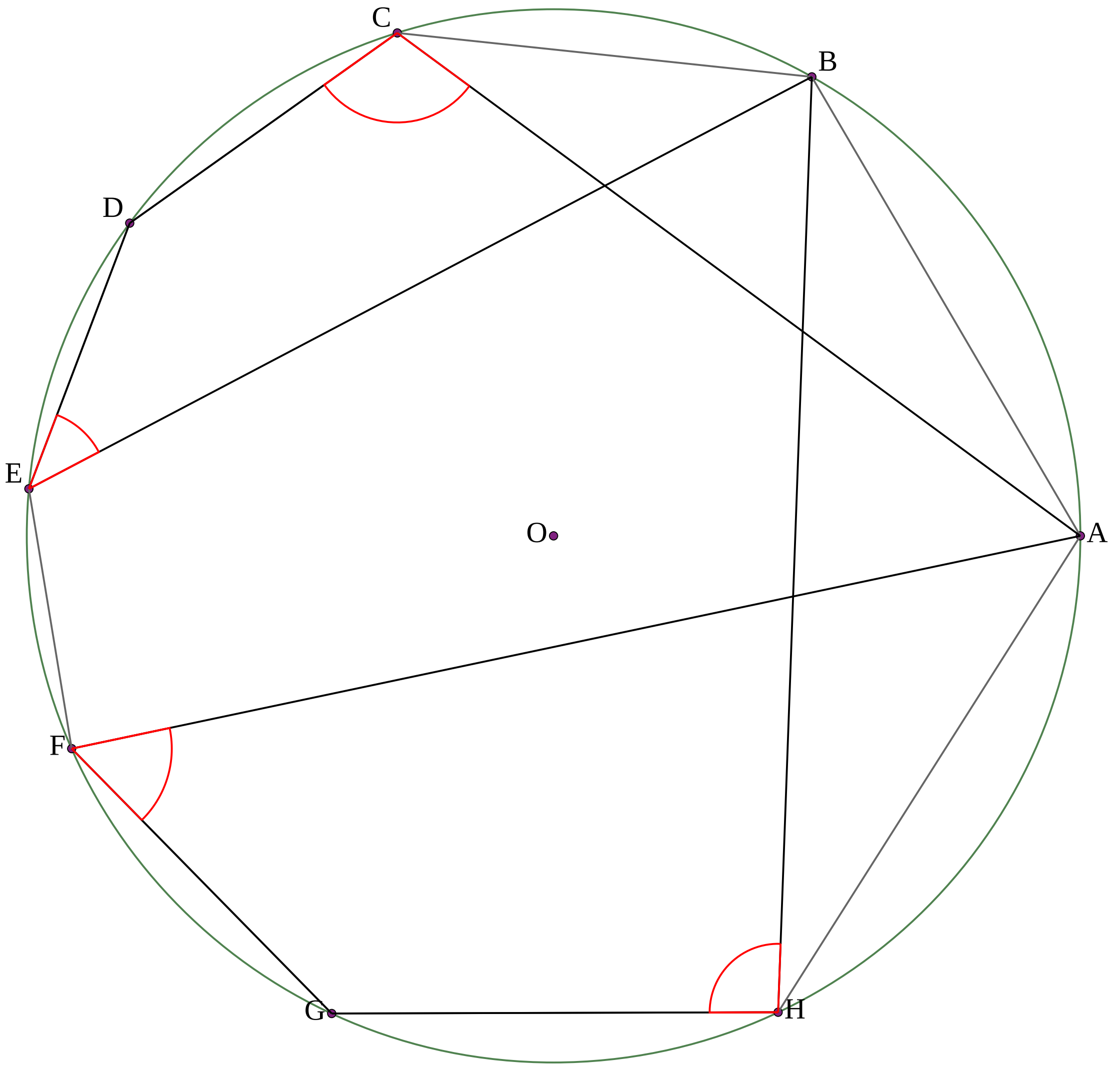
Let ABCDEFG be a cyclic heptagon with center O.
Angle DGE = 19°. Angle ECF = 31°. Angle FAB = 92°.
Find angle BDO.

Example 39



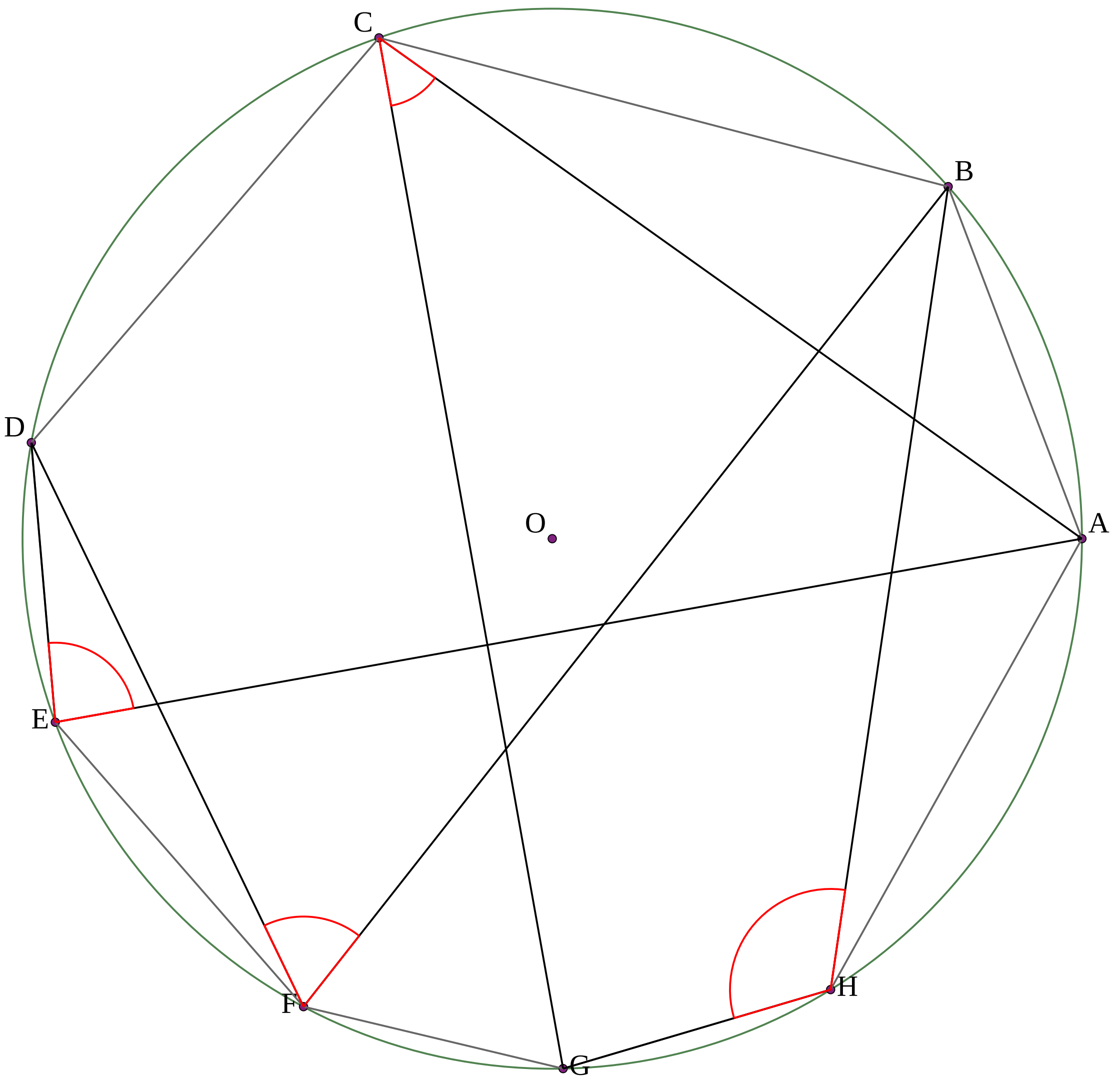
Let ABCDEFGH be a cyclic octagon with center O.
Angle ABC = 145°. Angle CDE = 131°. Angle EFG = 145°.
Find angle GHA.

Example 40



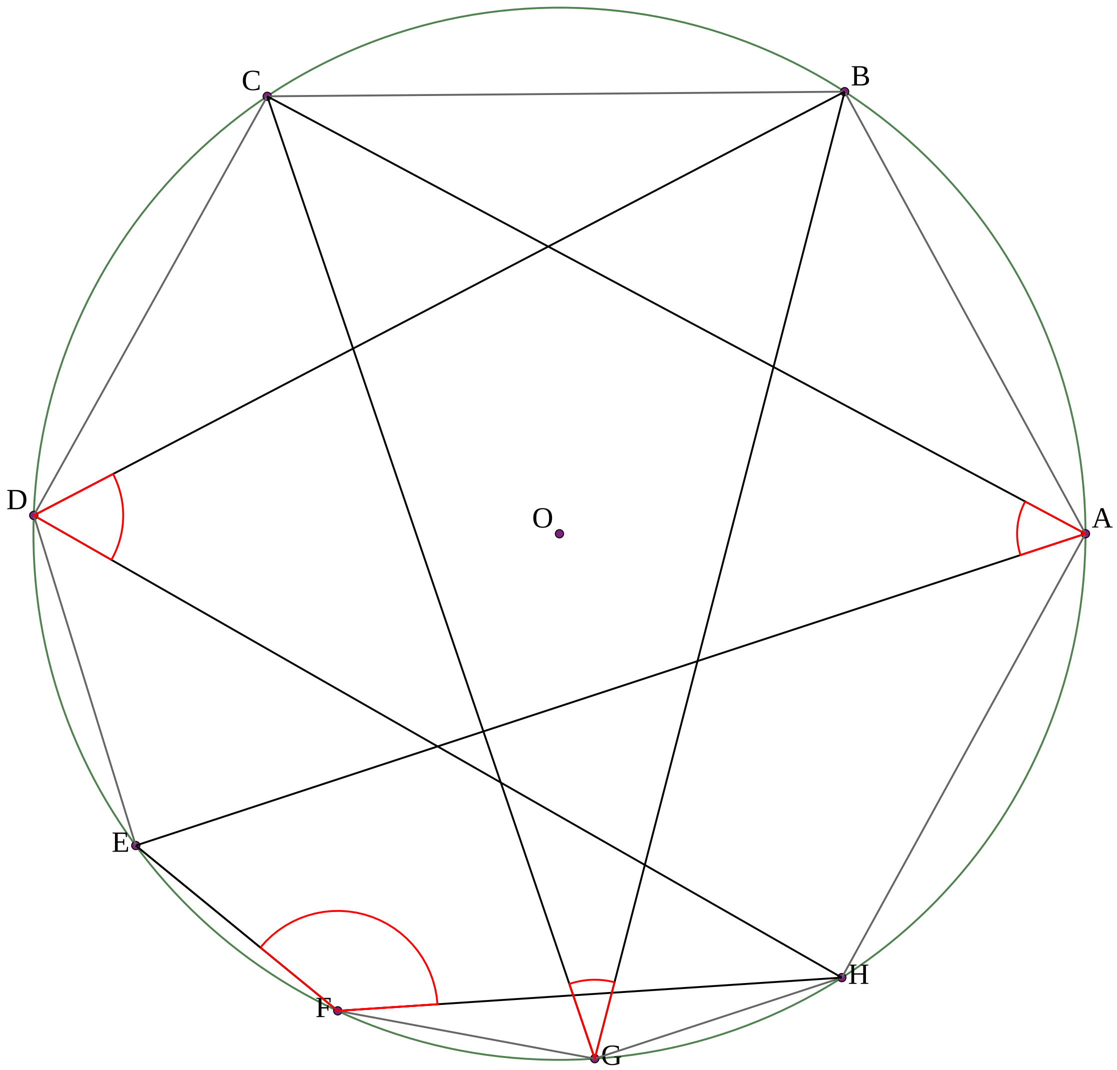
Let ABCDEFGH be a cyclic octagon with center O.
Prove that BHG+AFG = BED+ACD

Example 41



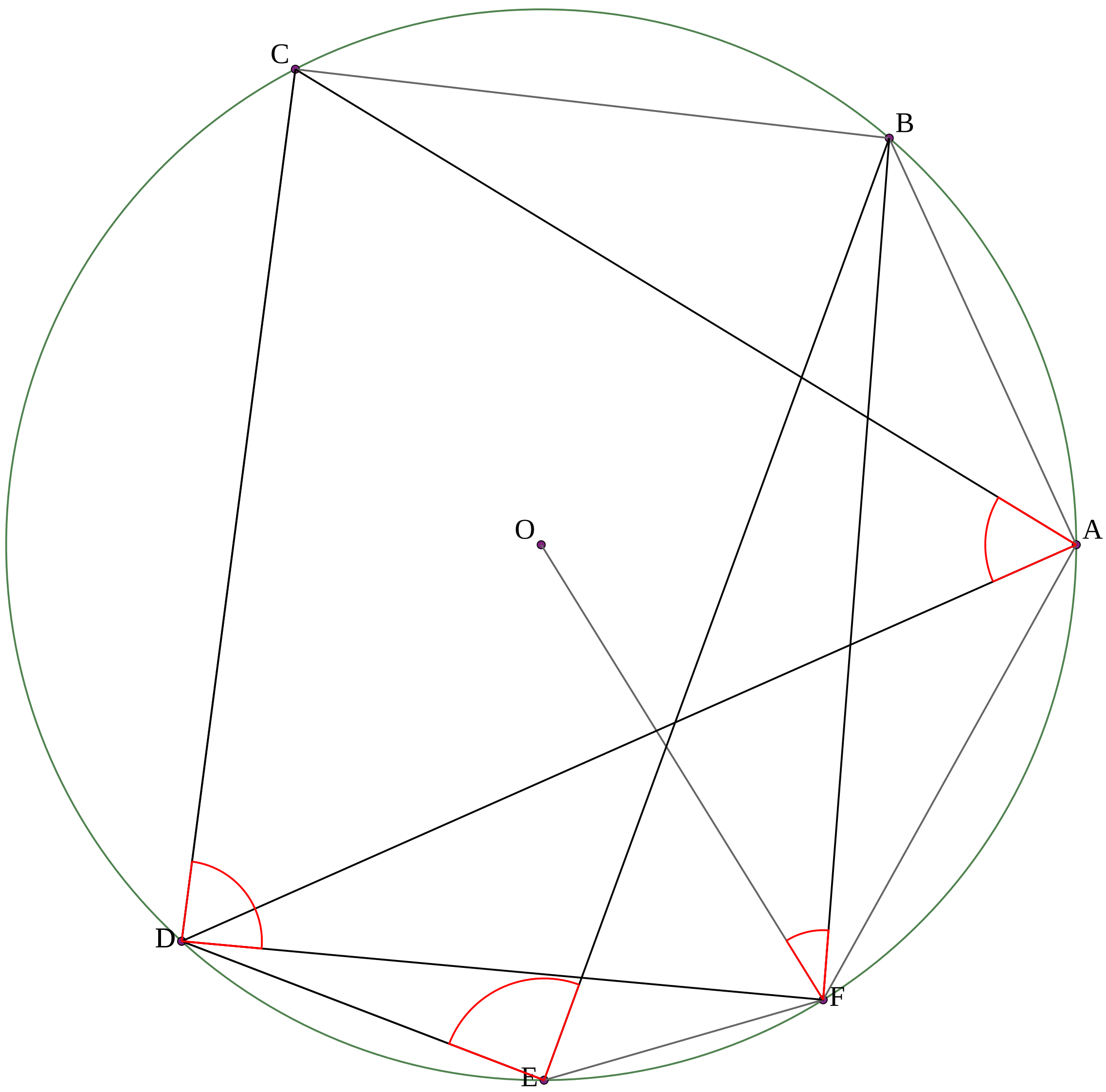
Let ABCDEFGH be a cyclic octagon with center O.
Prove that ACG+AED+BHG = BFD+180

Example 42



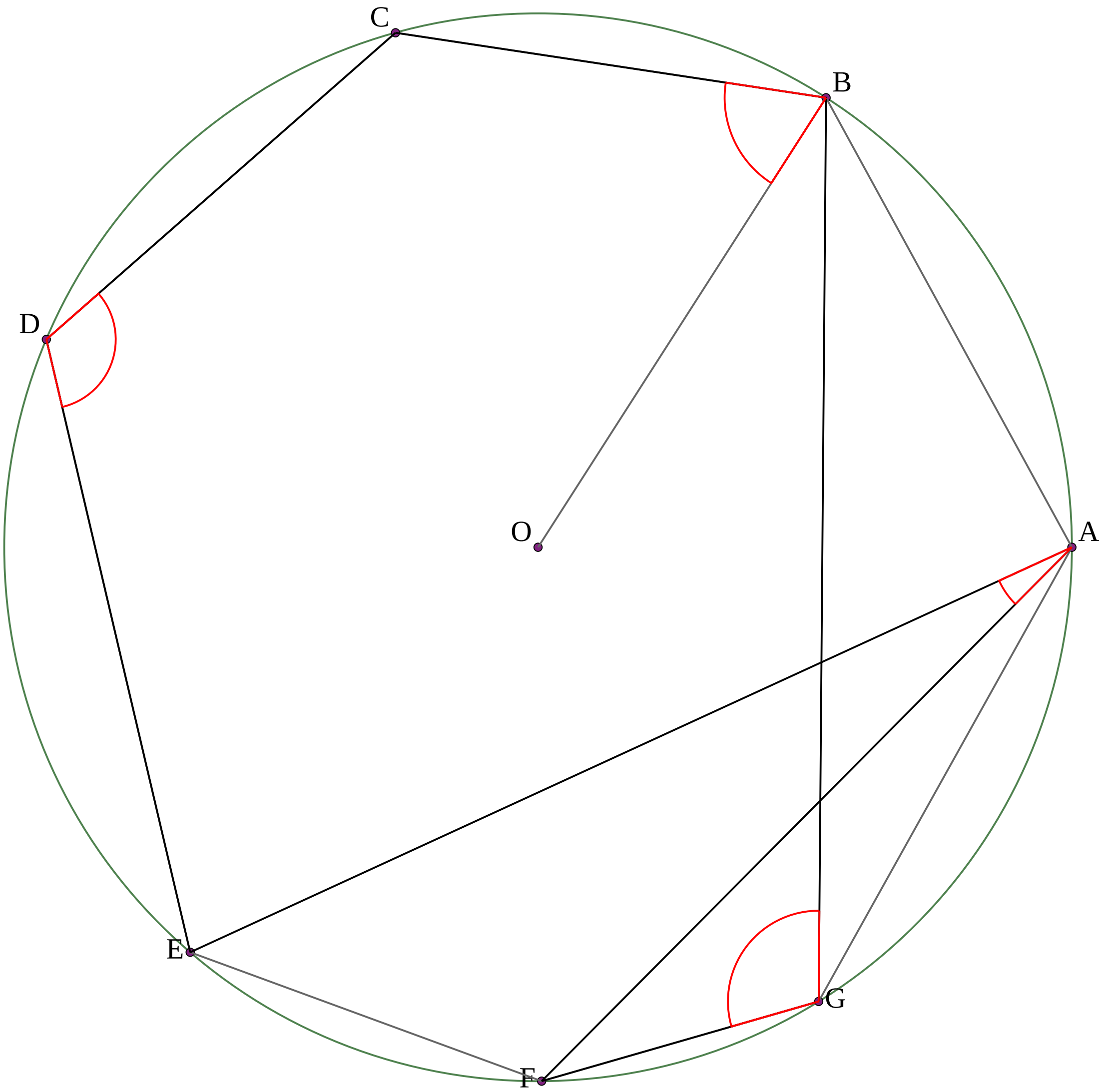
Let ABCDEFGH be a cyclic octagon with center O.
Prove that CAE+BGC+BDH = EFH

Example 43



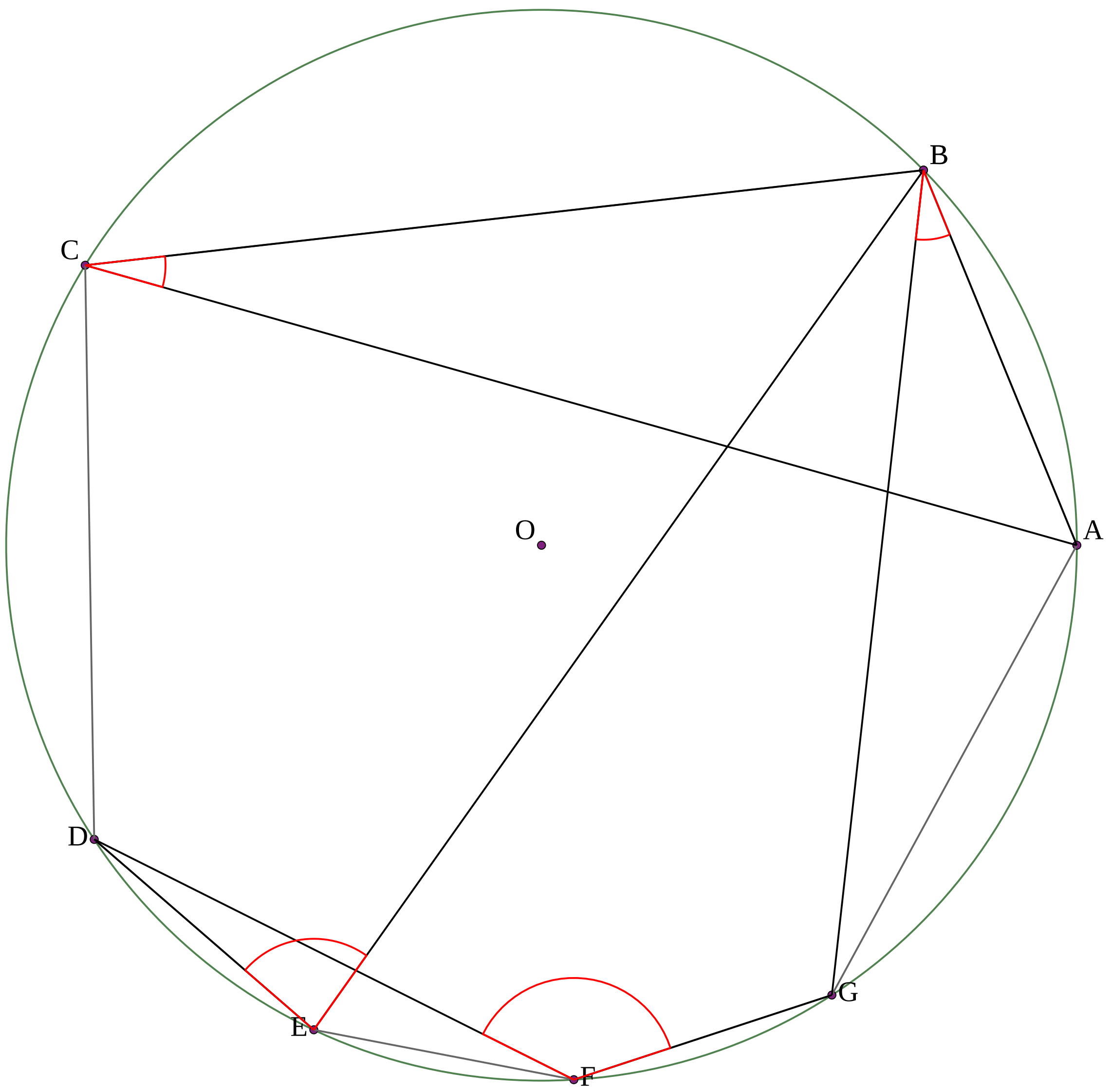
Let ABCDEF be a cyclic hexagon with center O.
Prove that BFO+CDF+CAD = BED+90

Example 44



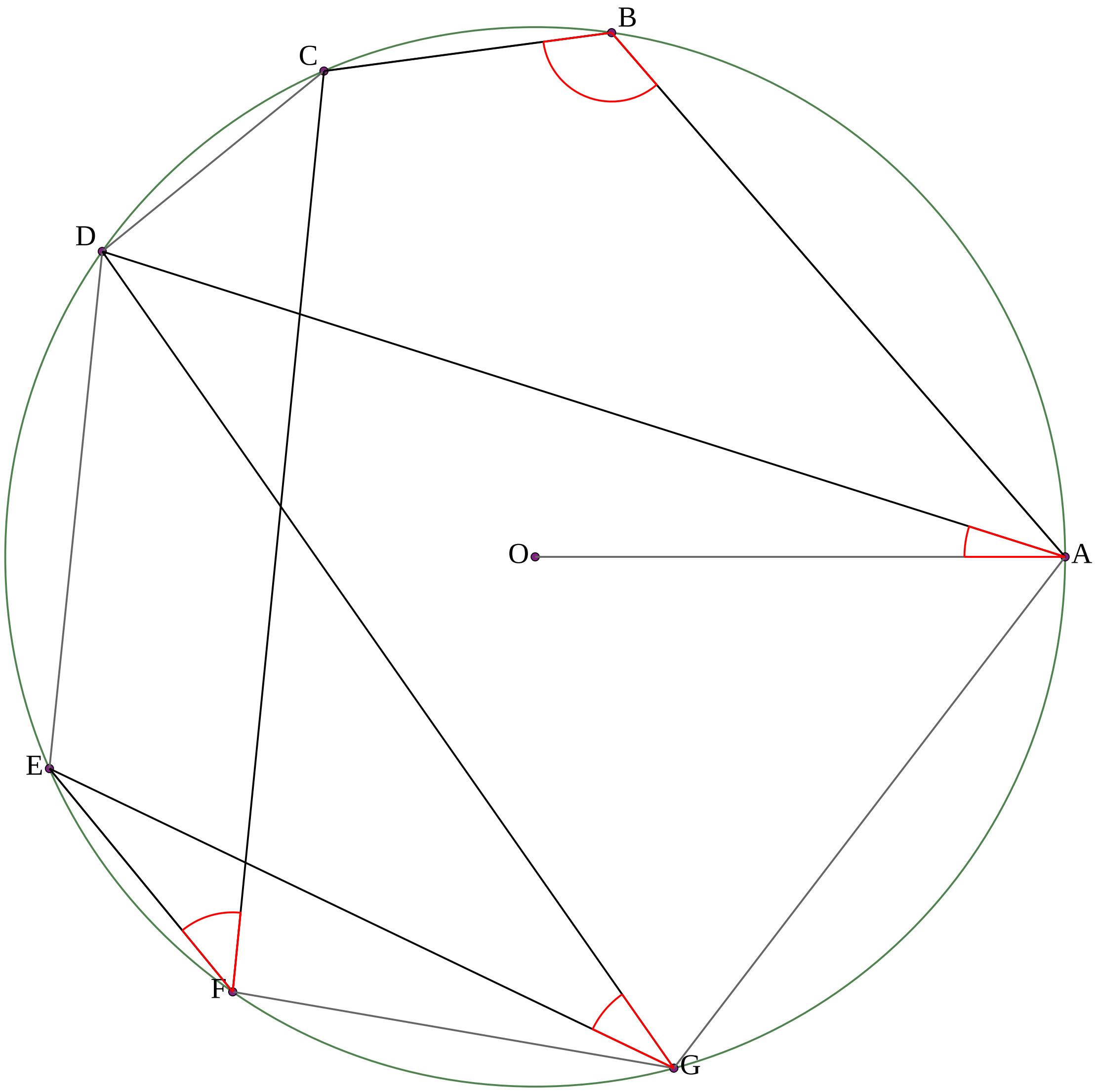
Let ABCDEFG be a cyclic heptagon with center O.
Angle EAF = x. Angle CDE = y. Angle FGB = z.
Find angle OBC.

Example 45



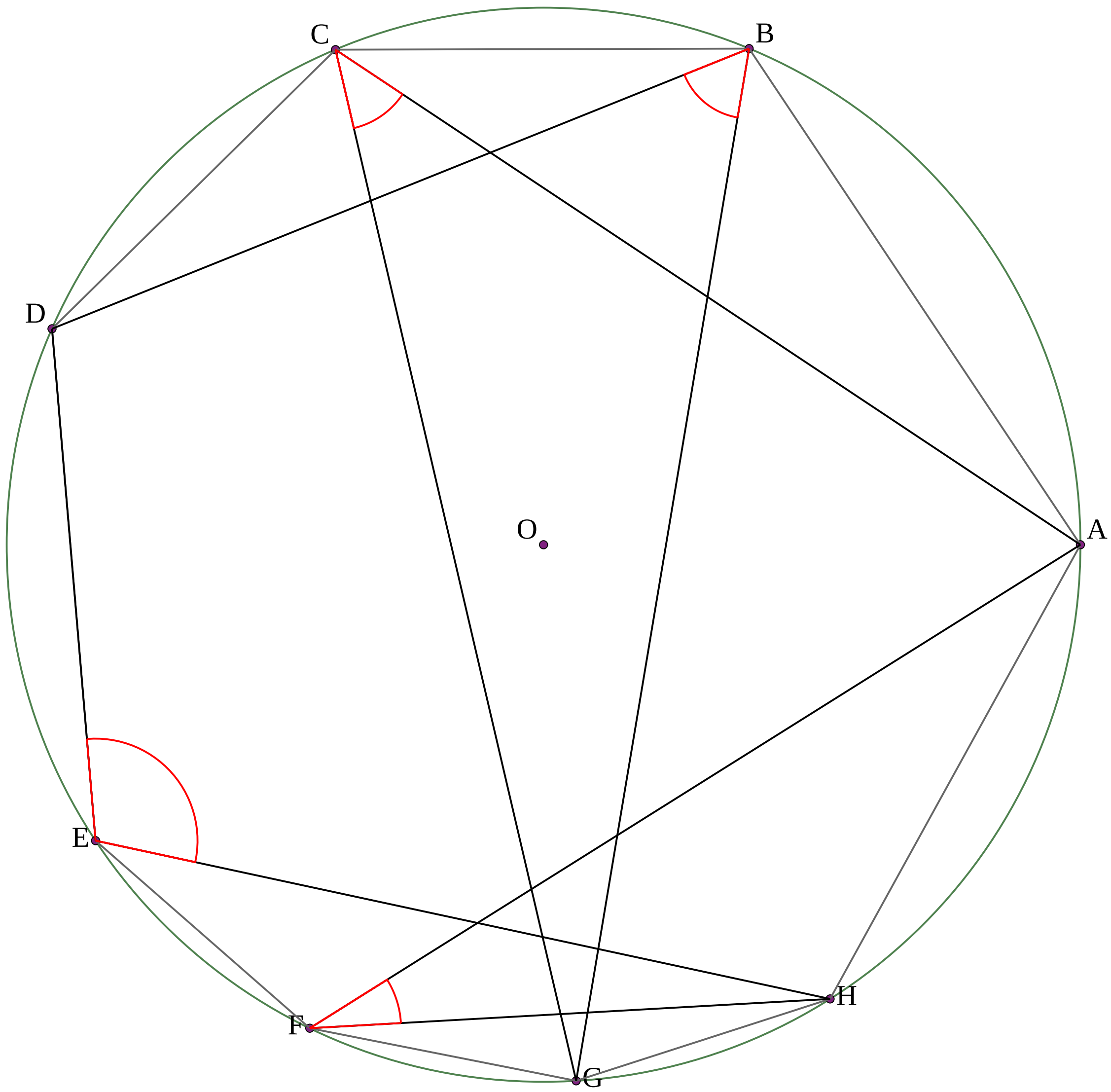
Let ABCDEFG be a cyclic heptagon with center O.
Prove that DFG = ABG+ACB+BED

Example 46



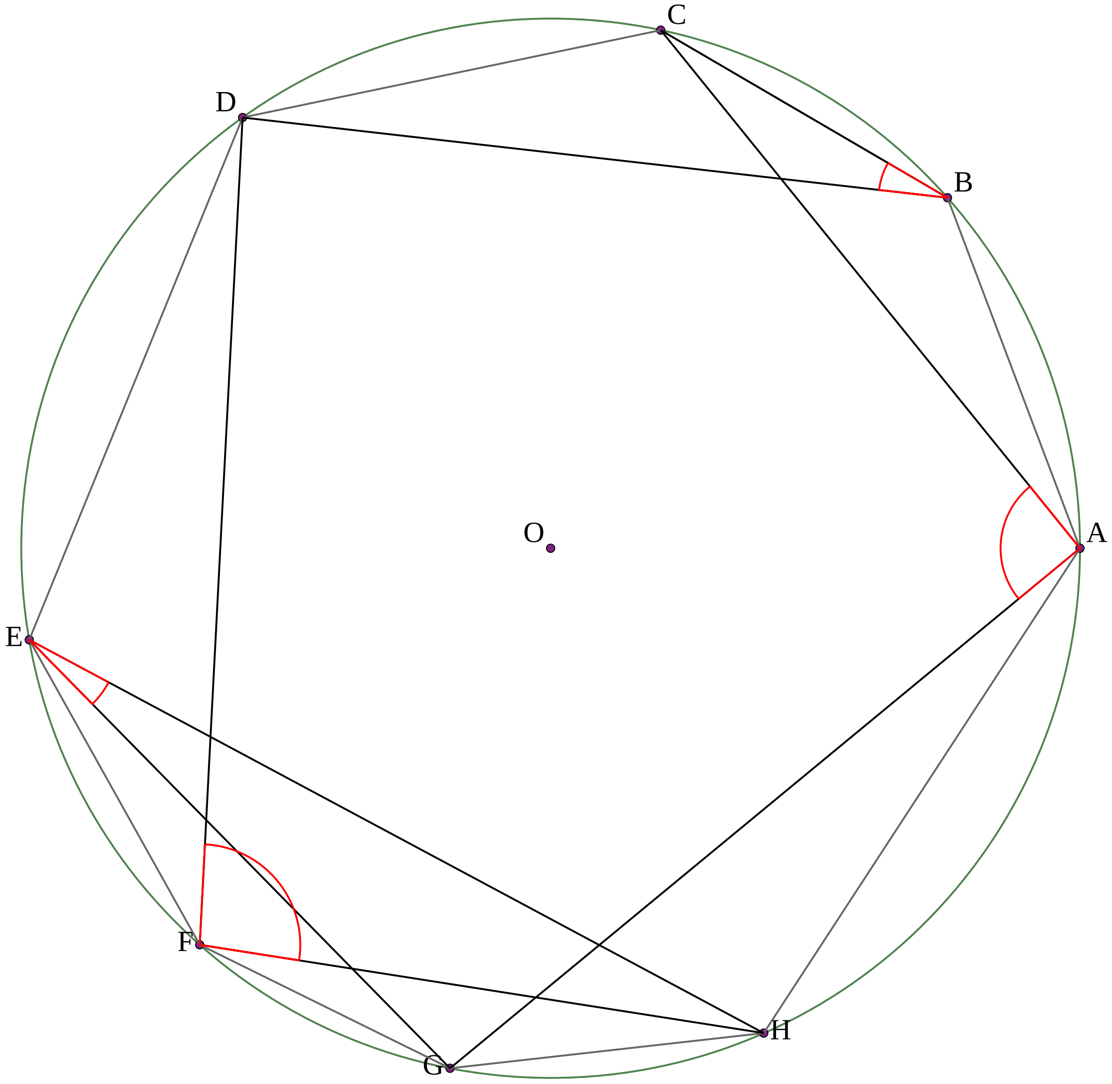
Let ABCDEFG be a cyclic heptagon with center O.
Angle ABC = x. Angle EGD = y. Angle CFE = z.
Find angle DAO.

Example 47



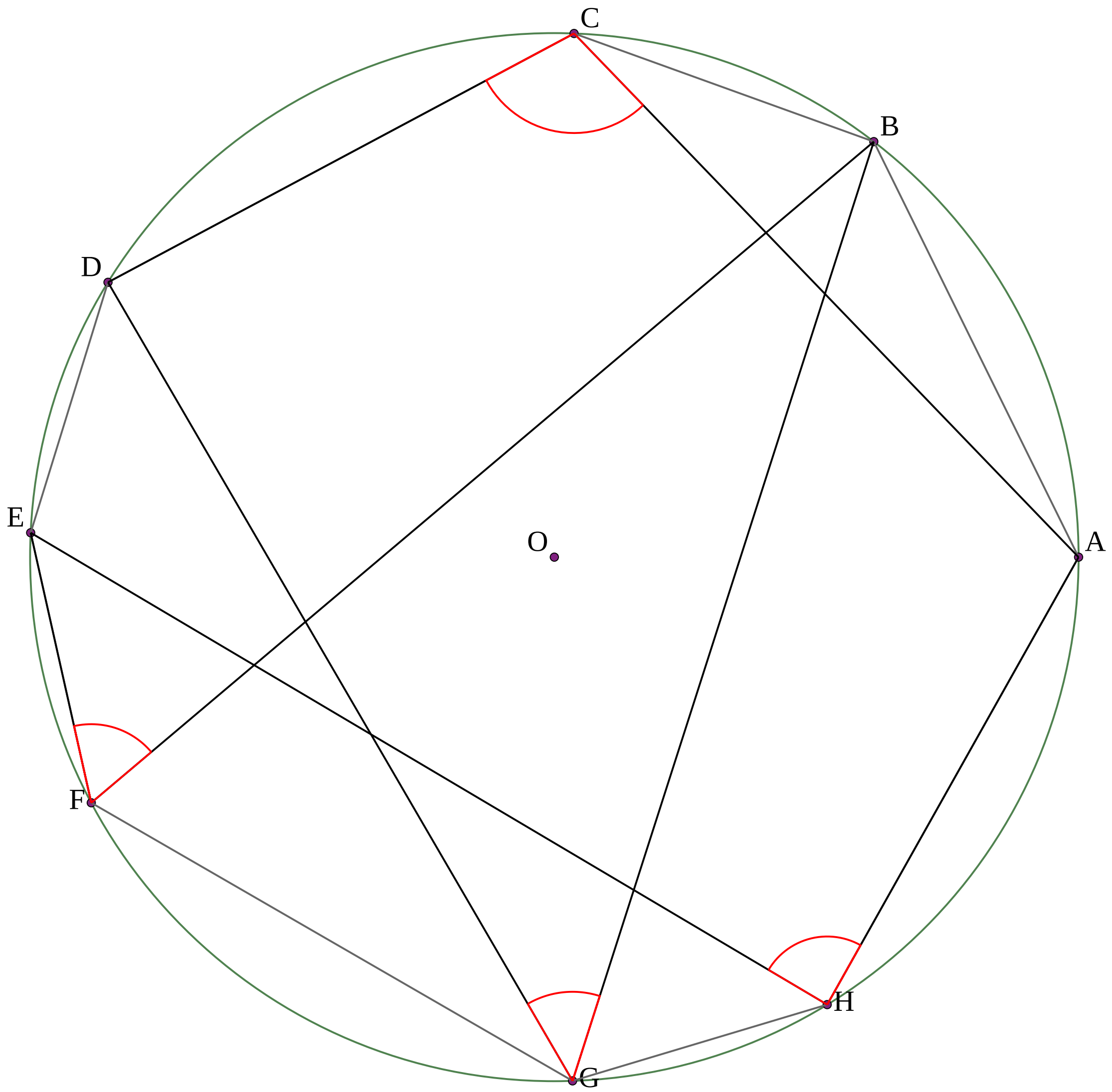
Let ABCDEFGH be a cyclic octagon with center O.
Prove that DBG+ACG+DEH = AFH+180

Example 48



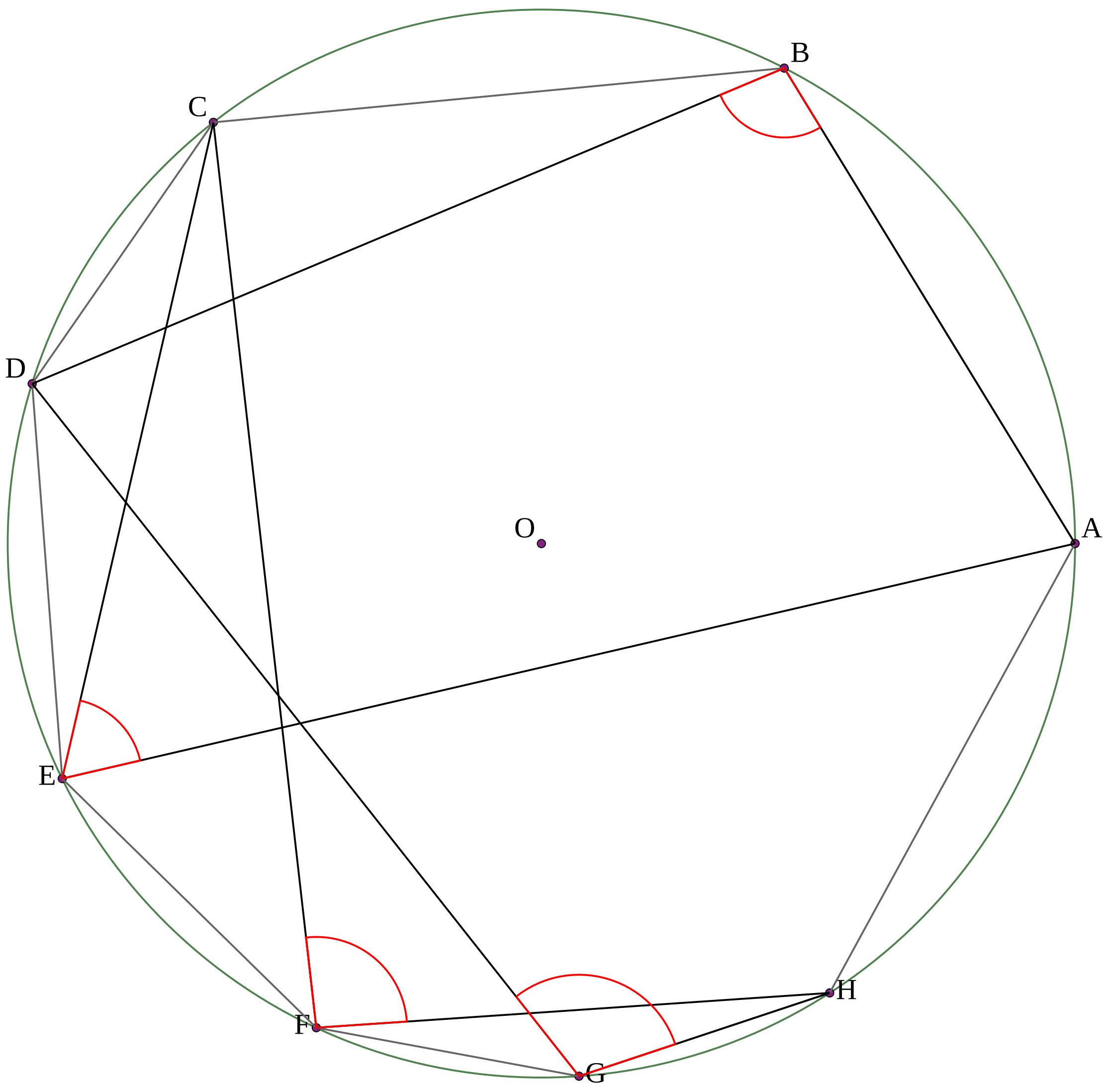
Let ABCDEFGH be a cyclic octagon with center O.
Prove that CAG+GEH+DFH = CBD+180

Example 49



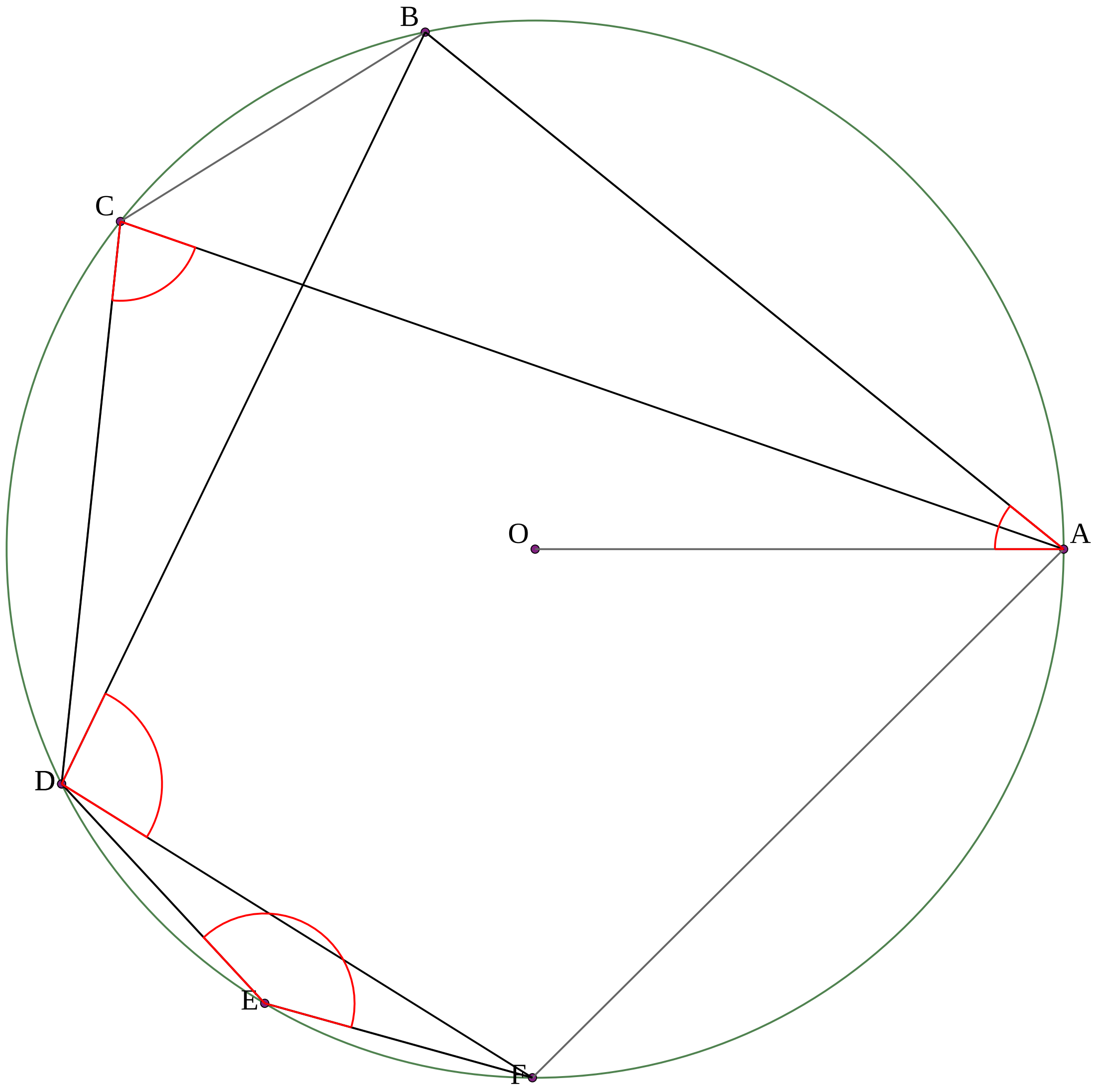
Let ABCDEFGH be a cyclic octagon with center O.
Prove that AHE+BGD+ACD = BFE+180

Example 50



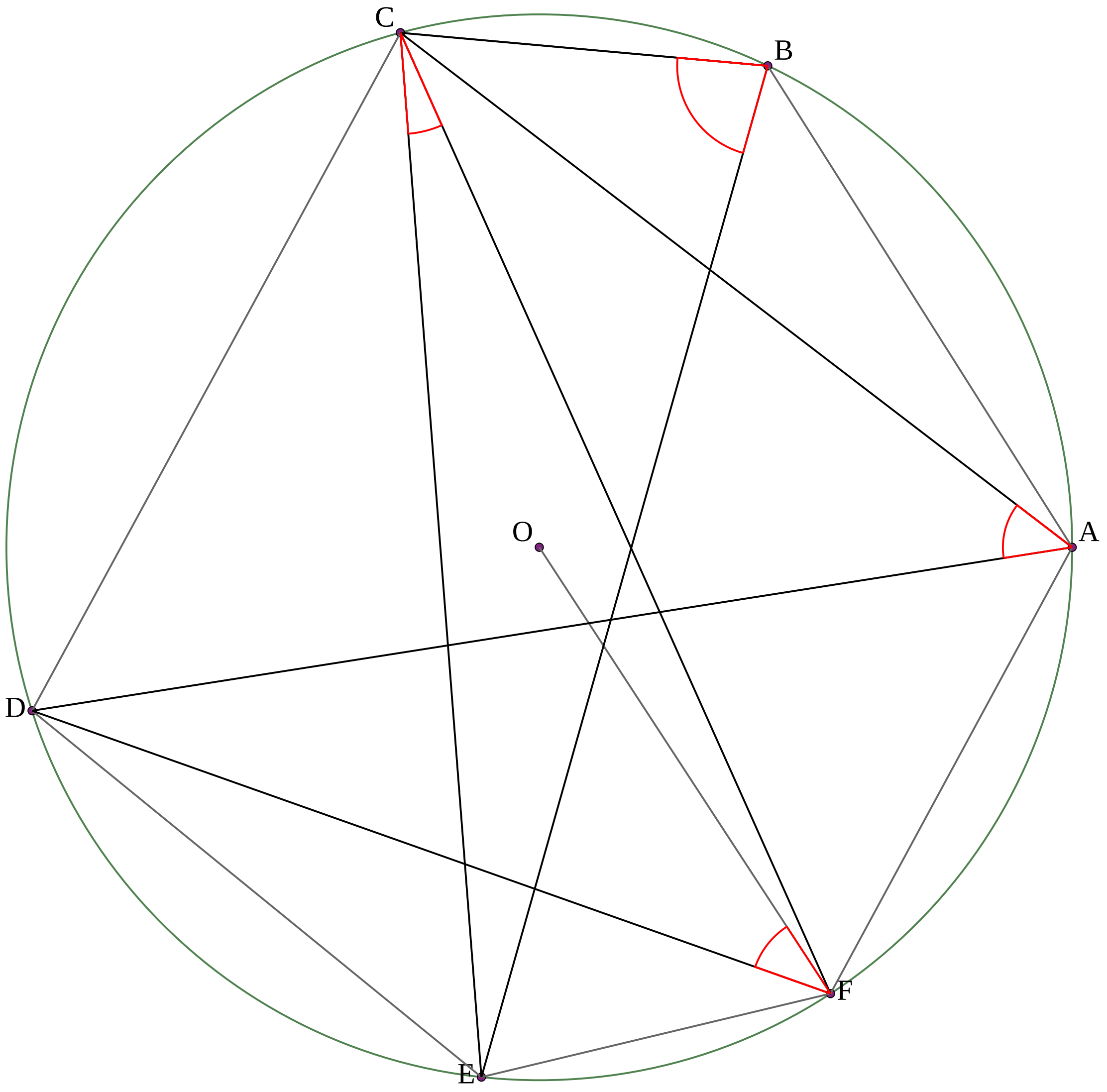
Let ABCDEFGH be a cyclic octagon with center O.
Prove that ABD+AEC+DGH = CFH+180

Example 51



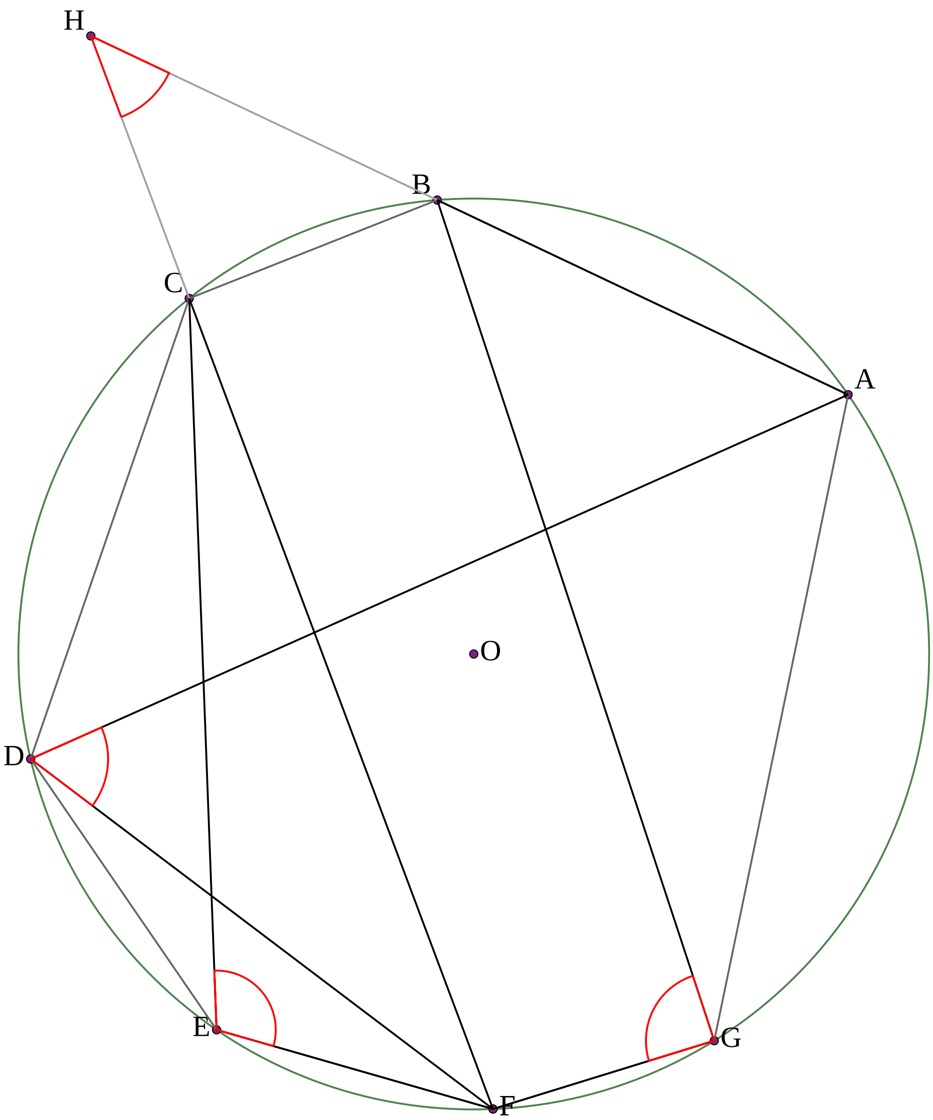
Let ABCDEF be a cyclic hexagon with center O.
Prove that ACD+DEF = BAO+BDF+90

Example 52



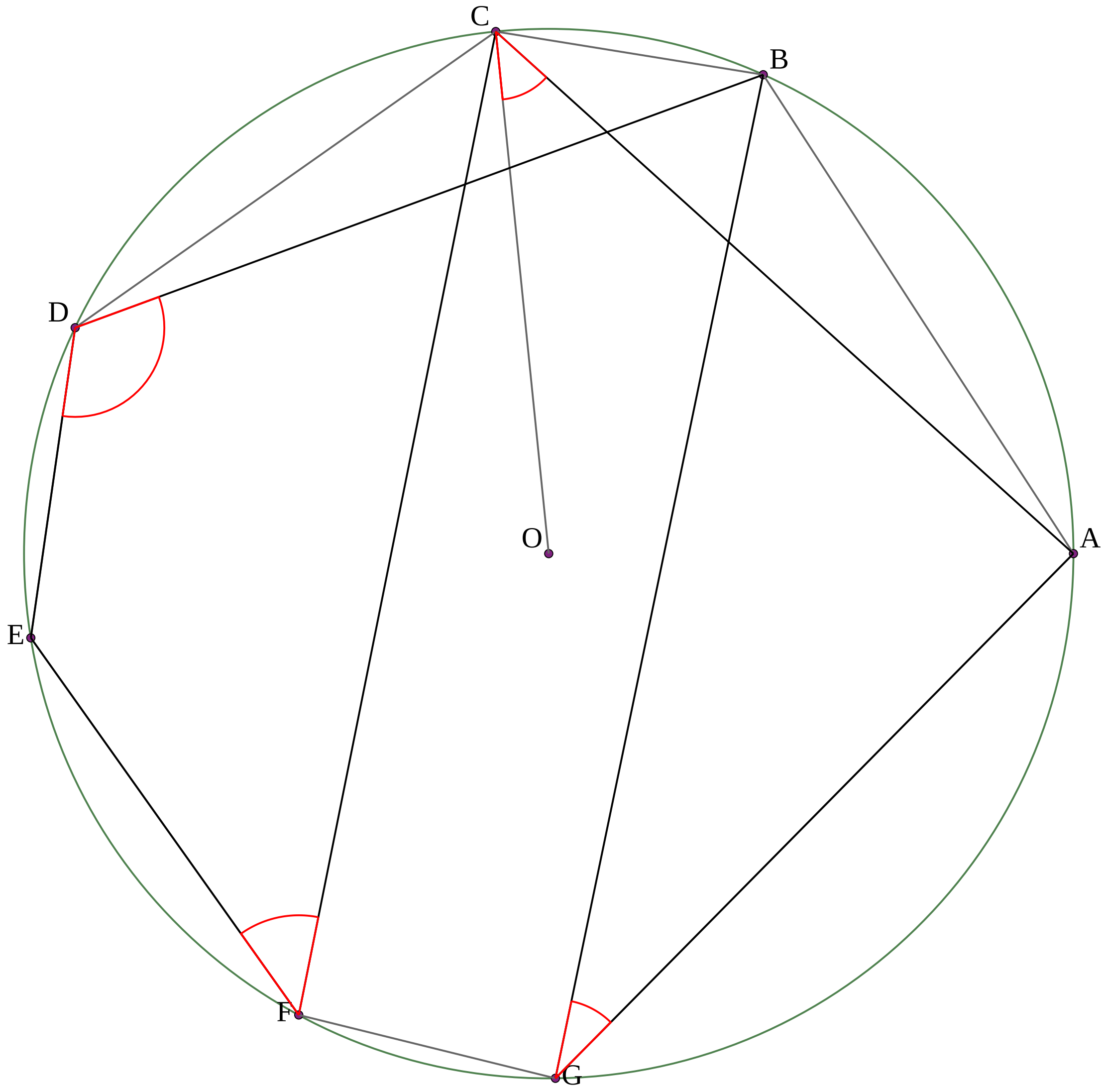
Let ABCDEF be a cyclic hexagon with center O.
Prove that DFO+CBE+ECF = CAD+90

Example 53



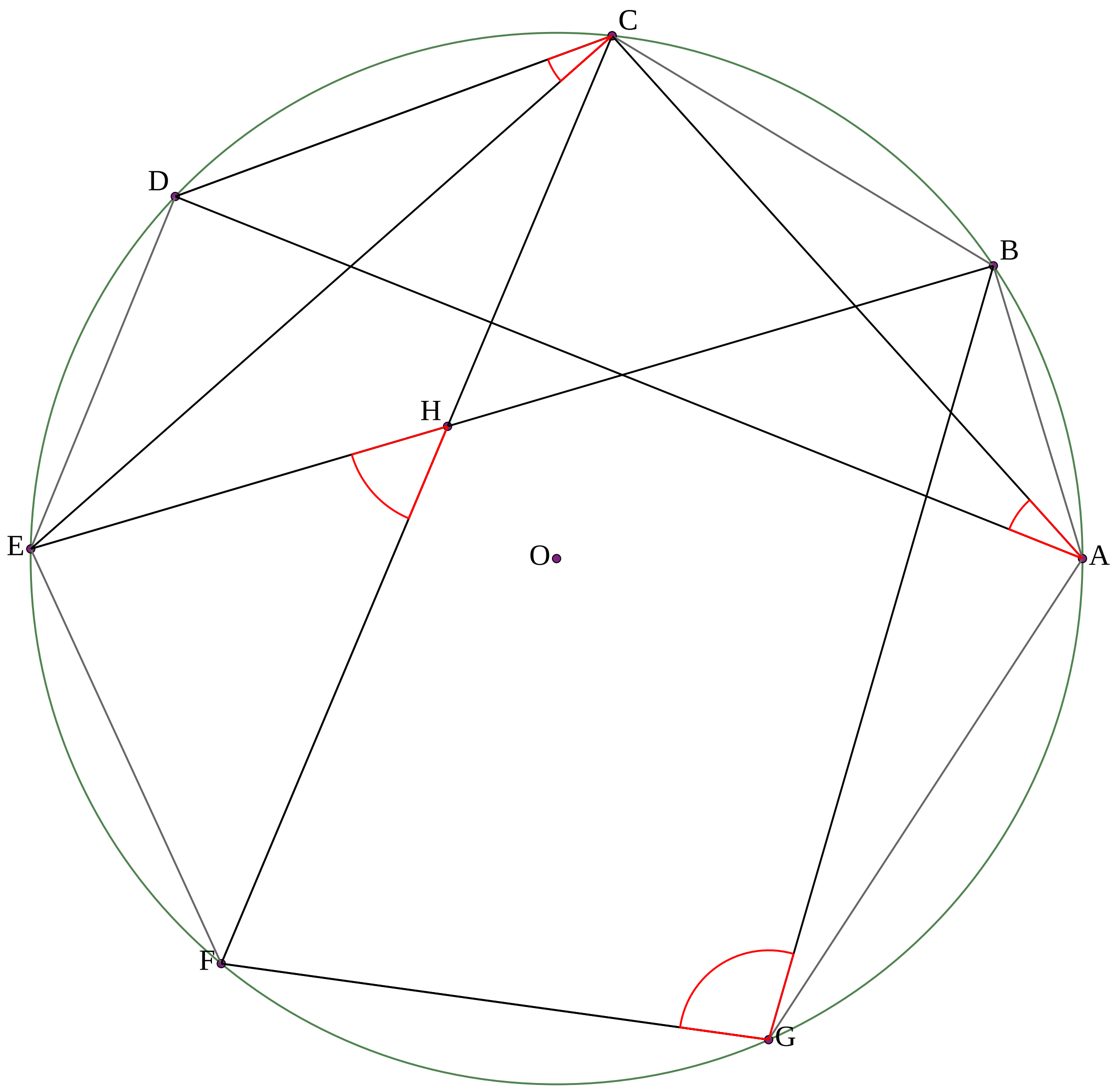
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of BA and FC.
Angle ADF = 61°. Angle FGB = 89°. Angle BHC = 44°.
Find angle CEF.

Example 54



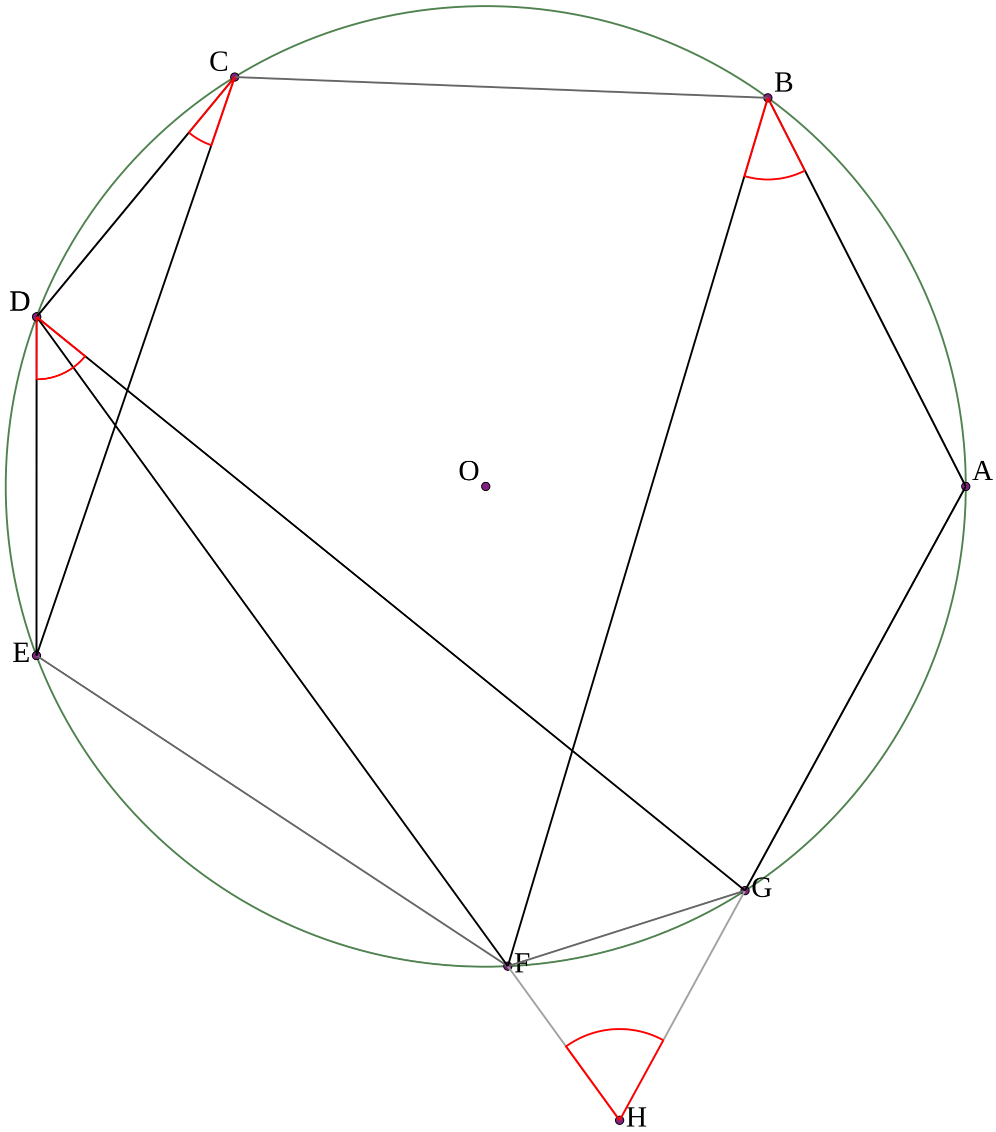
Let ABCDEFG be a cyclic heptagon with center O.
Prove that BDE+CFE = ACO+AGB+90

Example 55



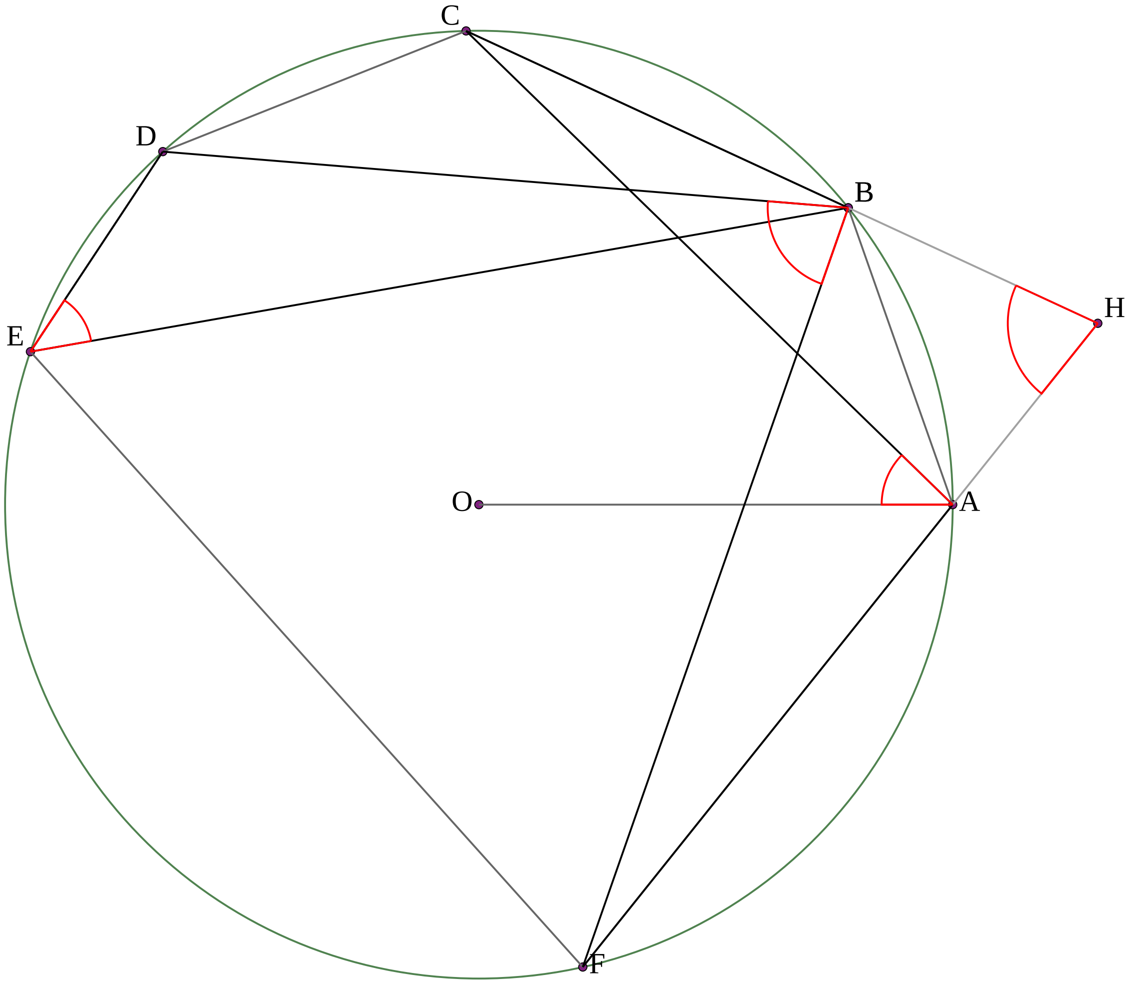
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CF and BE.
Angle FGB = 98°. Angle DAC = 26°. Angle FHE = 51°.
Find angle ECD.

Example 56



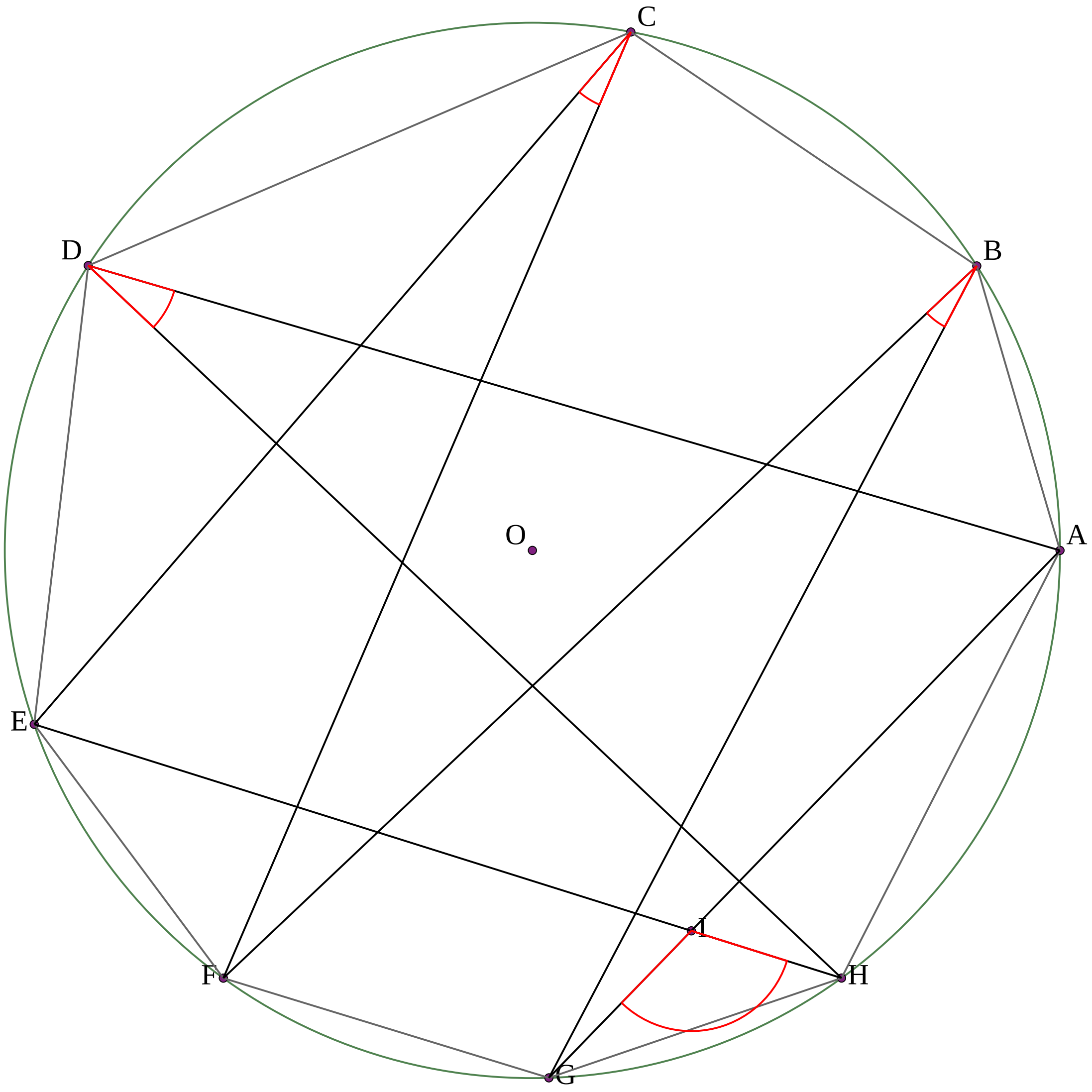
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of DF and AG.
Angle GDE = 51°. Angle FBA = 44°. Angle FHG = 65°.
Find angle ECD.

Example 57



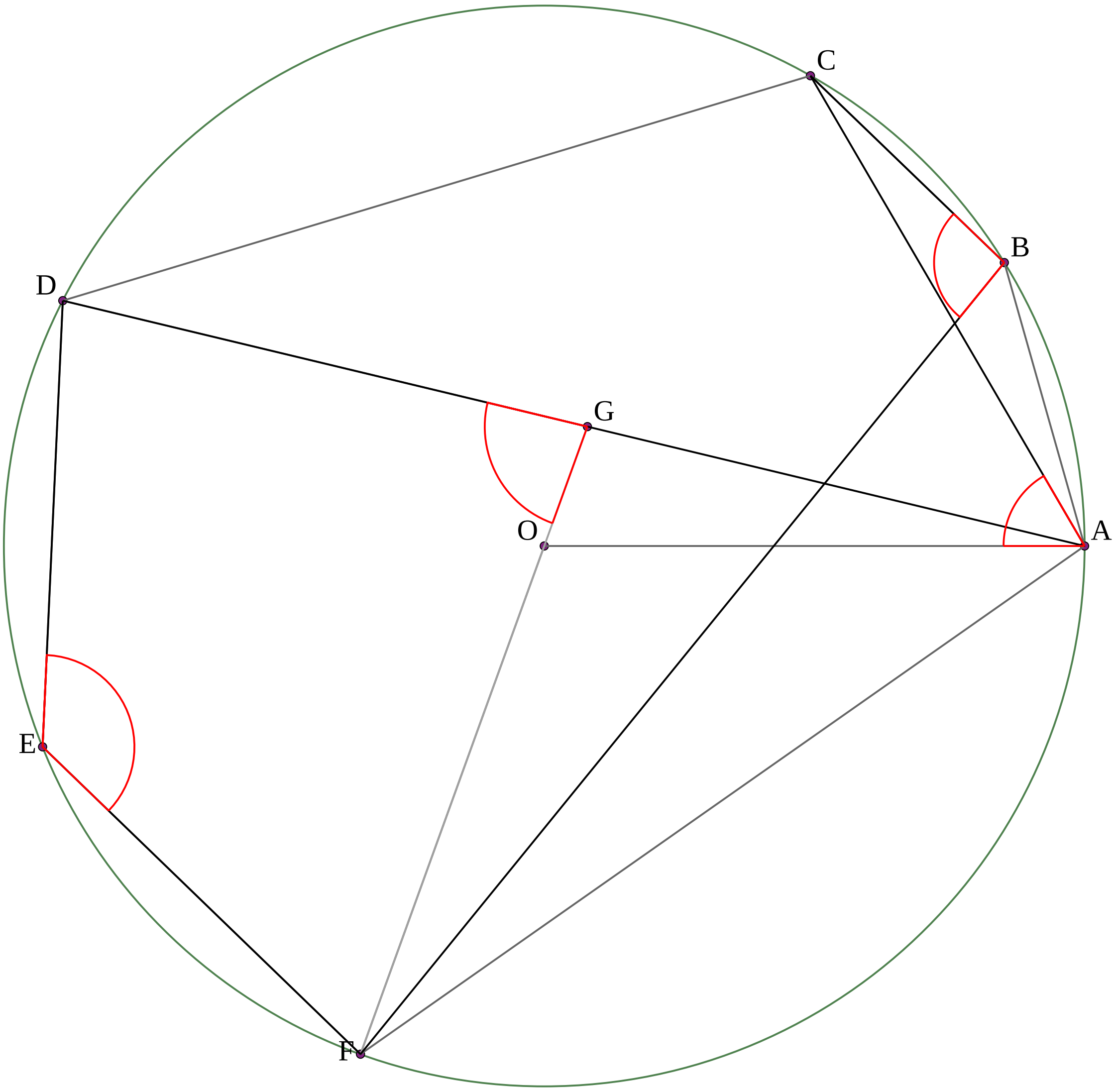
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of FA and CB.
Angle DEB = 47°. Angle OAC = 44°. Angle FBD = 75°.
Find angle AHB.

Example 58



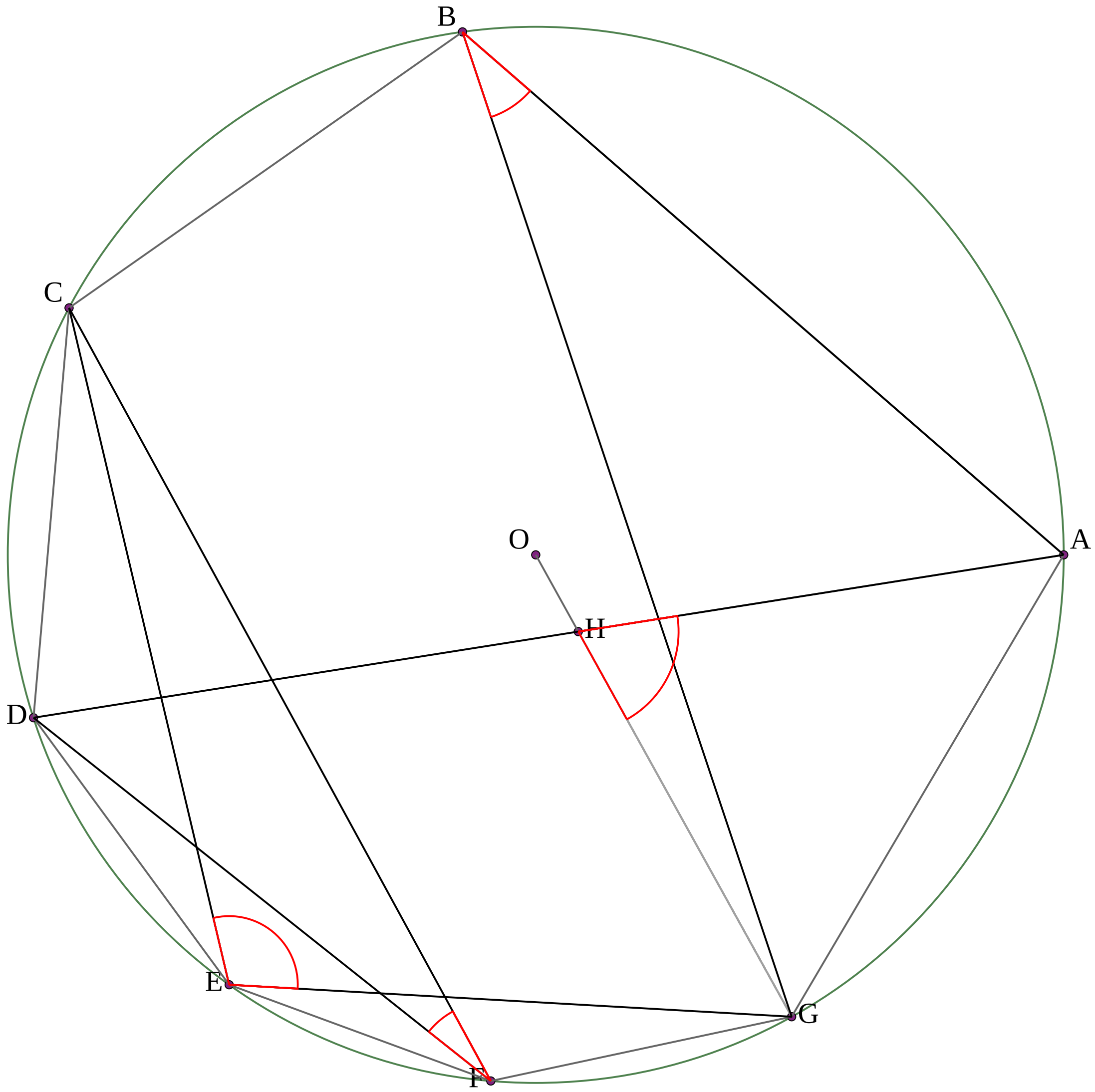
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of EH and AG.
Angle GBF = 19°. Angle HDA = 27°. Angle HIG = 117°.
Find angle FCE.

Example 59



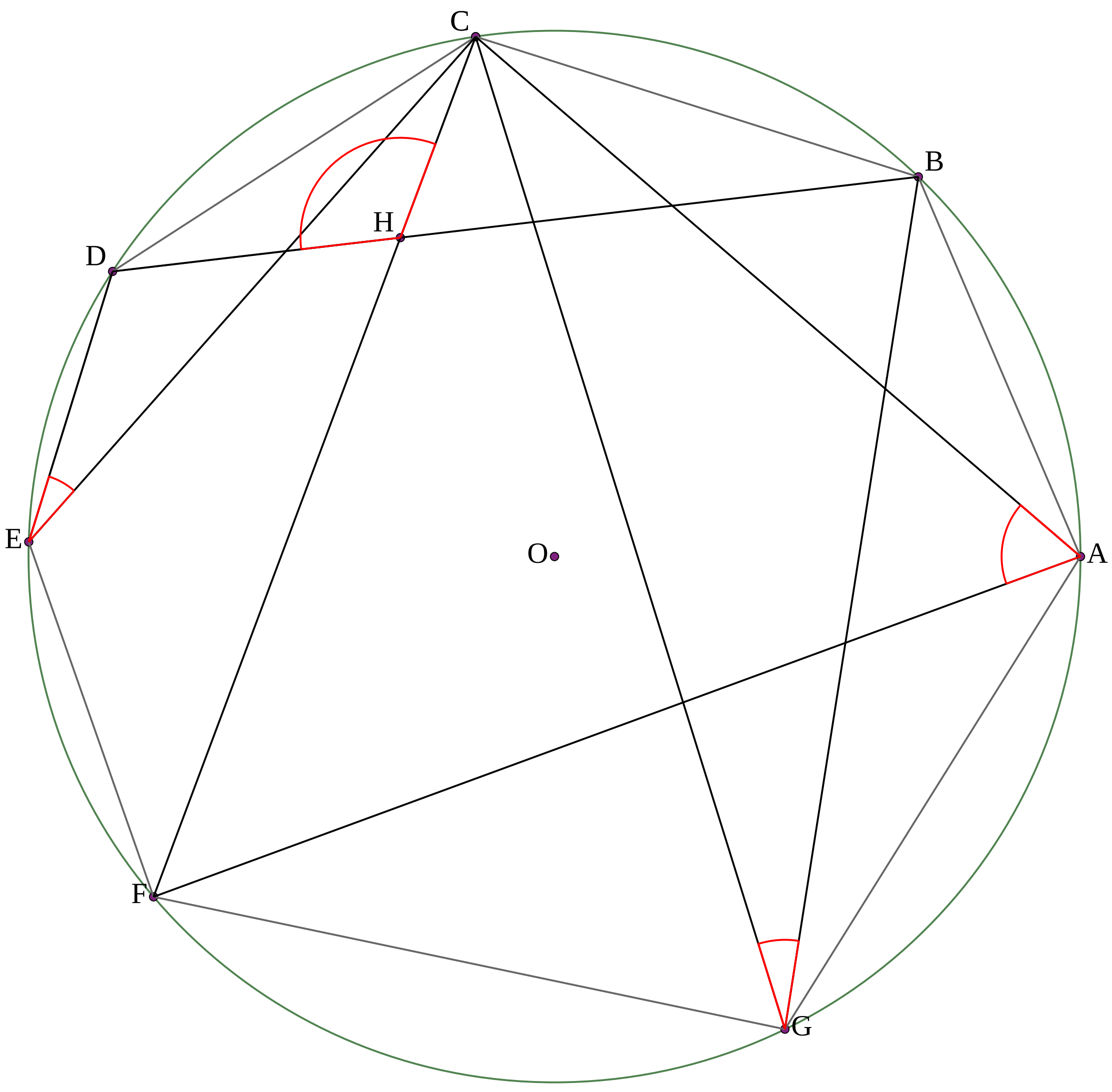
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AD and FO.
Angle FBC = 95°. Angle CAO = 60°. Angle DGF = 84°.
Find angle DEF.

Example 60



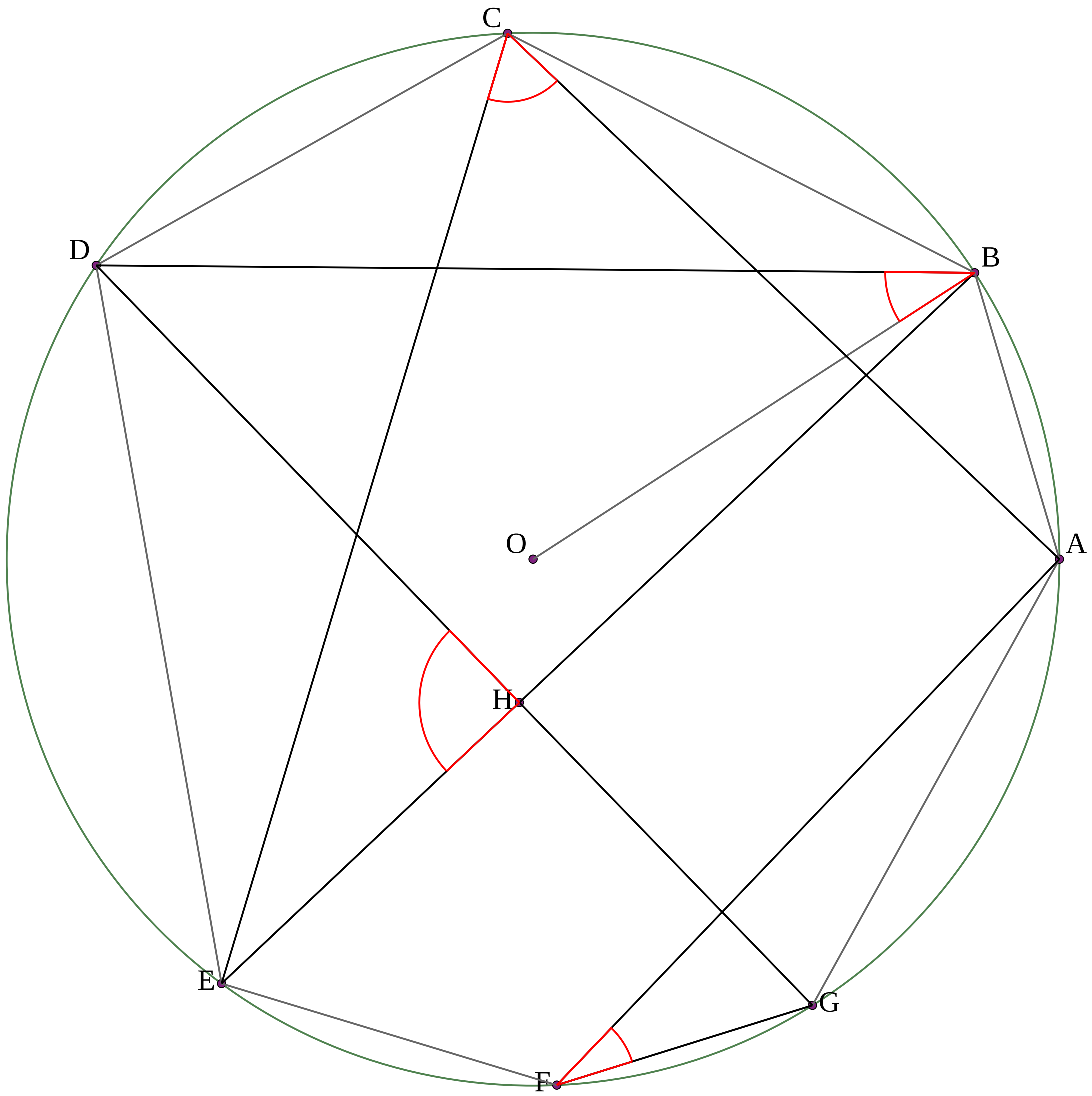
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of DA and GO.
Angle GEC = x. Angle CFD = y. Angle ABG = z.
Find angle AHG.

Example 61



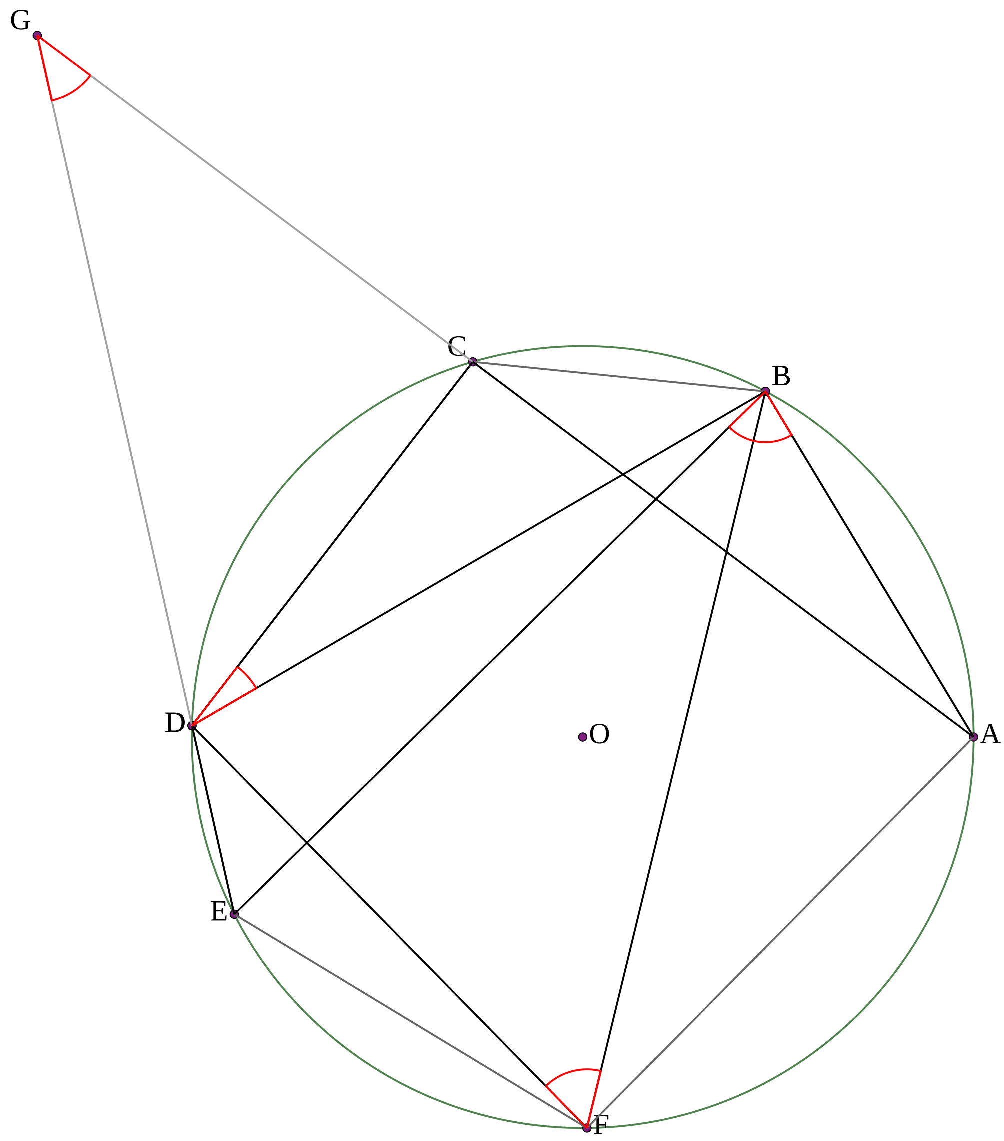
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FC and BD.
Angle DEC = 24°. Angle CGB = 26°. Angle CHD = 117°.
Find angle CAF.

Example 62



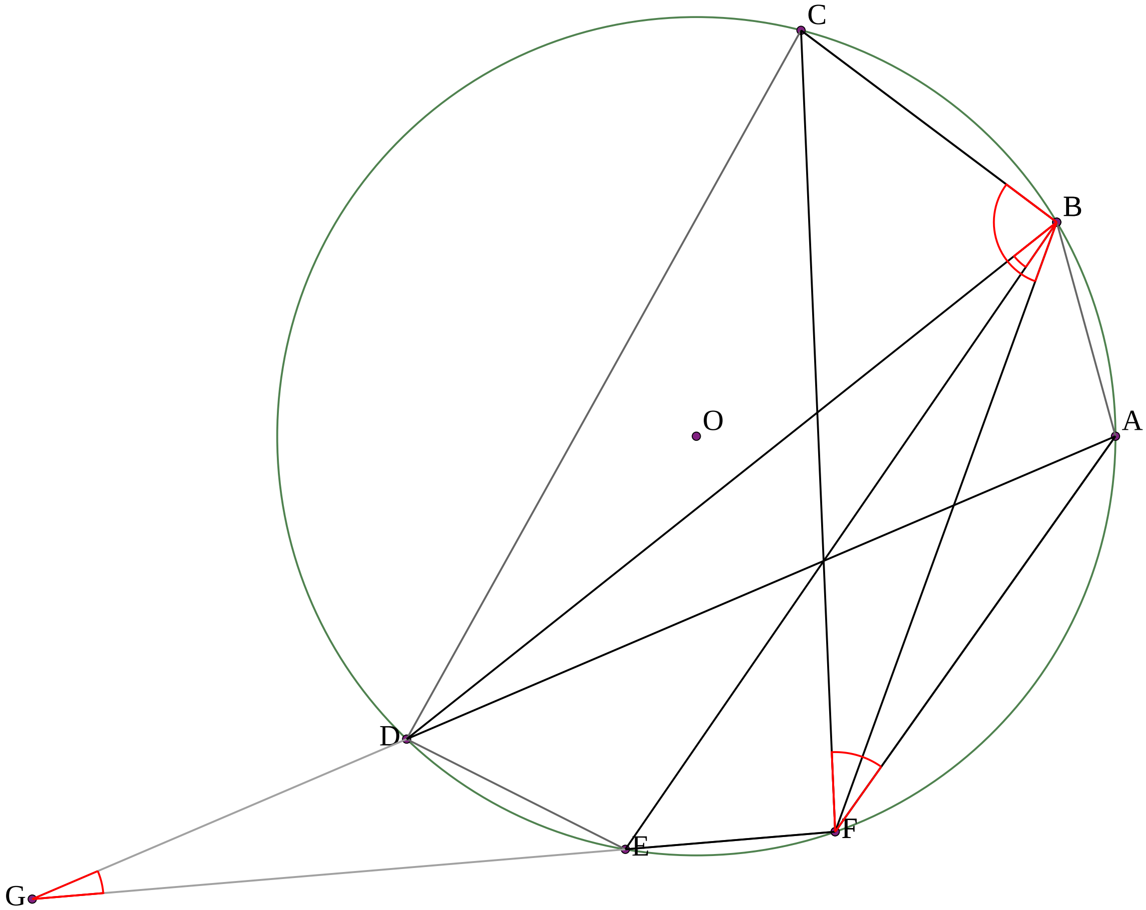
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GD and BE.
Angle ECA = 63°. Angle DHE = 89°. Angle AFG = 29°.
Find angle DBO.

Example 63



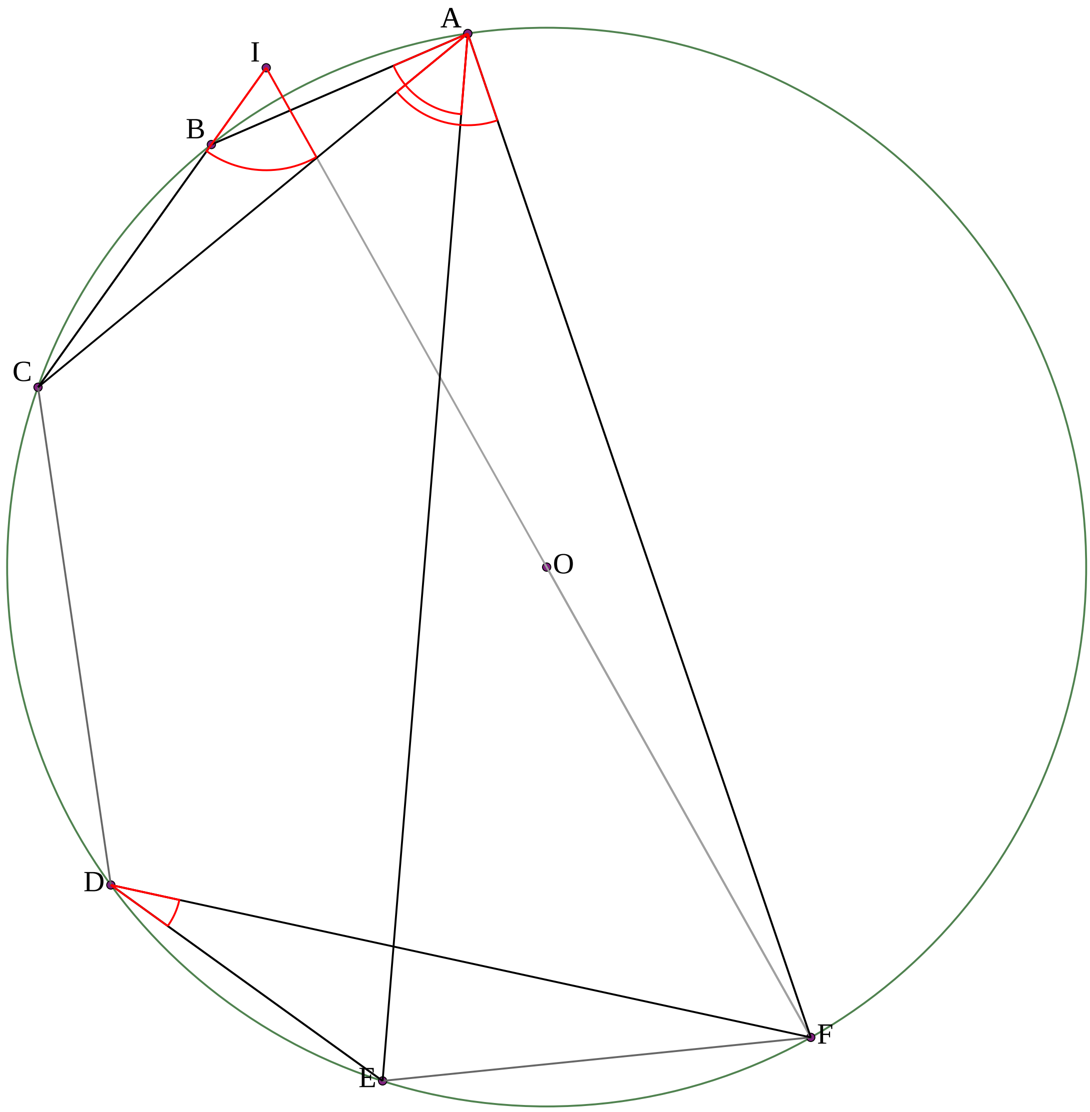
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and CA.
Angle BFD = 58°. Angle DGC = 41°. Angle ABE = 77°.
Find angle BDC.

Example 64



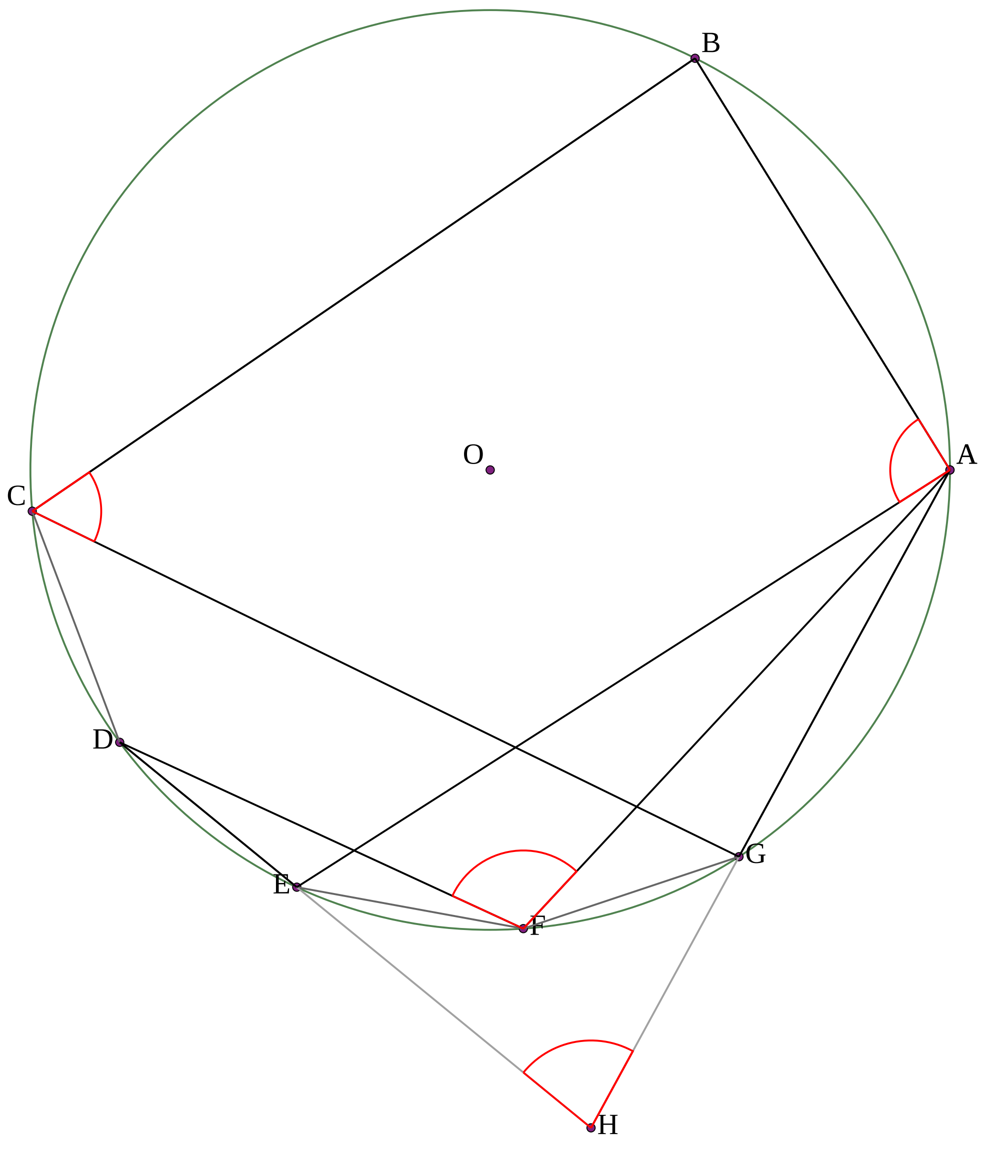
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and AD.
Angle DBE = 17°. Angle EGD = 18°. Angle CFA = 38°.
Find angle CBF.

Example 65



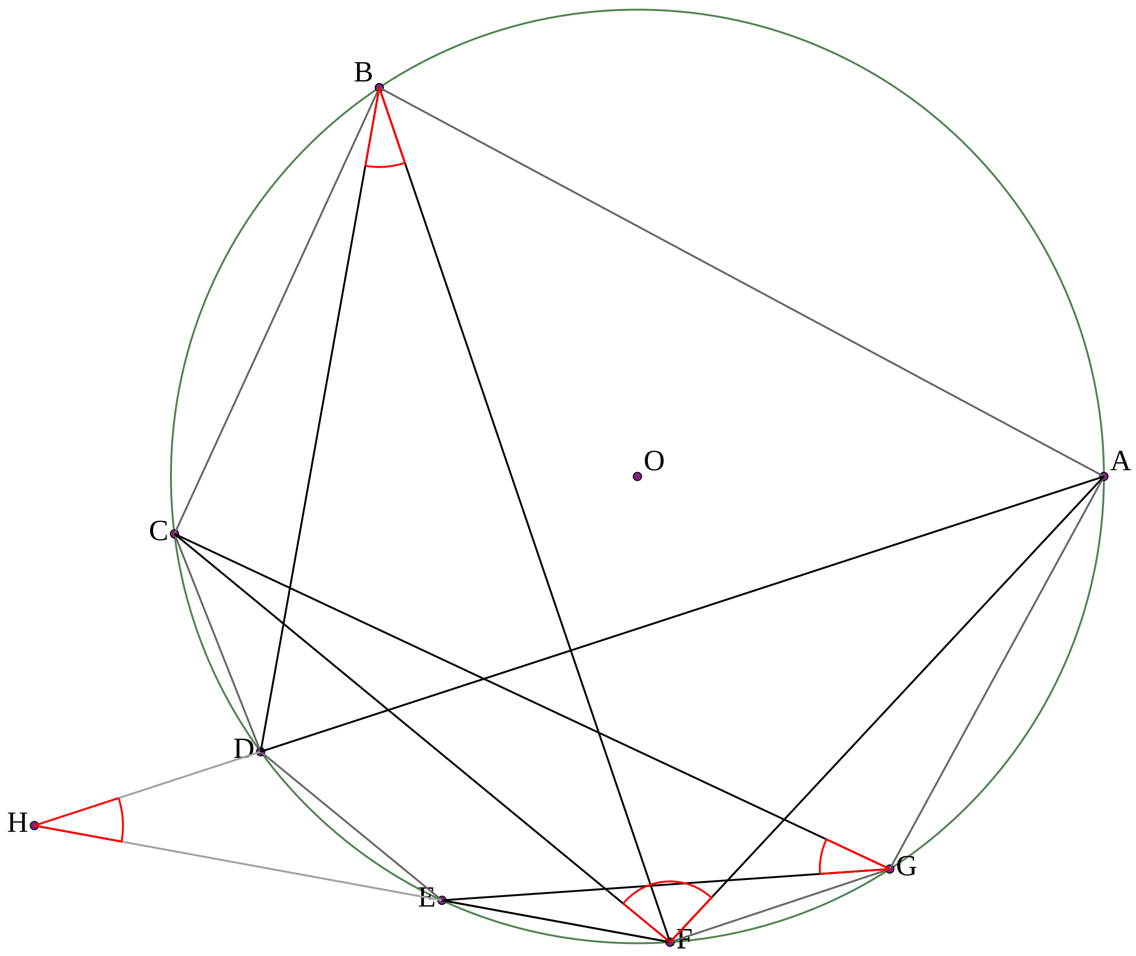
Let ABCDEF be a cyclic hexagon with center O. Let I be the intersection of CB and FO.
Angle FDE = x. Angle EAB = y. Angle CAF = z.
Find angle BIF.

Example 66



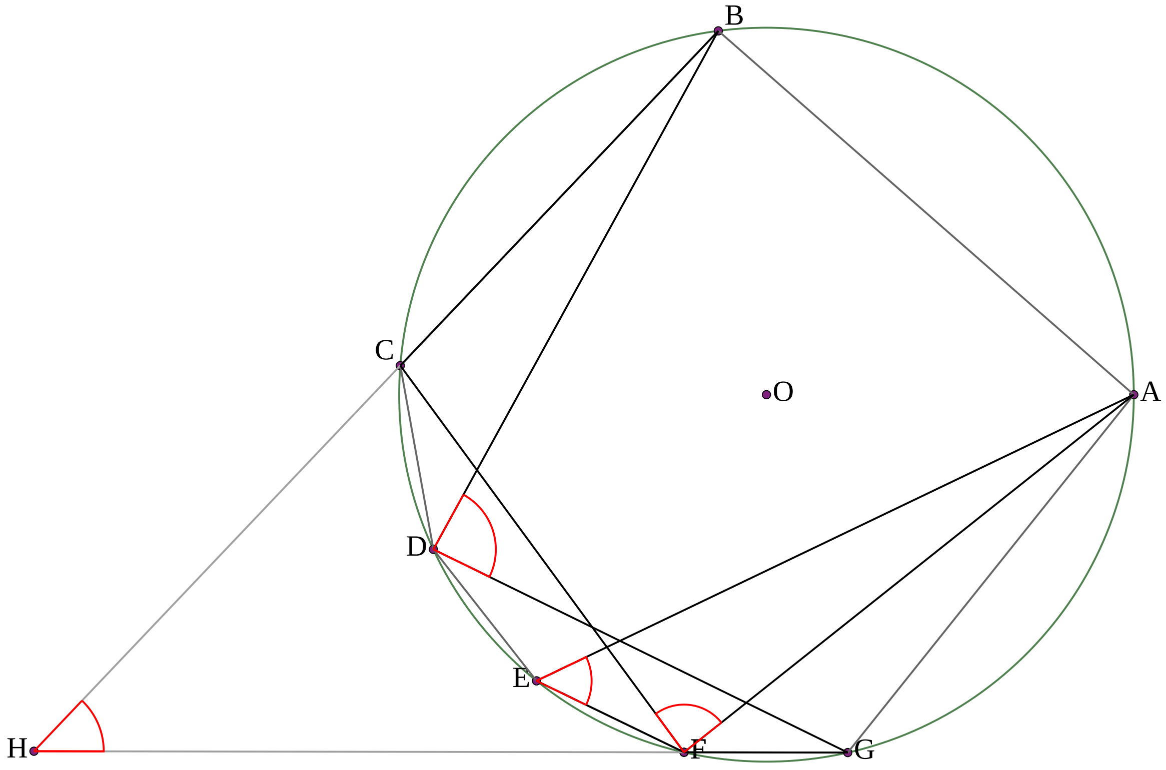
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GA and DE.
Prove that BAE+BCG+AFD = EHG+180

Example 67



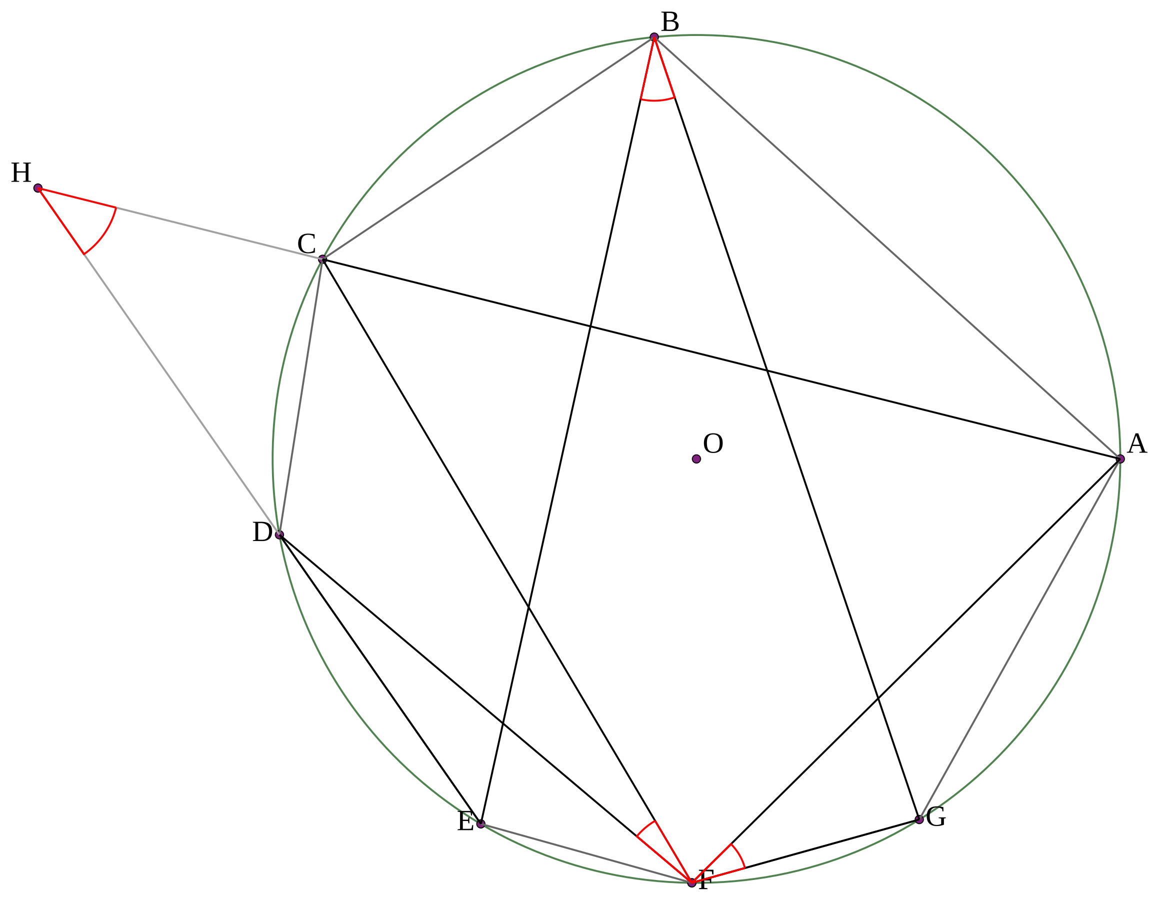
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and DA.
Angle AFC = 94°. Angle CGE = 29°. Angle EHD = 28°.
Find angle FBD.

Example 68



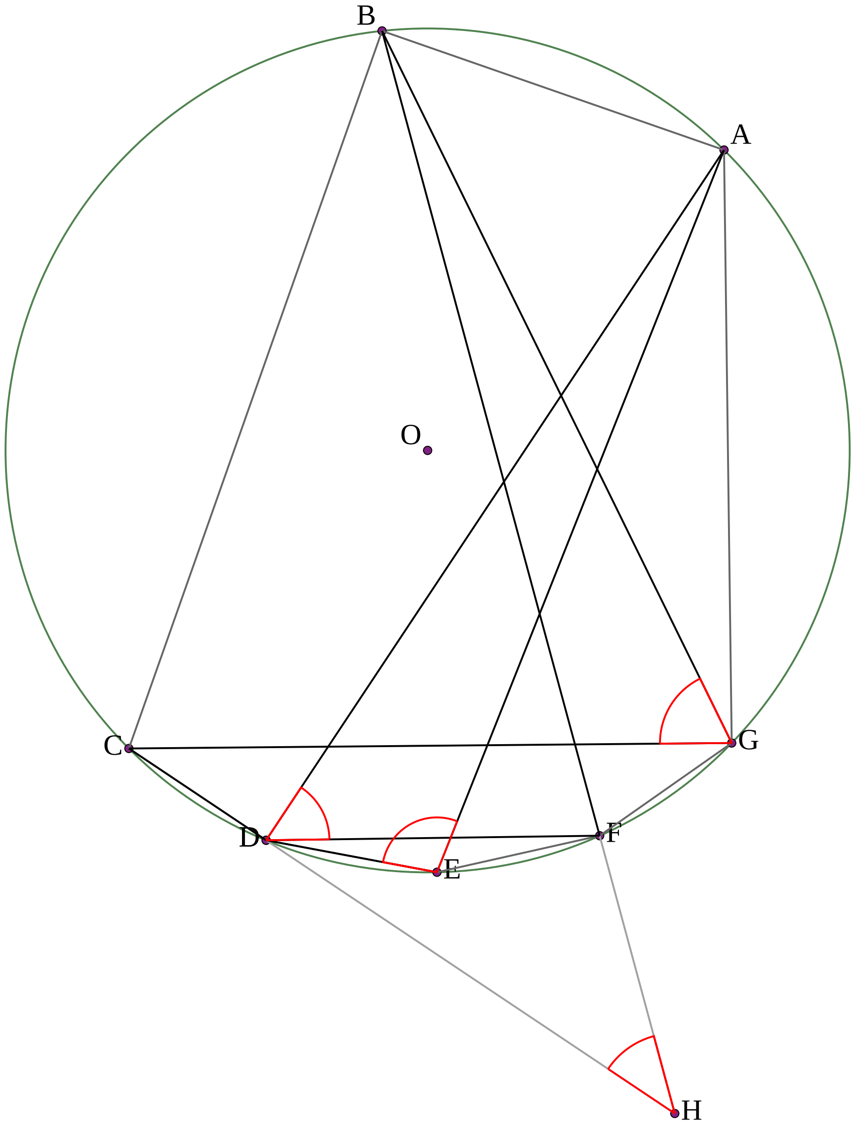
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and BC.
Angle FHC = x. Angle AEF = y. Angle GDB = z.
Find angle CFA.

Example 69



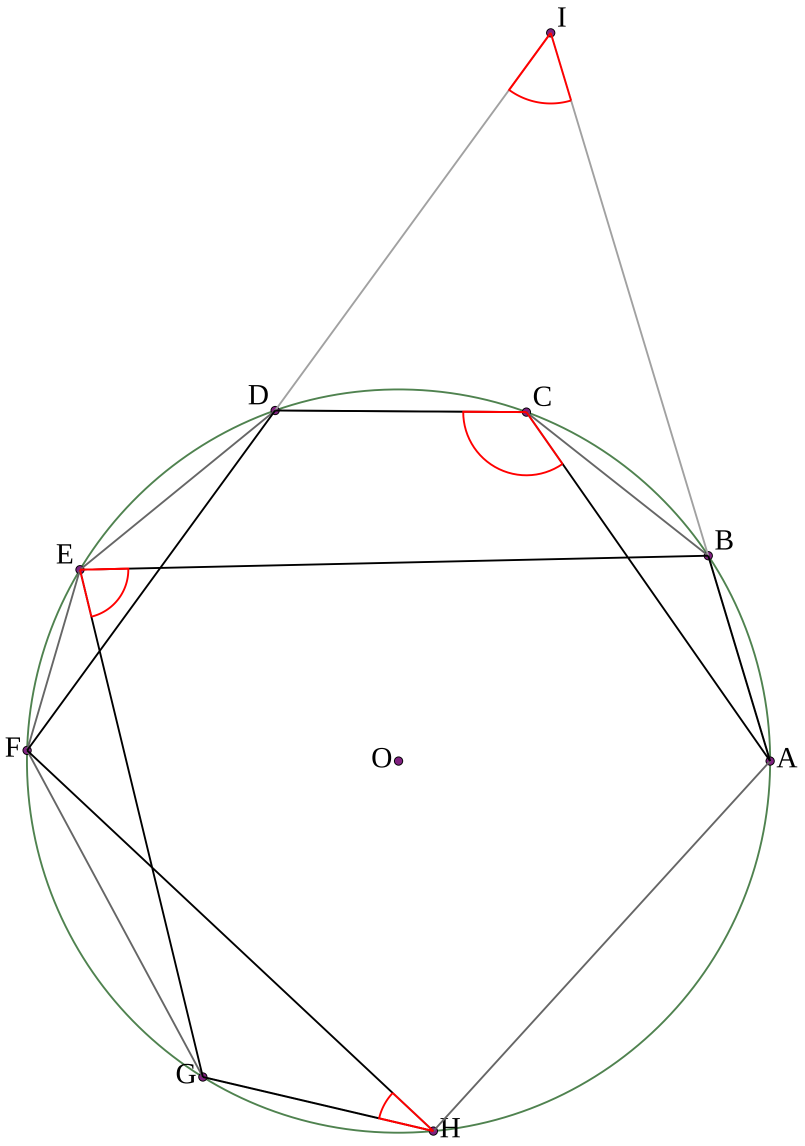
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of ED and CA.
Angle AFG = x. Angle GBE = y. Angle DHC = z.
Find angle DFC.

Example 70



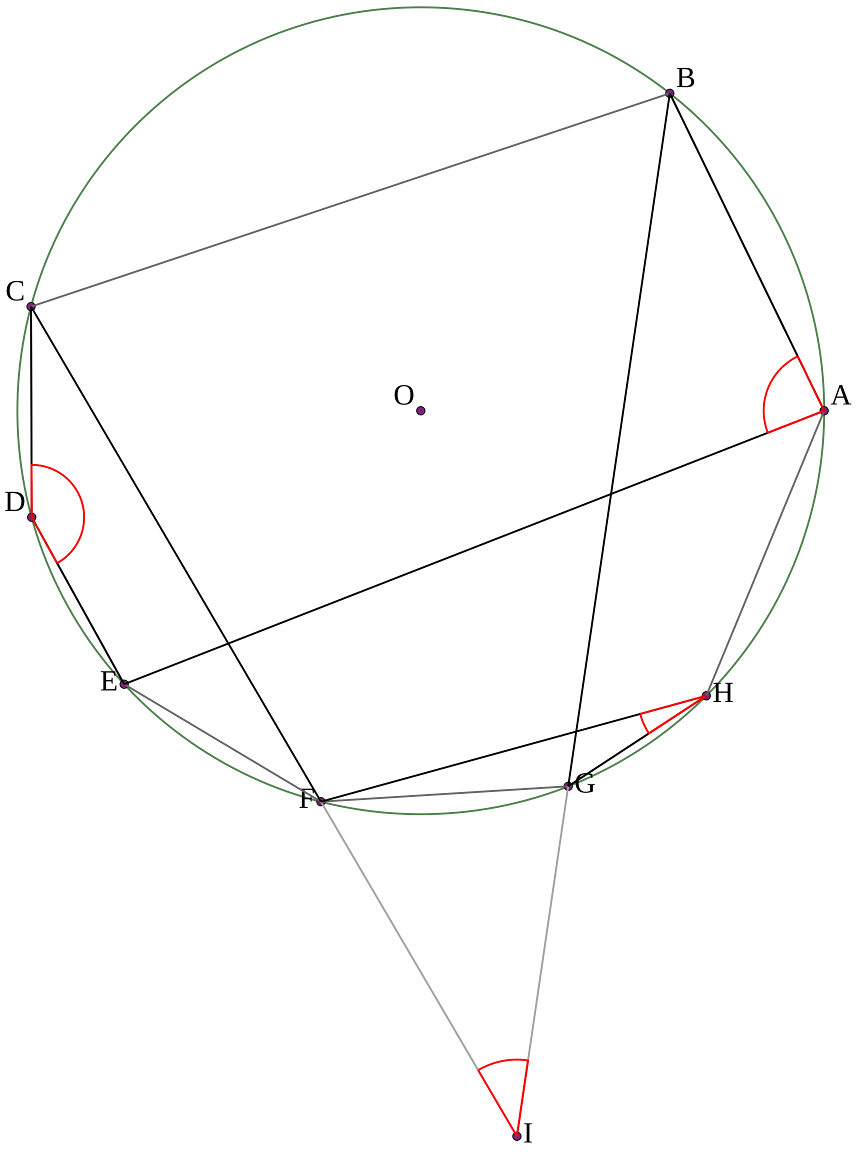
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FB and CD.
Angle DEA = 101°. Angle ADF = 56°. Angle BGC = 64°.
Find angle FHD.

Example 71



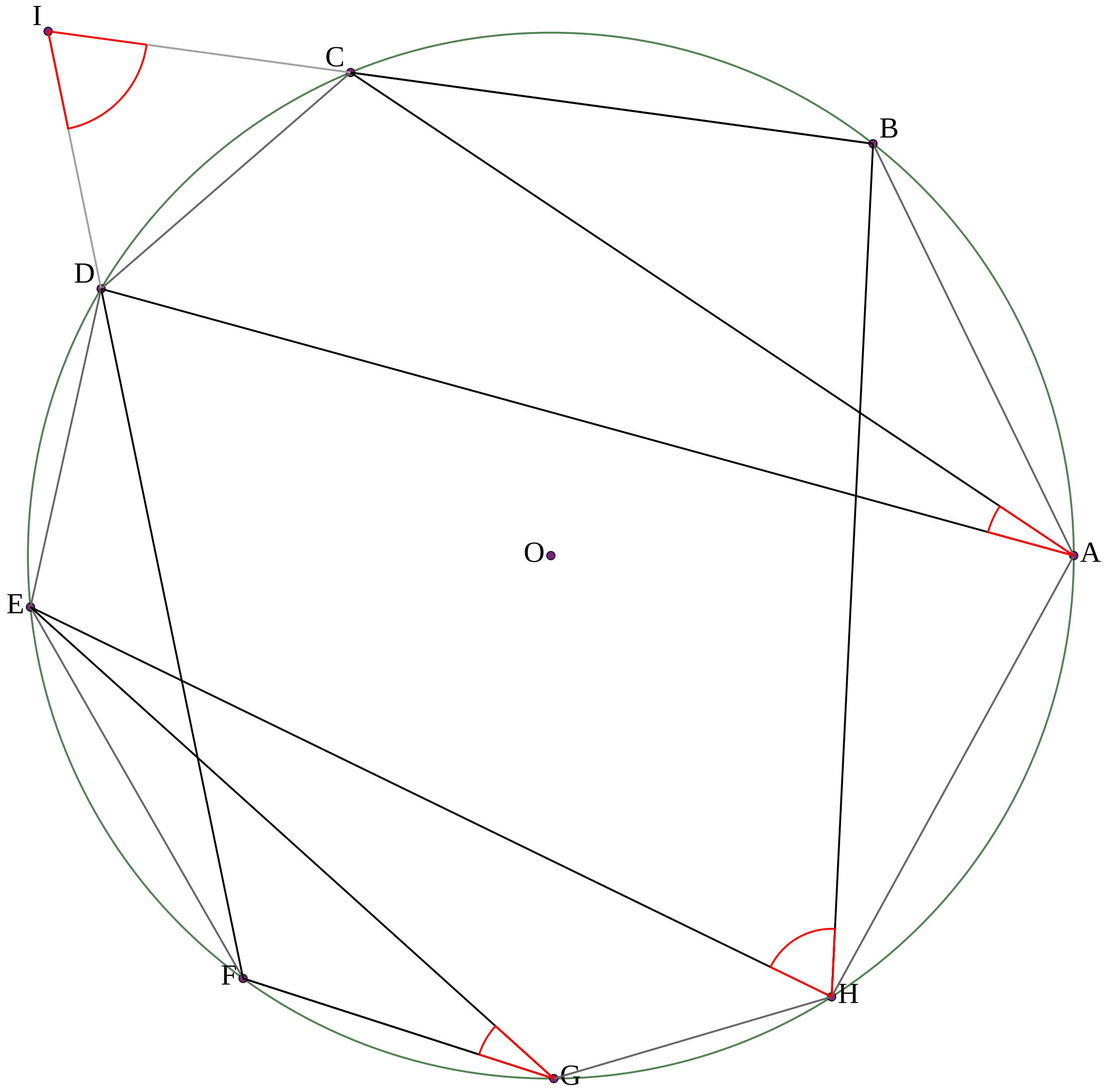
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FD and AB.
Prove that BEG+FHG+ACD = BID+180

Example 72



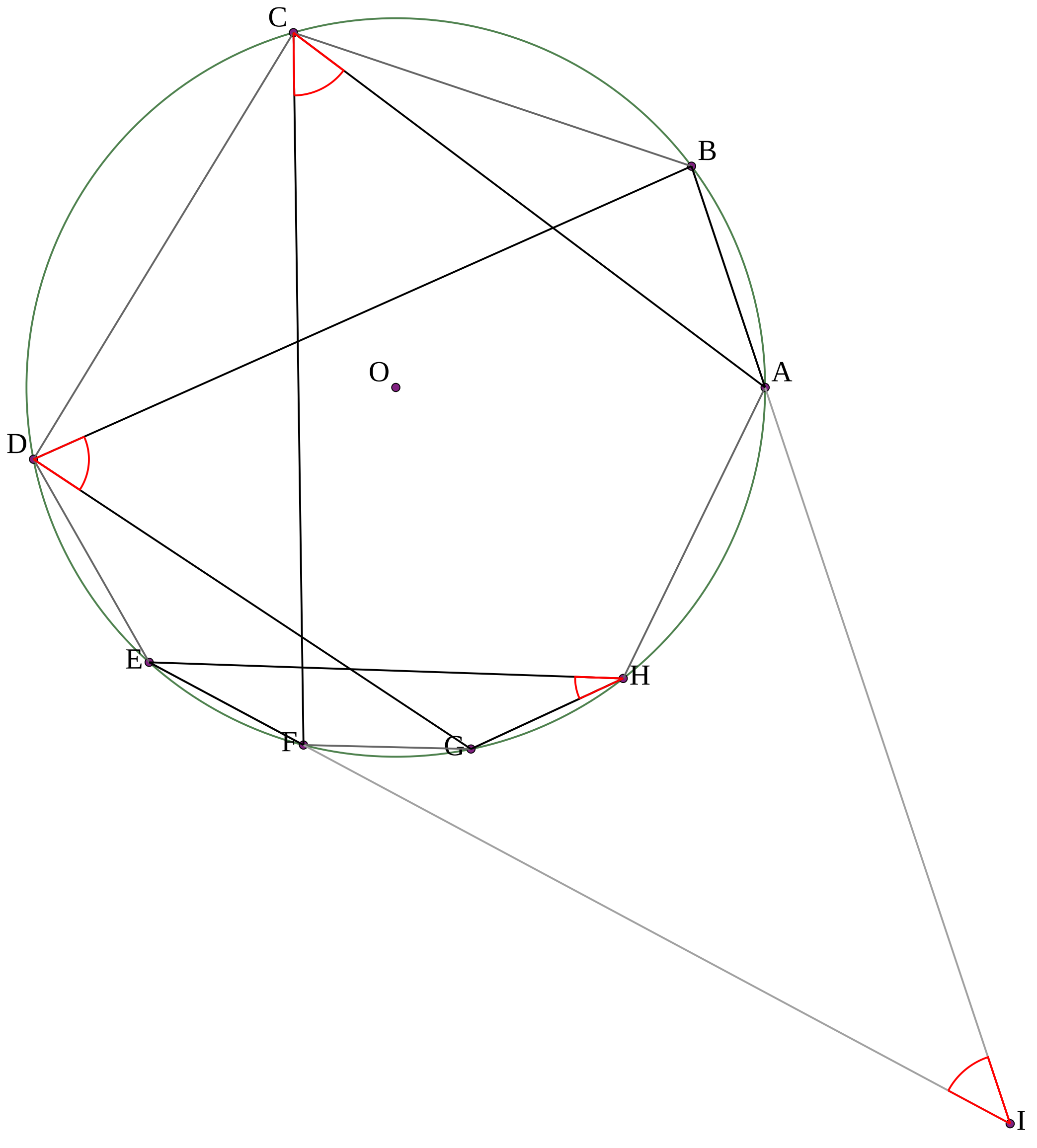
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of BG and FC.
Angle CDE = x. Angle EAB = y. Angle GIF = z.
Find angle GHF.

Example 73



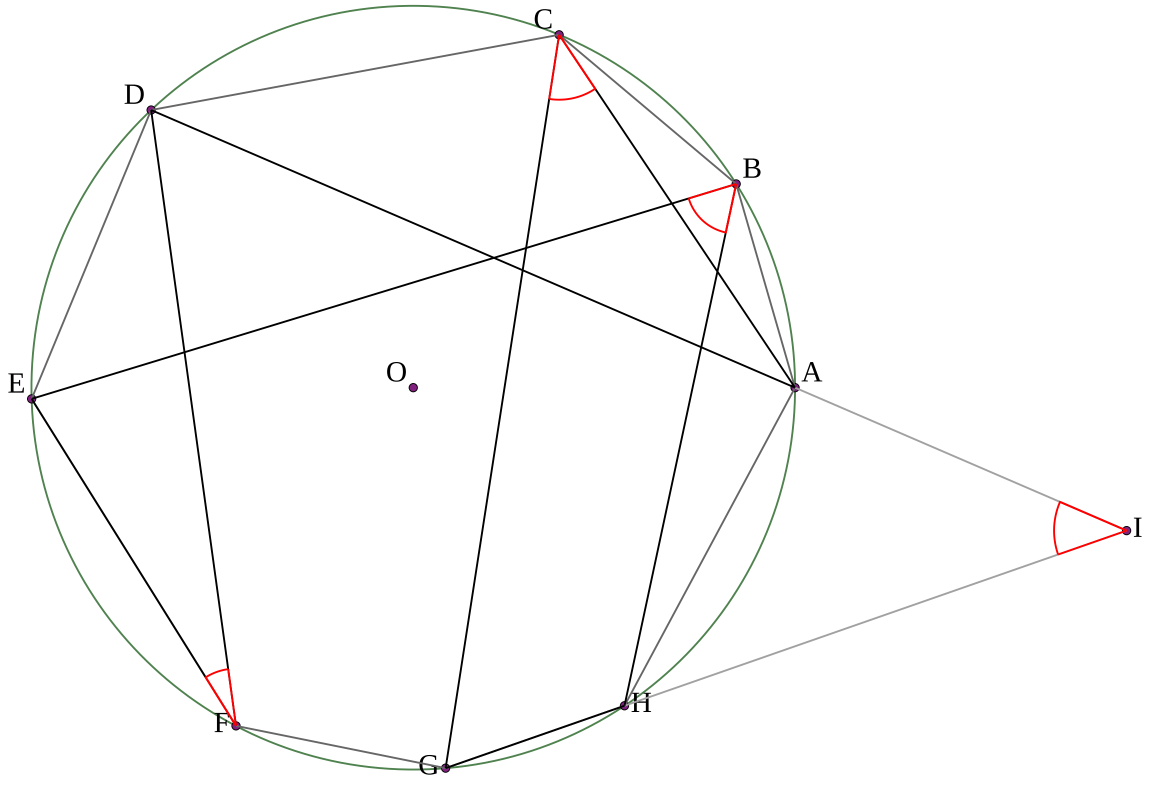
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FD and CB.
Angle EGF = 24°. Angle DAC = 18°. Angle DIC = 71°.
Find angle BHE.

Example 74



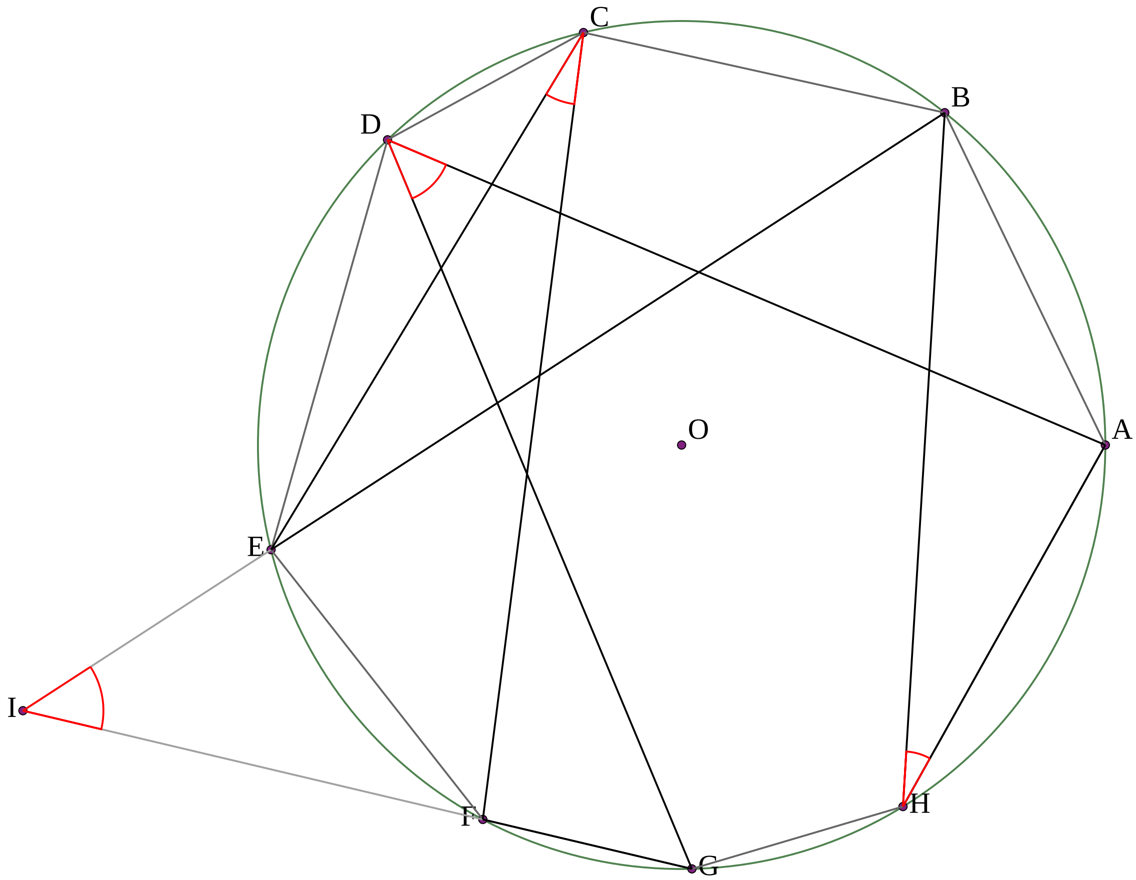
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of BA and FE.
Angle GDB = 58°. Angle ACF = 52°. Angle AIF = 43°.
Find angle EHG.

Example 75



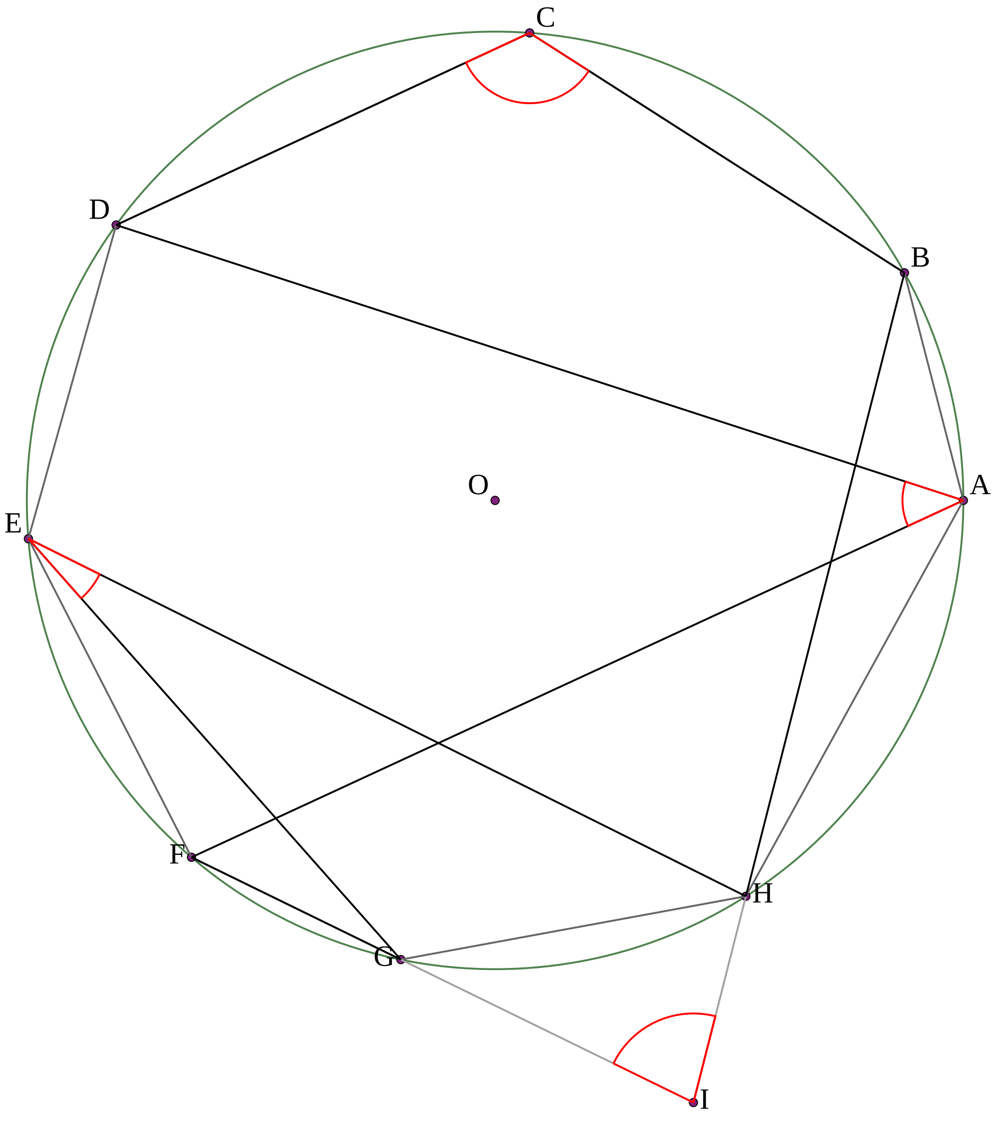
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DA and GH.
Prove that EBH+DFE = ACG+AIH

Example 76



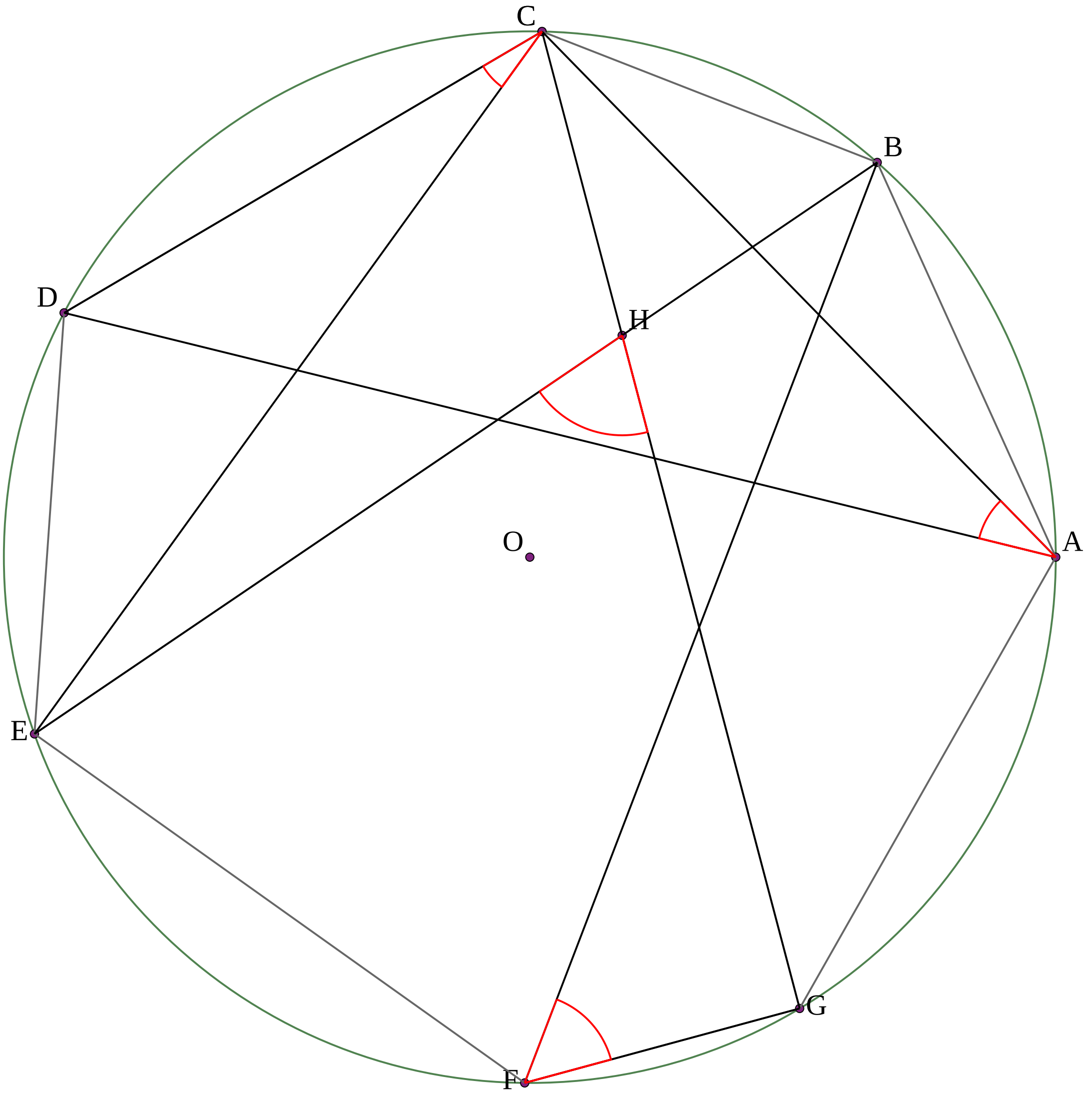
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GF and EB.
Angle BHA = x. Angle ADG = y. Angle FIE = z.
Find angle FCE.

Example 77



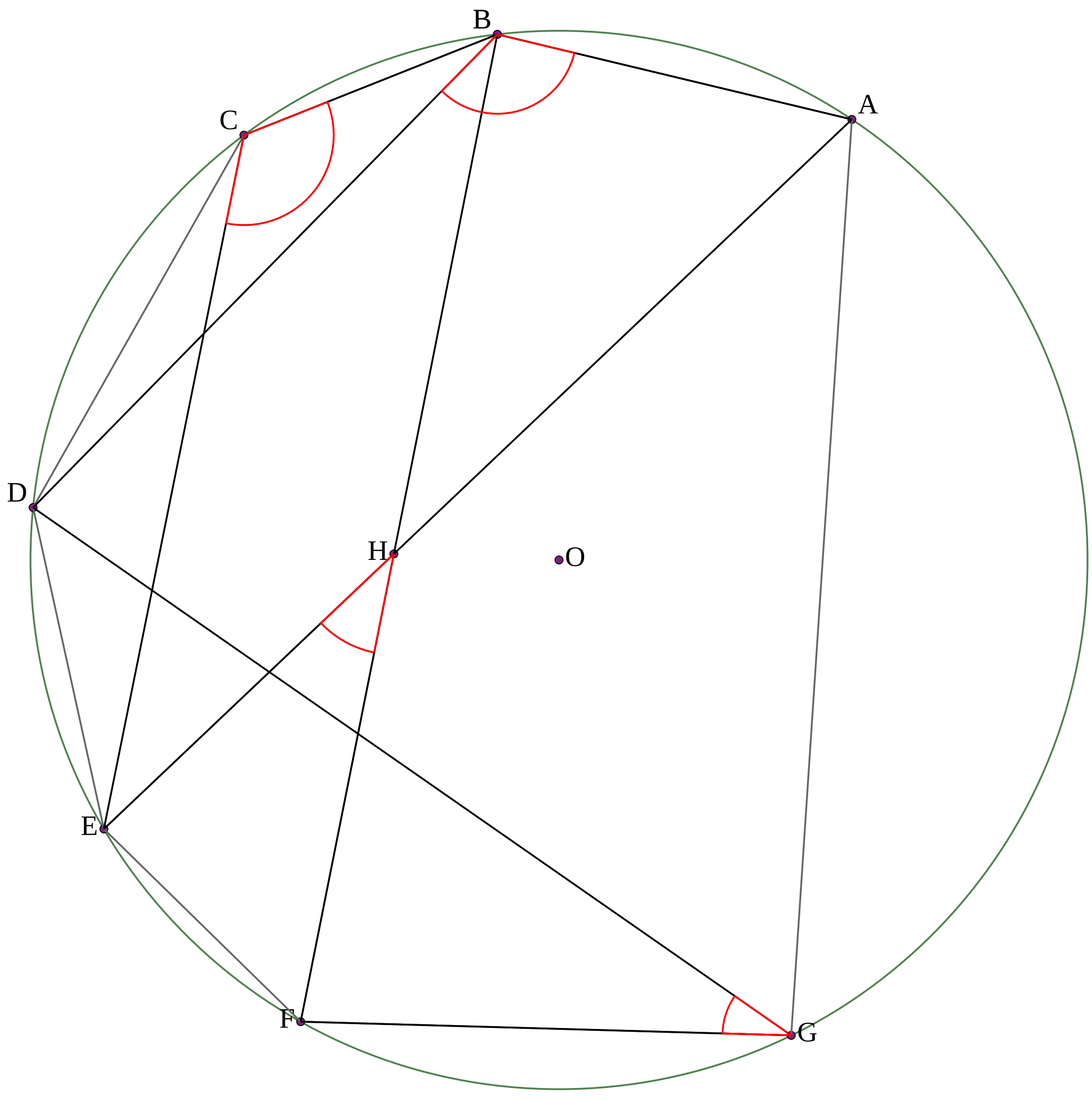
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of BH and GF.
Angle FAD = x. Angle HEG = y. Angle HIG = z.
Find angle DCB.

Example 78



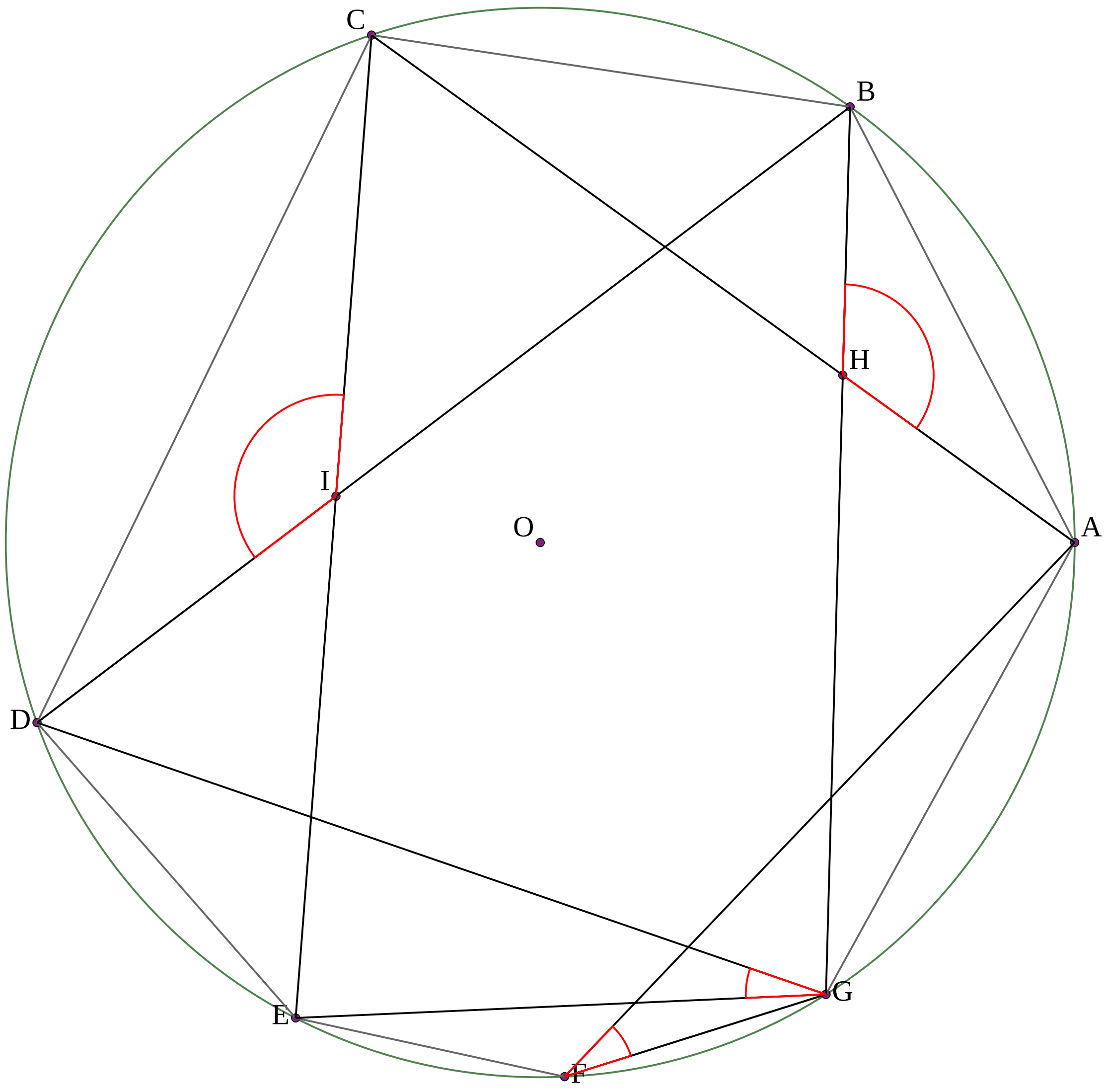
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CG and BE.
Angle ECD = x. Angle GFB = y. Angle GHE = z.
Find angle DAC.

Example 79



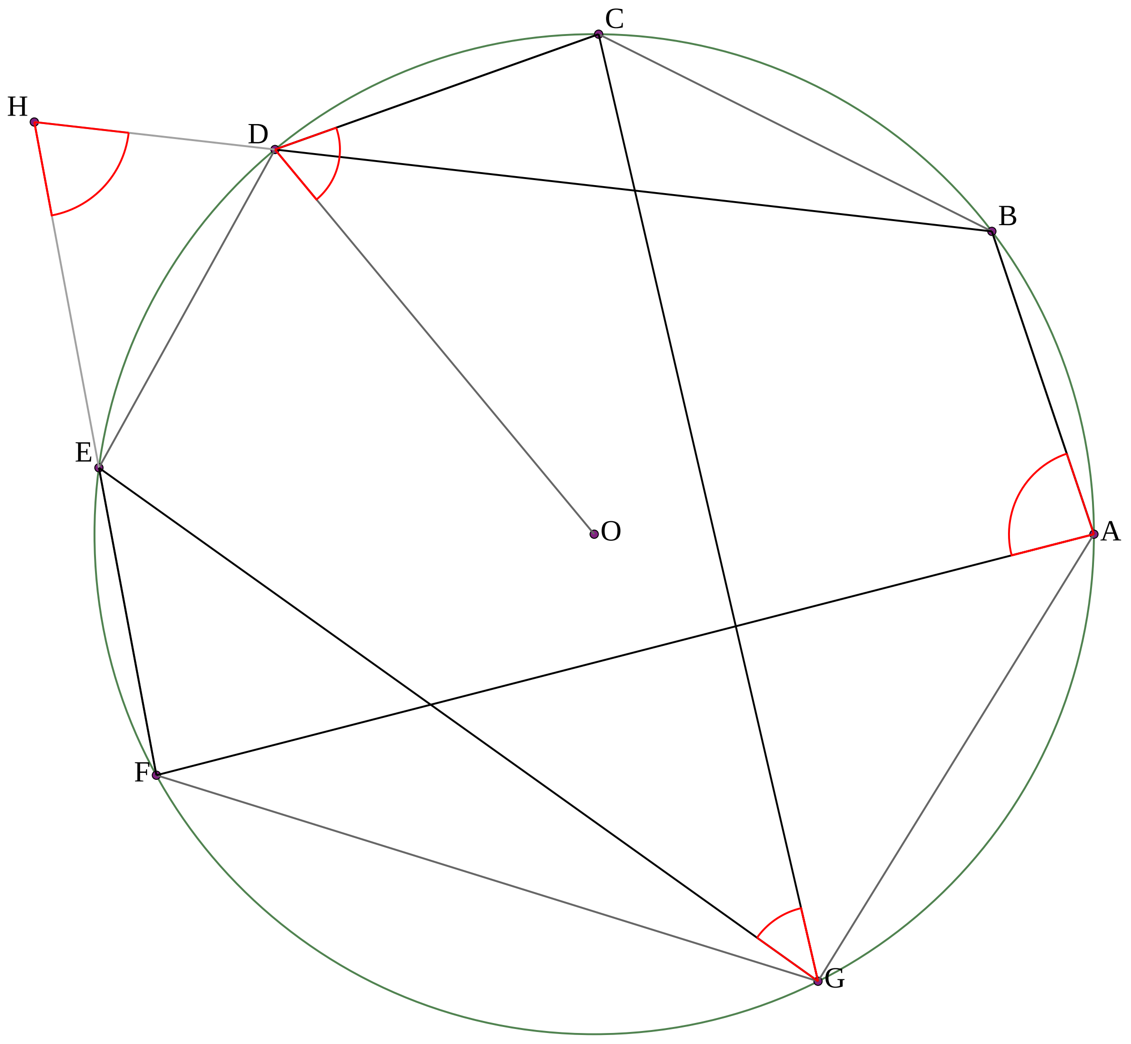
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AE and BF.
Prove that DGF+BCE = ABD+EHF

Example 80



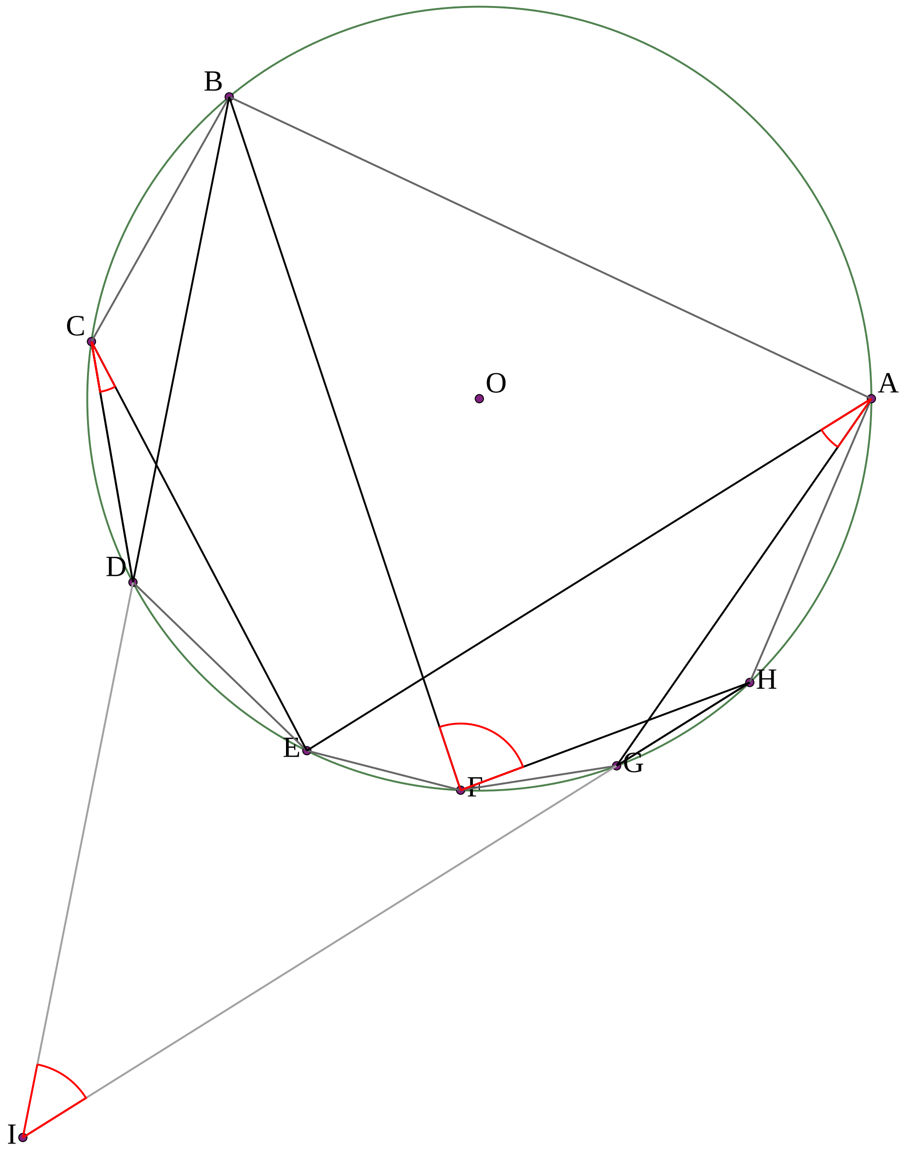
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GB and CA. Let I be the intersection of BD and EC.
Angle AFG = x. Angle BHA = y. Angle DIC = z.
Find angle DGE.

Example 81



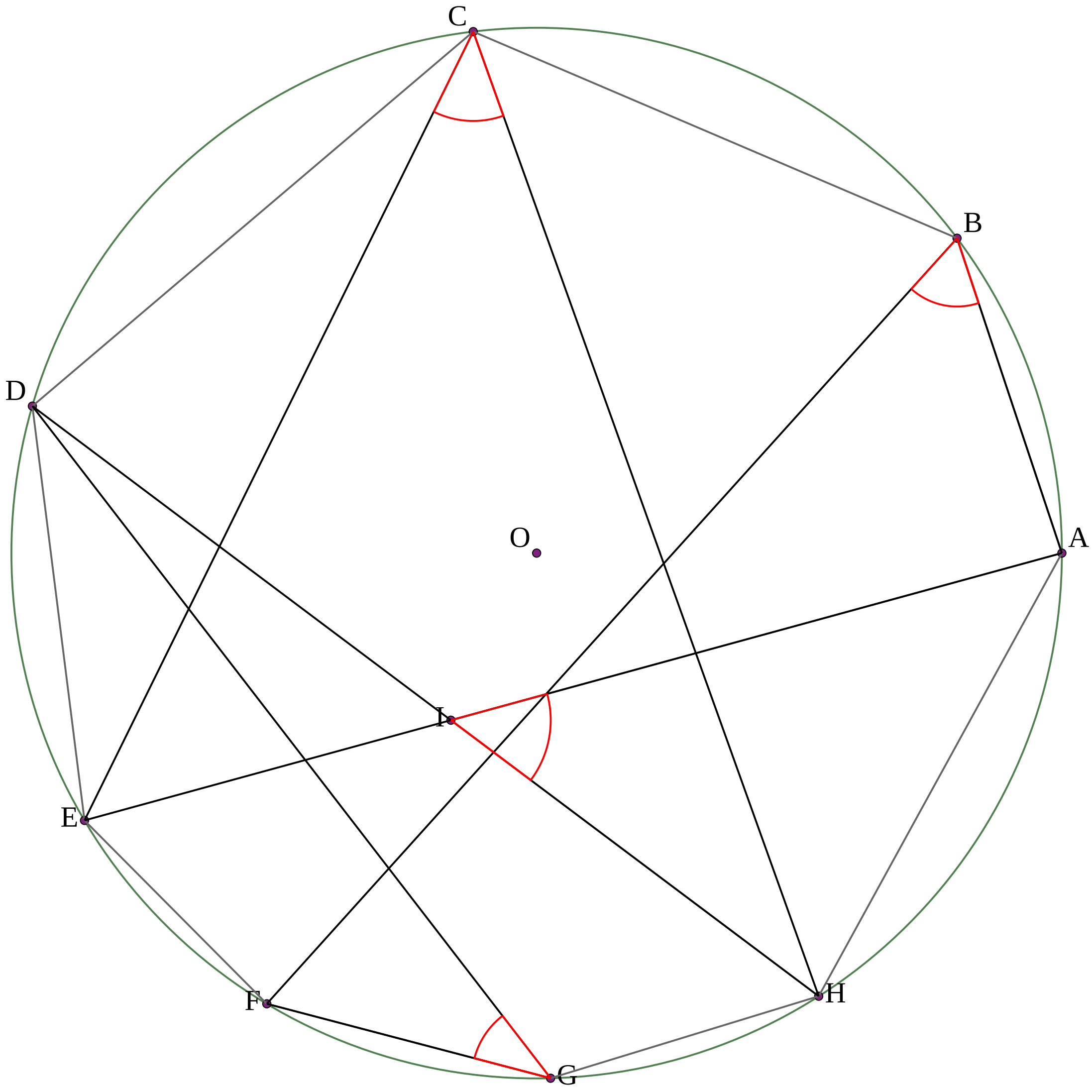
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and BD.
Angle ODC = x. Angle FAB = y. Angle EHD = z.
Find angle CGE.

Example 82



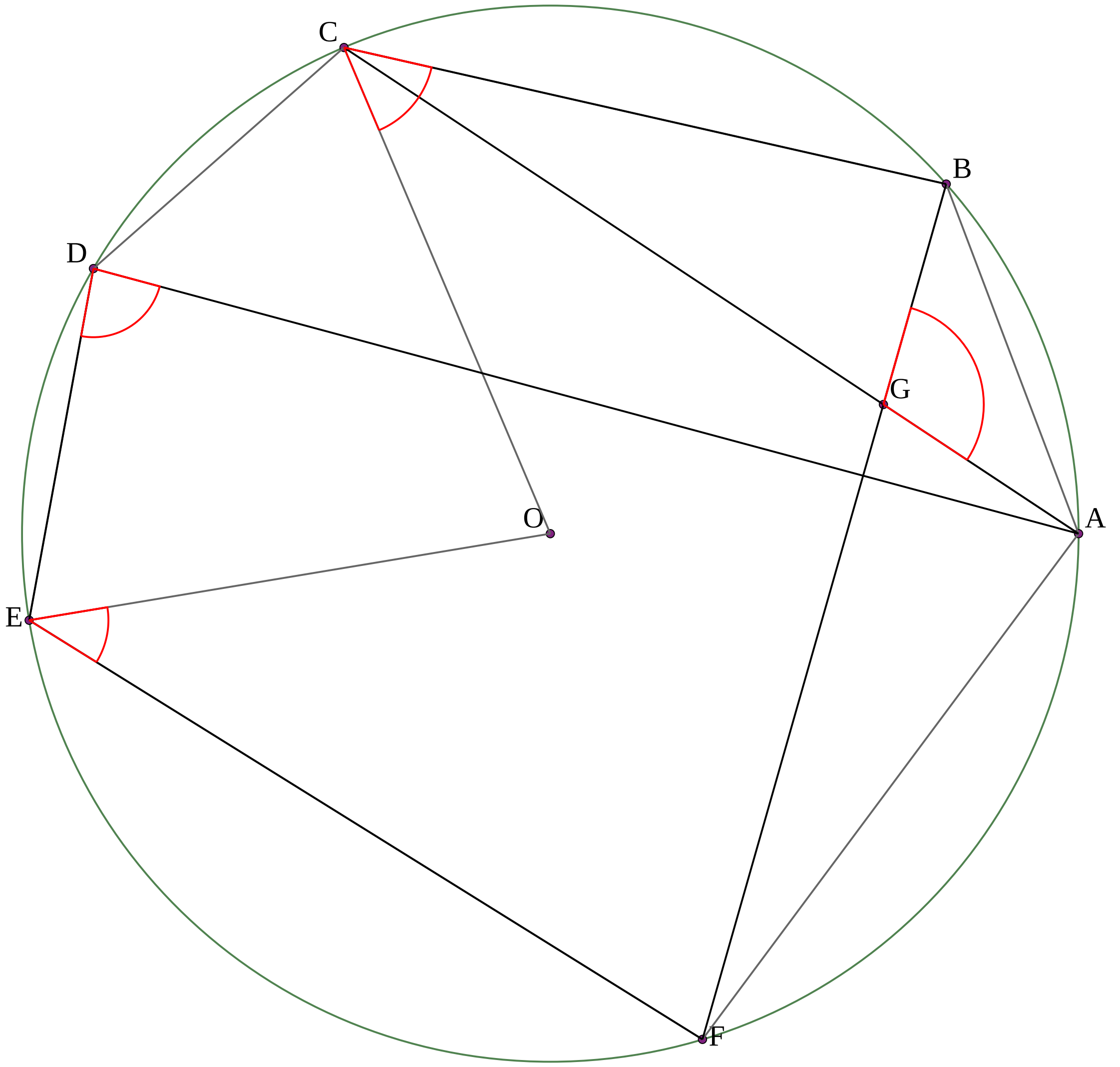
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GH and BD.
Angle HFB = x. Angle EAG = y. Angle GID = z.
Find angle DCE.

Example 83



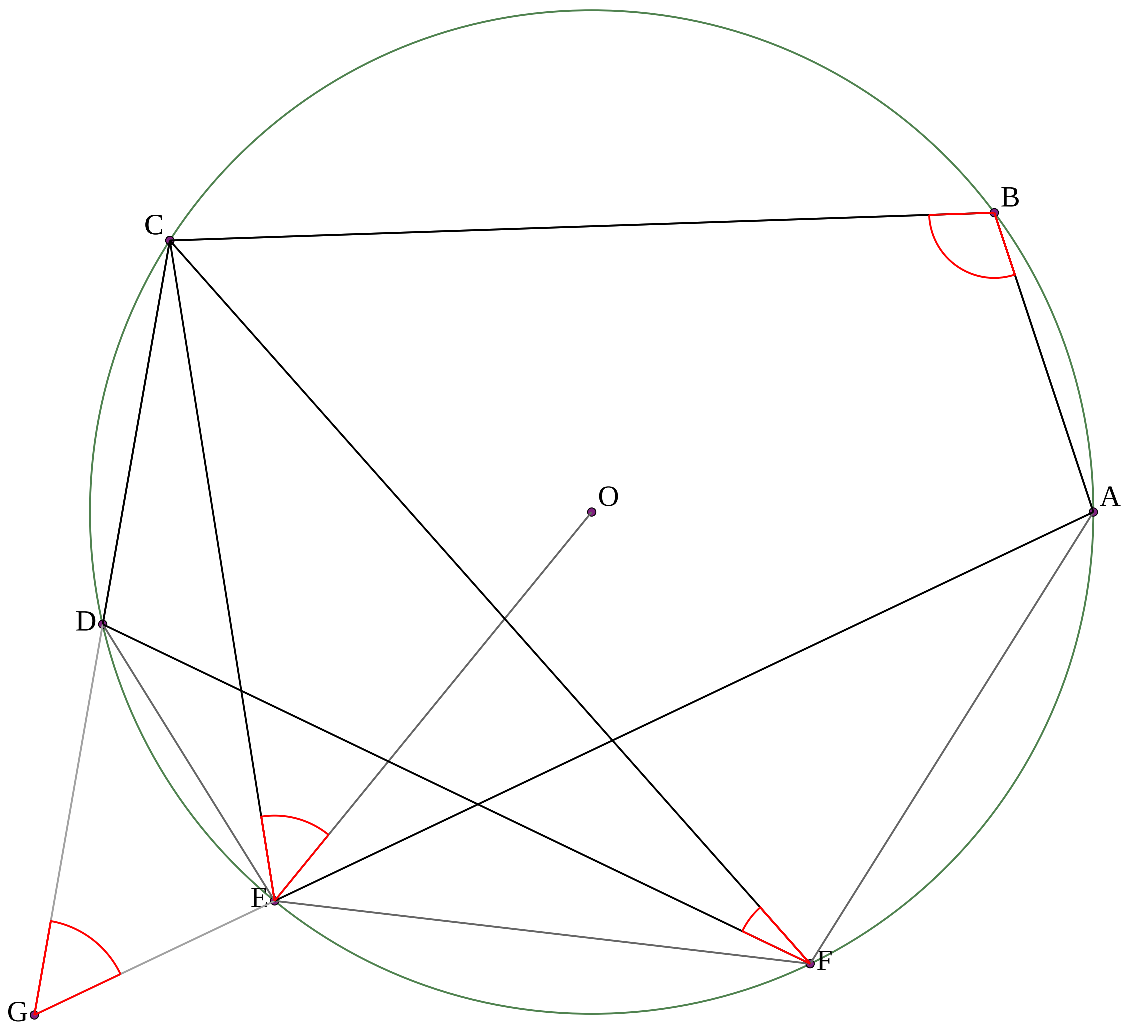
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DH and EA.
Angle HCE = 46°. Angle HIA = 52°. Angle FGD = 38°.
Find angle ABF.

Example 84



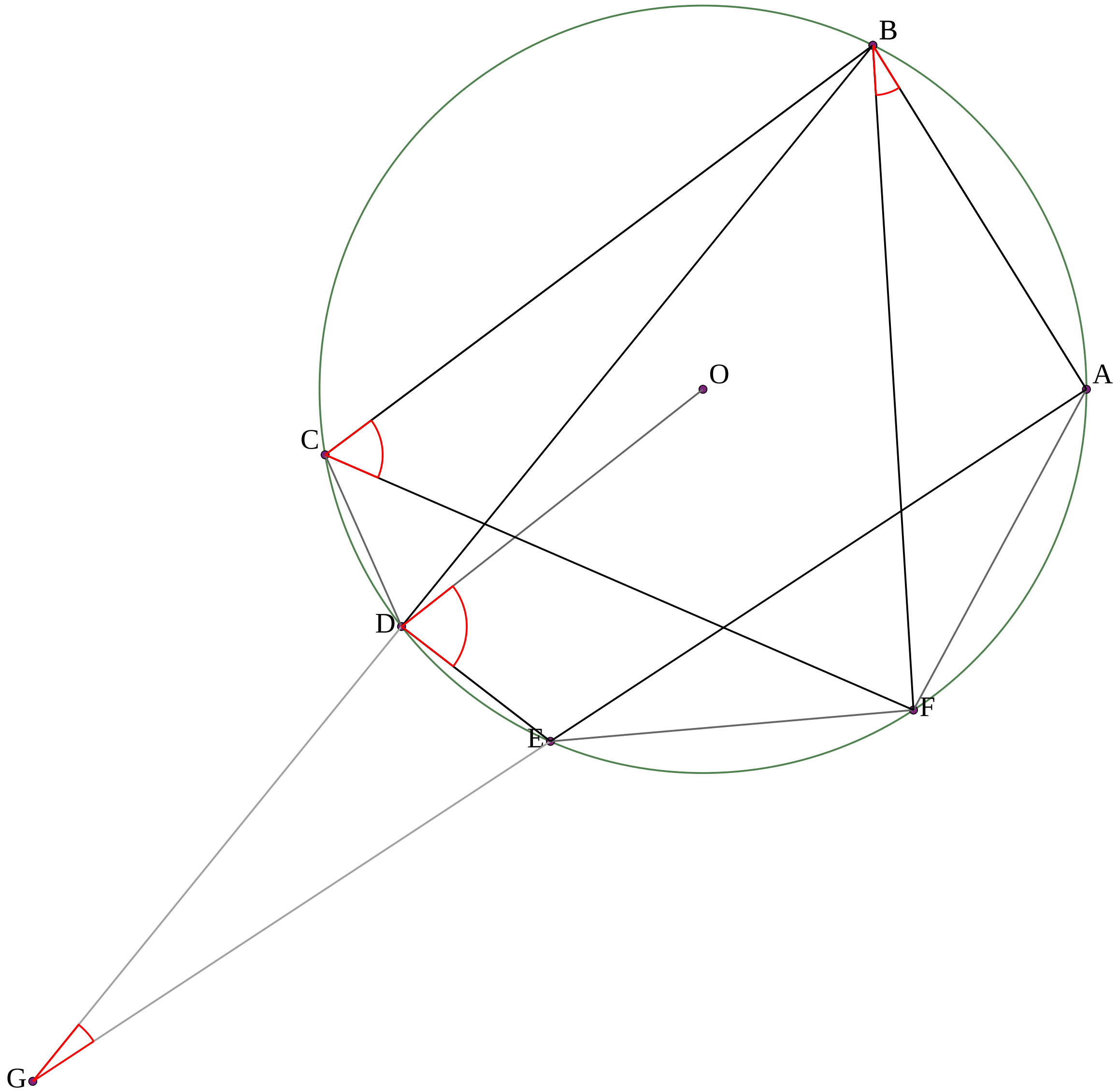
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FB and CA.
Prove that ADE+FEO+AGB = BCO+180

Example 85



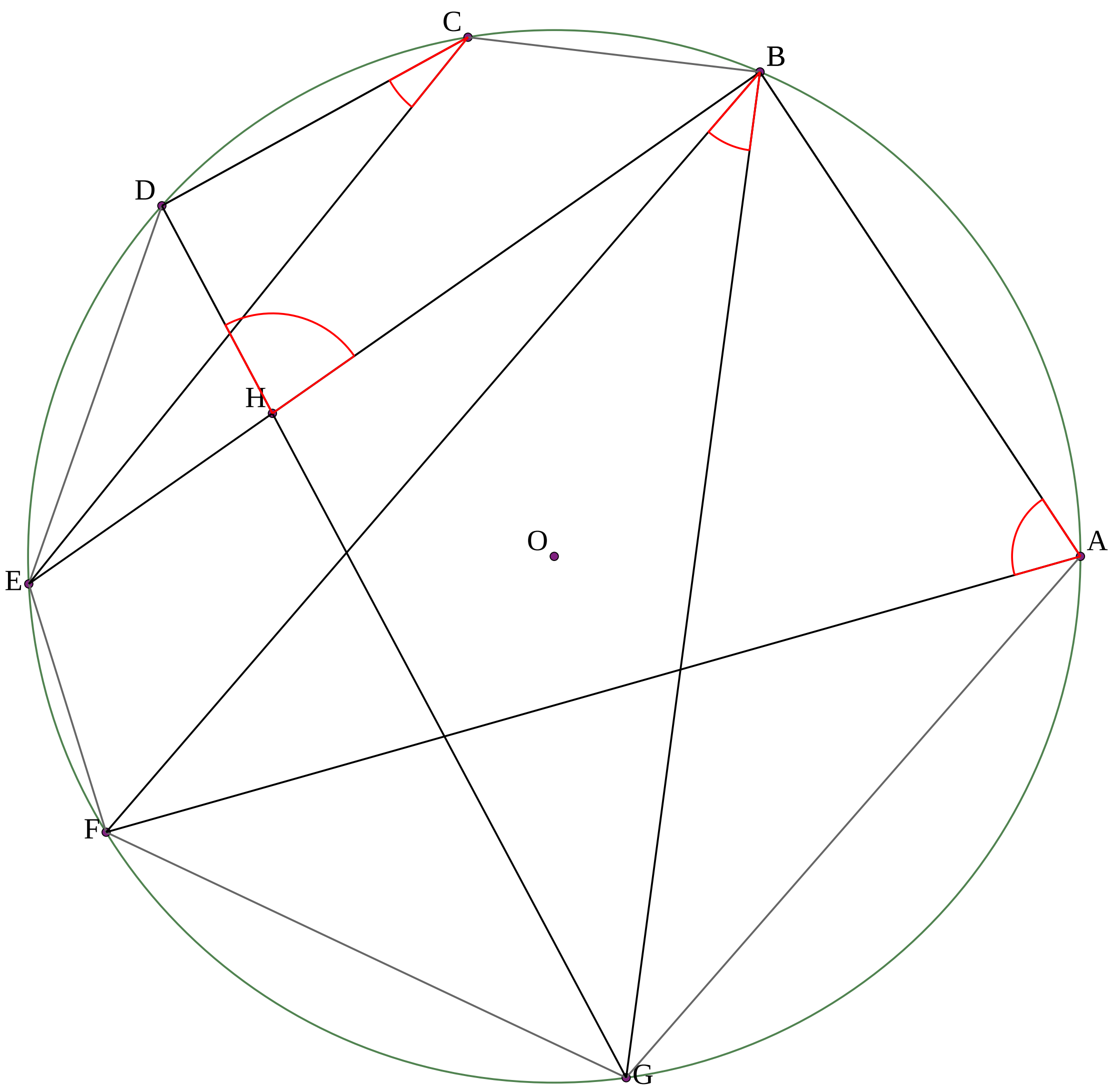
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DC and EA.
Prove that ABC+DGE = CFD+CEO+90

Example 86



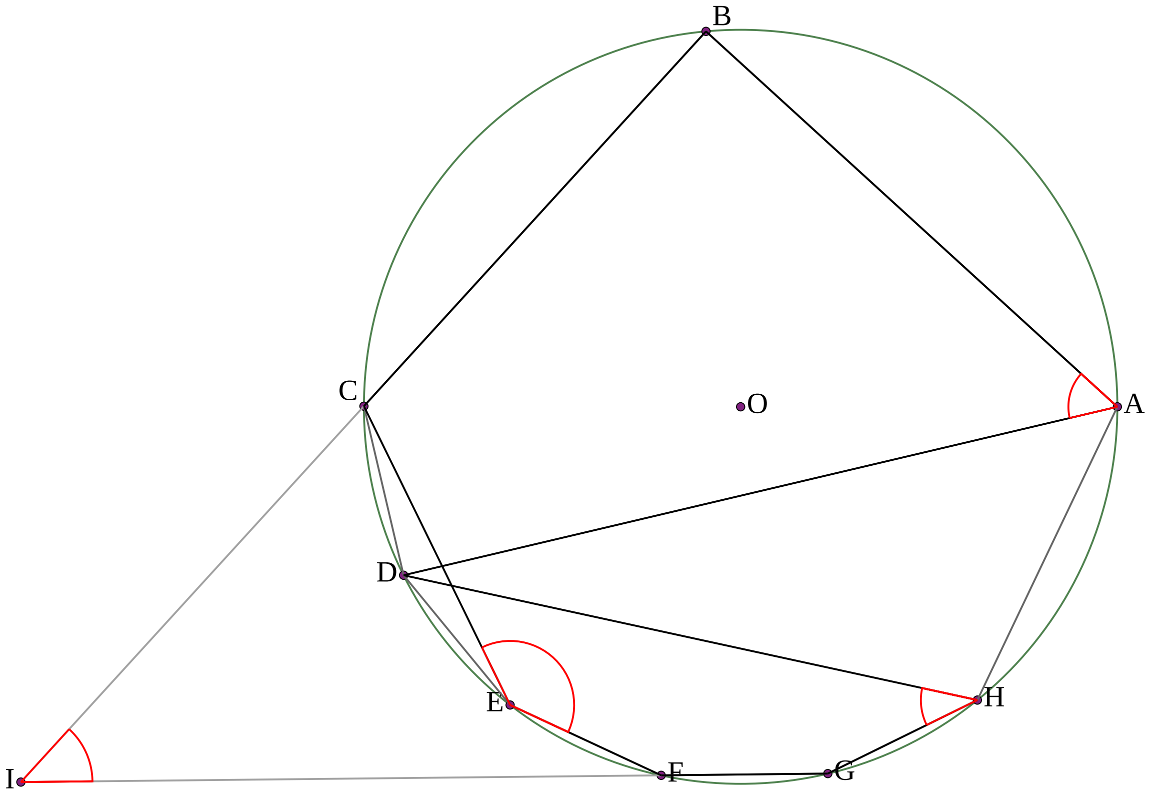
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BD and EA.
Angle ABF = x. Angle DGE = y. Angle FCB = z.
Find angle ODE.

Example 87



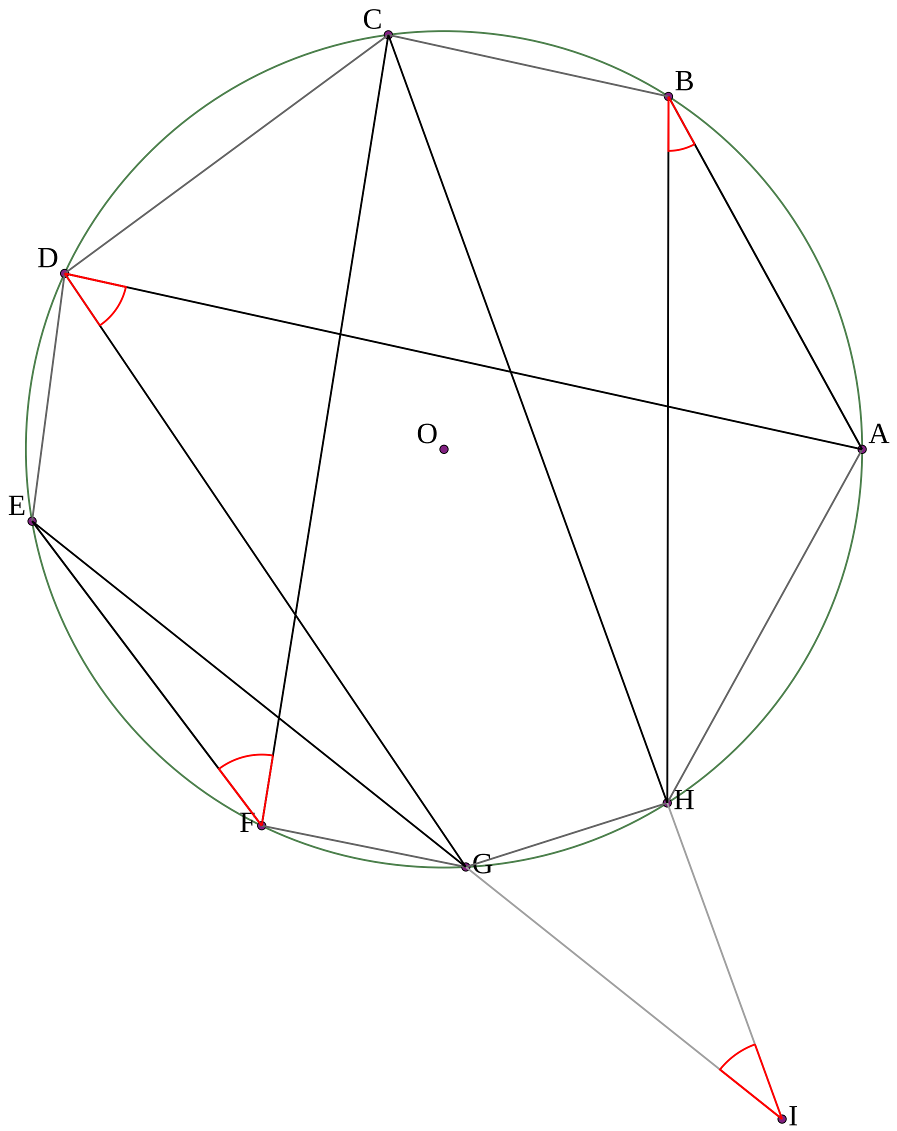
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GD and EB.
Angle DCE = x. Angle DHB = y. Angle FBG = z.
Find angle BAF.

Example 88



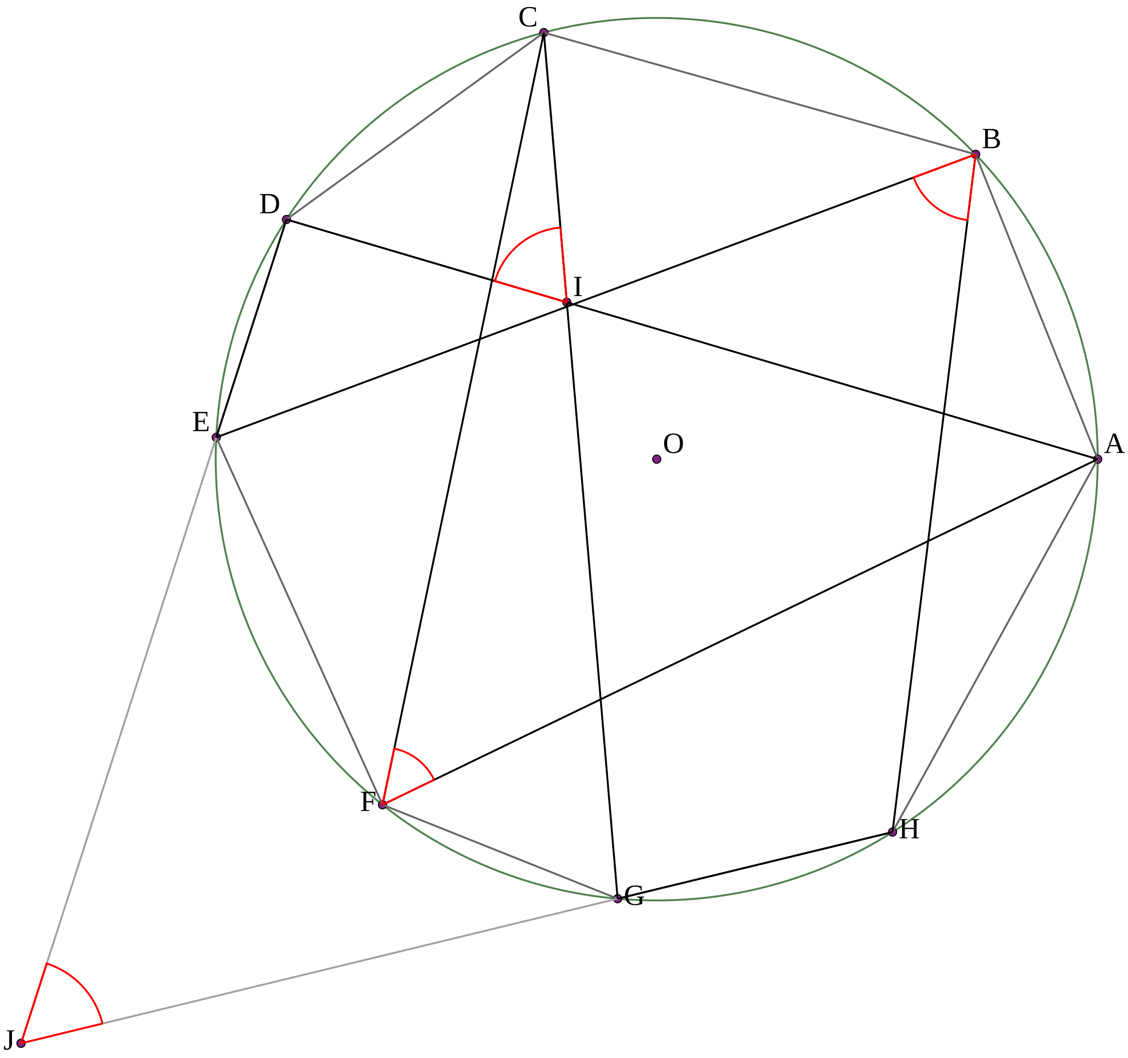
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GF and CB.
Prove that CEF = BAD+DHG+CIF

Example 89



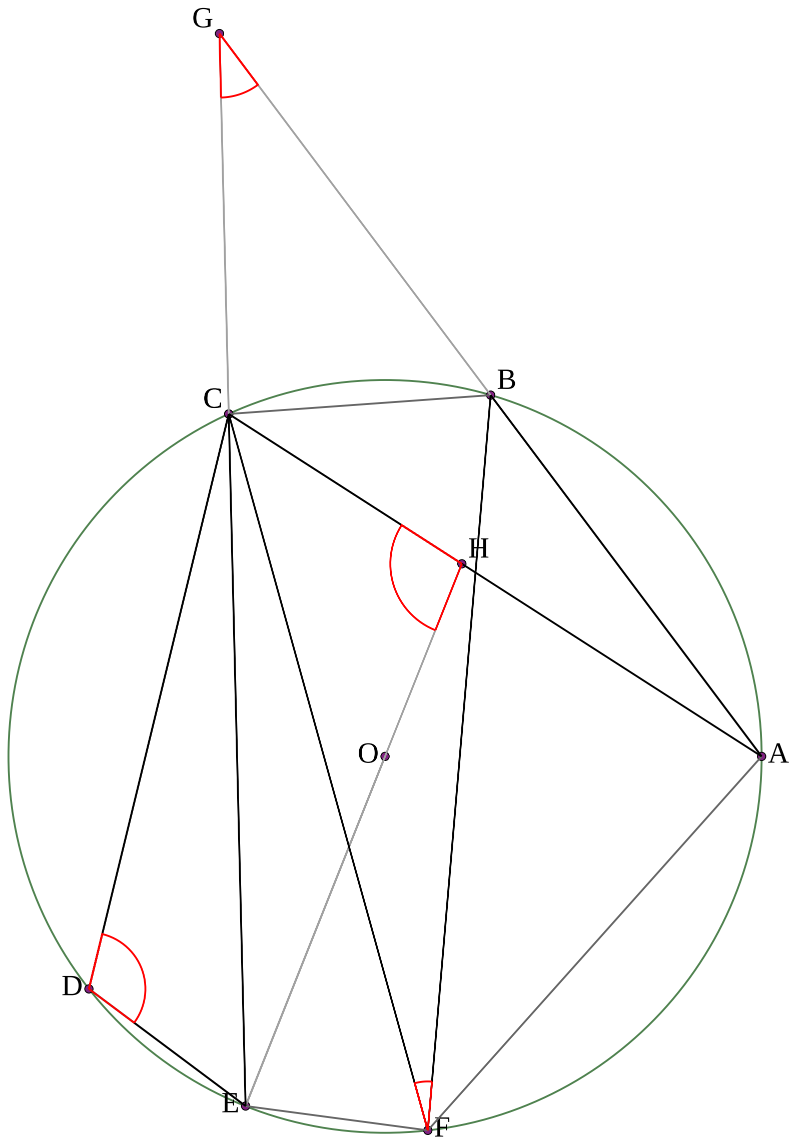
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GE and CH.
Angle HBA = 29°. Angle EFC = 46°. Angle GIH = 31°.
Find angle ADG.

Example 90



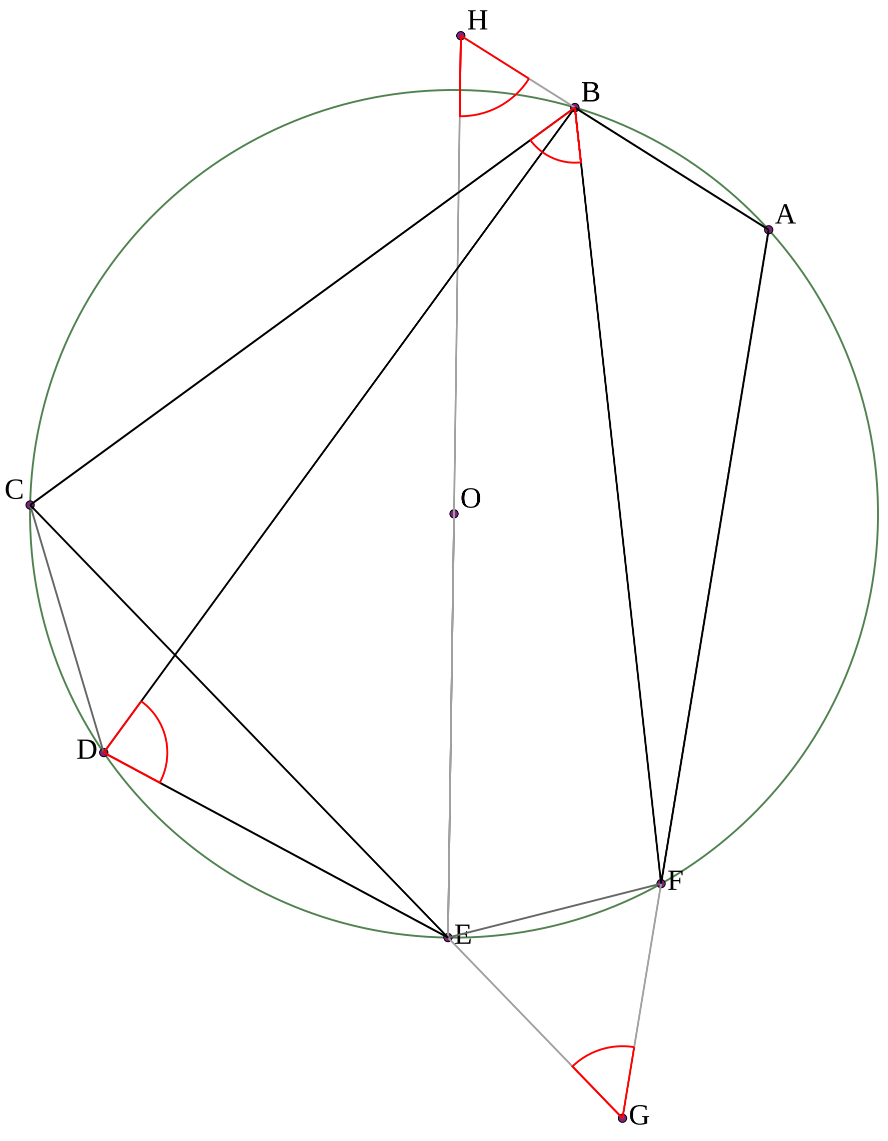
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of AD and GC. Let J be the intersection of DE and HG.
Angle CFA = x. Angle DIC = y. Angle EBH = z.
Find angle EJG.

Example 91



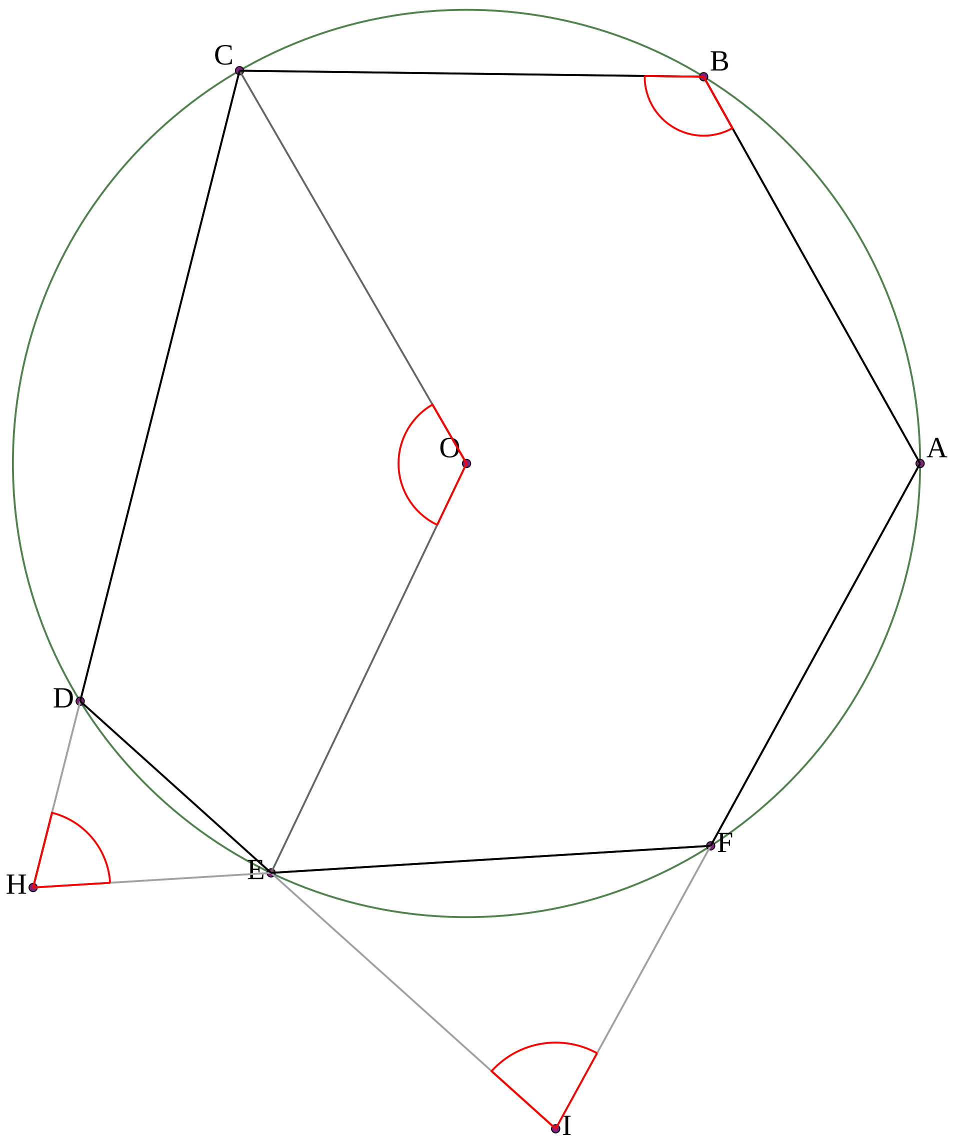
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BA and EC. Let H be the intersection of AC and EO.
Angle CFB = x. Angle BGC = y. Angle CHE = z.
Find angle CDE.

Example 92



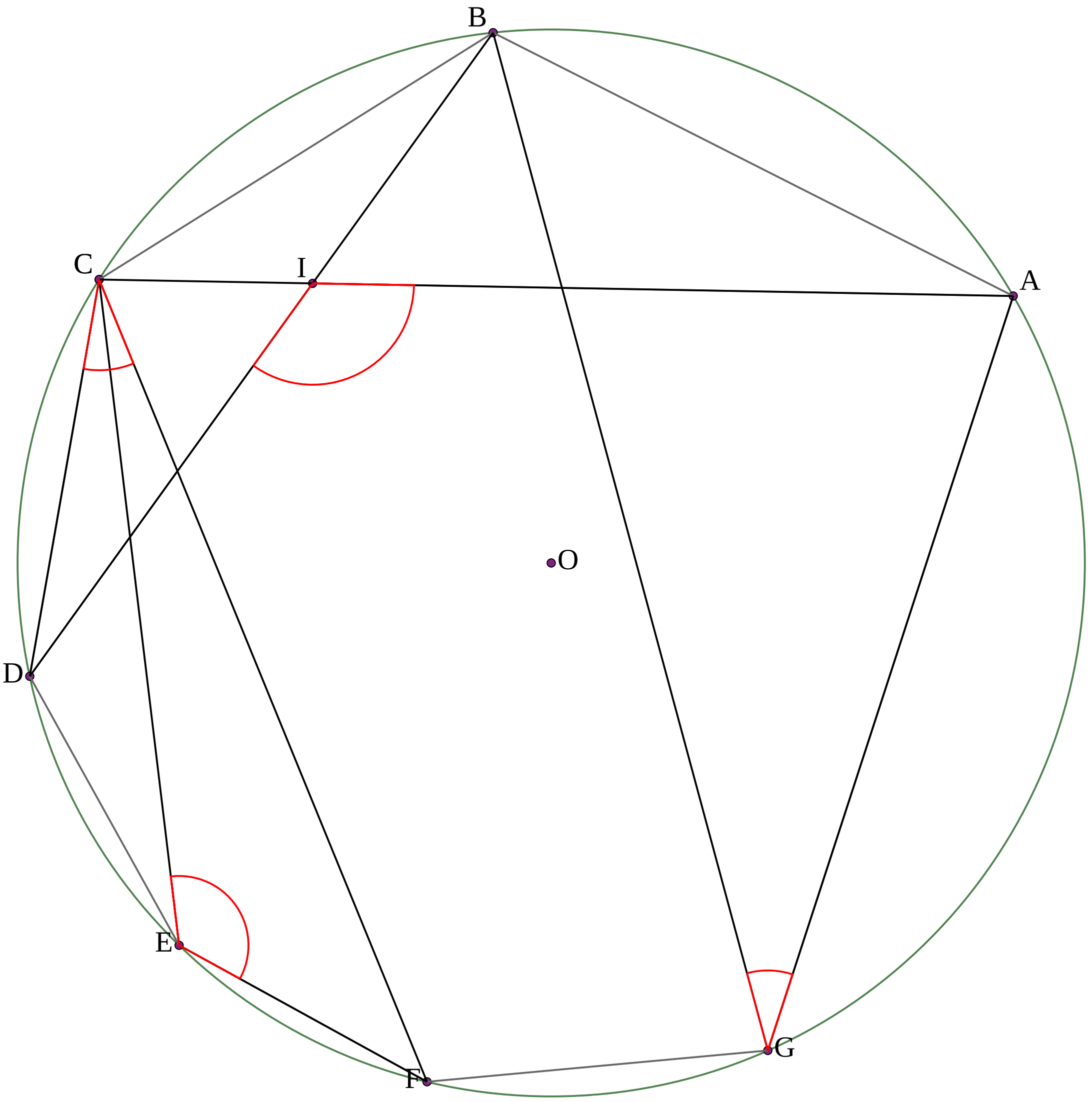
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CE and AF. Let H be the intersection of OE and BA.
Angle FBC = x. Angle EGF = y. Angle EHB = z.
Find angle EDB.

Example 93



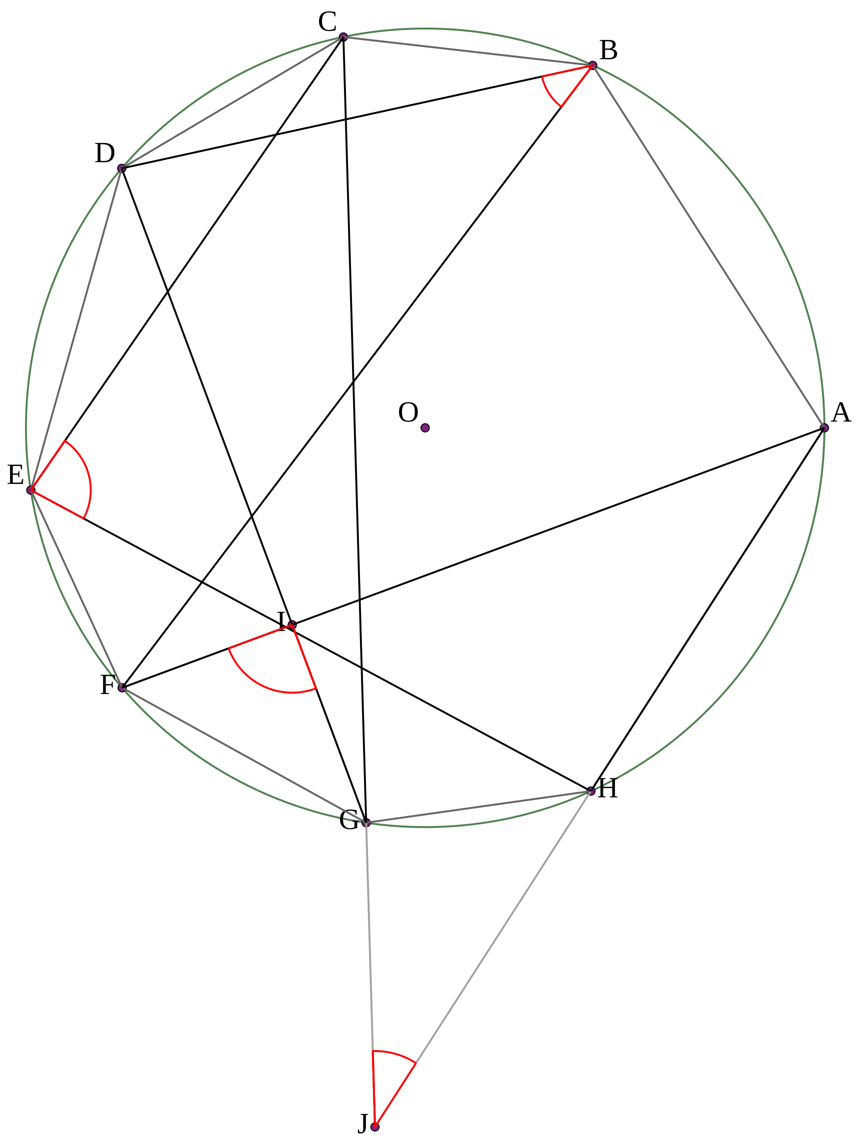
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of CD and EF. Let I be the intersection of DE and FA.
Angle COE = 124°. Angle DHE = 72°. Angle ABC = 120°.
Find angle EIF.

Example 94



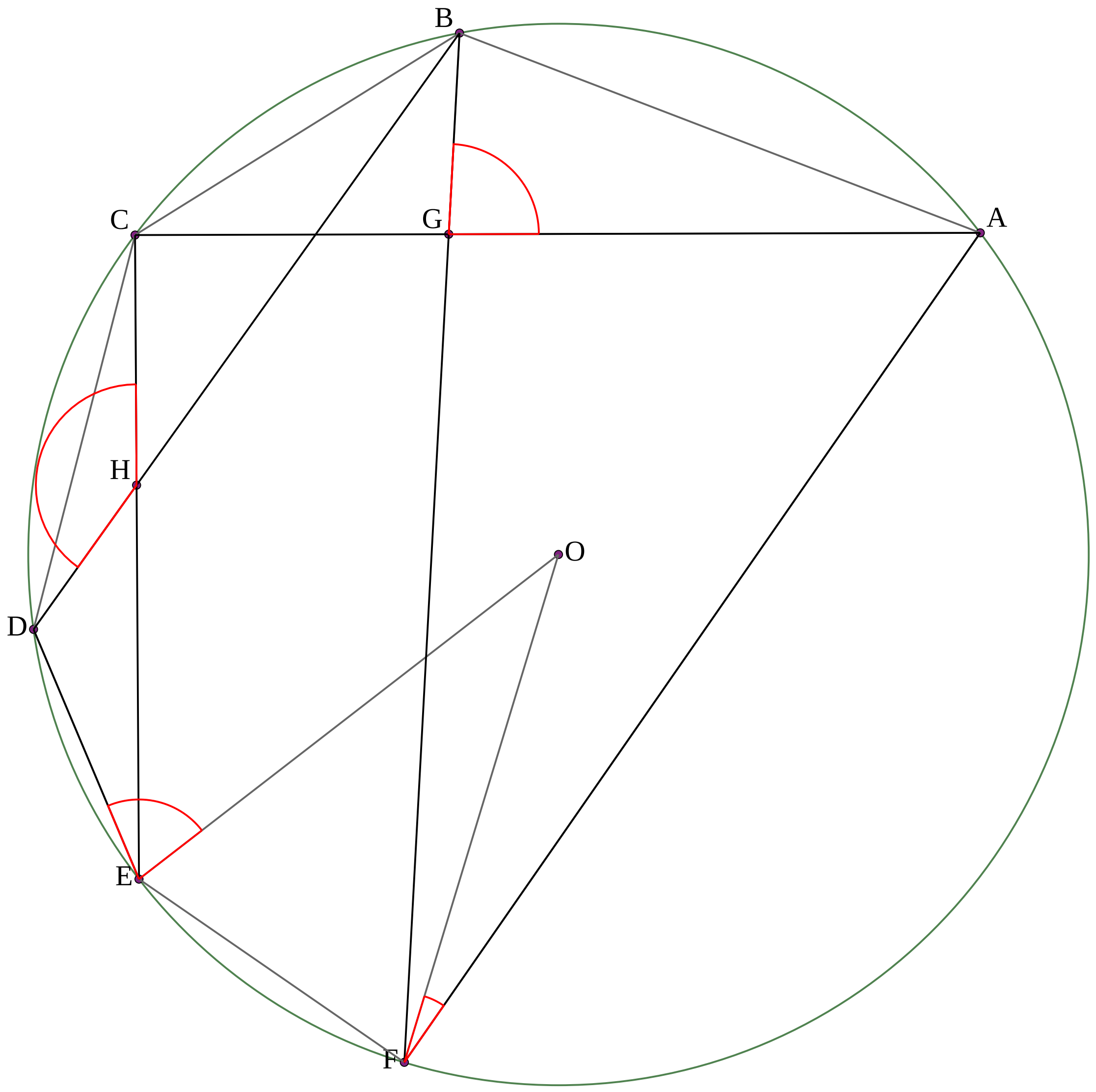
Let ABCDEFG be a cyclic heptagon with center O. Let I be the intersection of CA and BD.
Prove that CEF+DCF = AGB+AID

Example 95



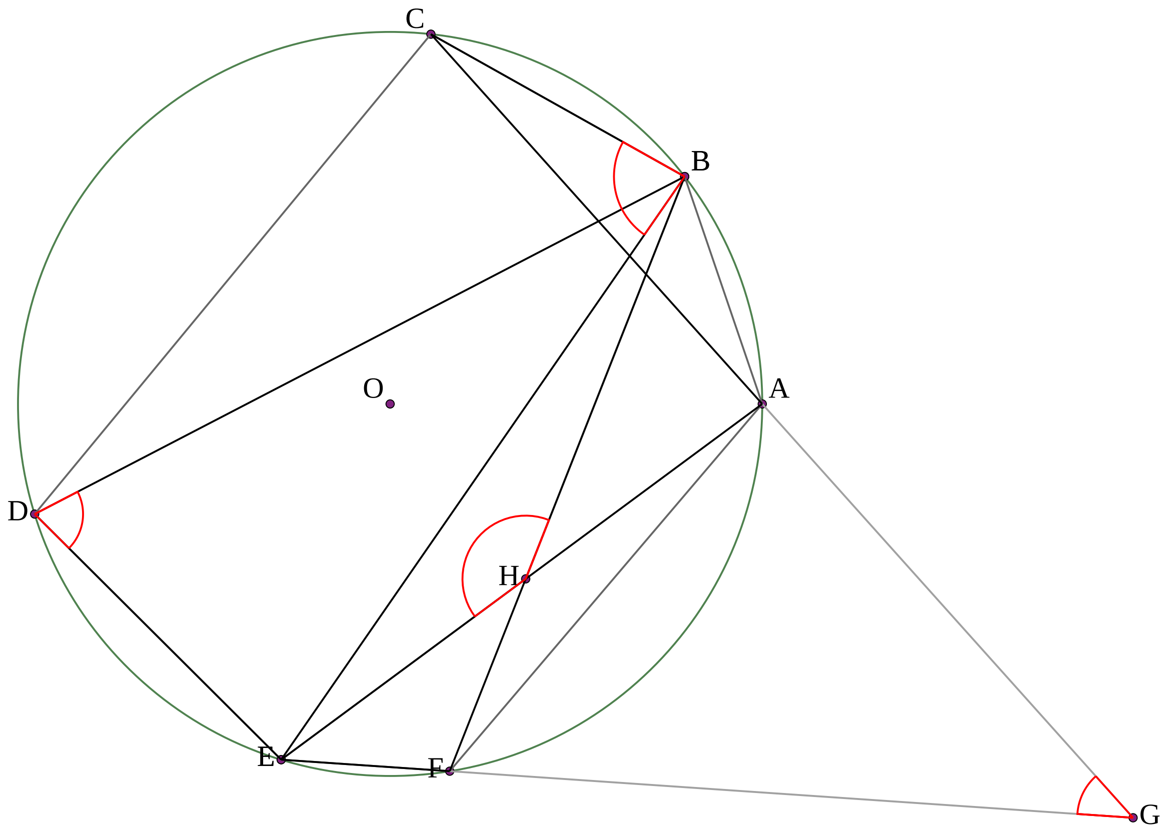
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DG and AF. Let J be the intersection of GC and HA.
Prove that DBF+CEH+FIG = GJH+180

Example 96



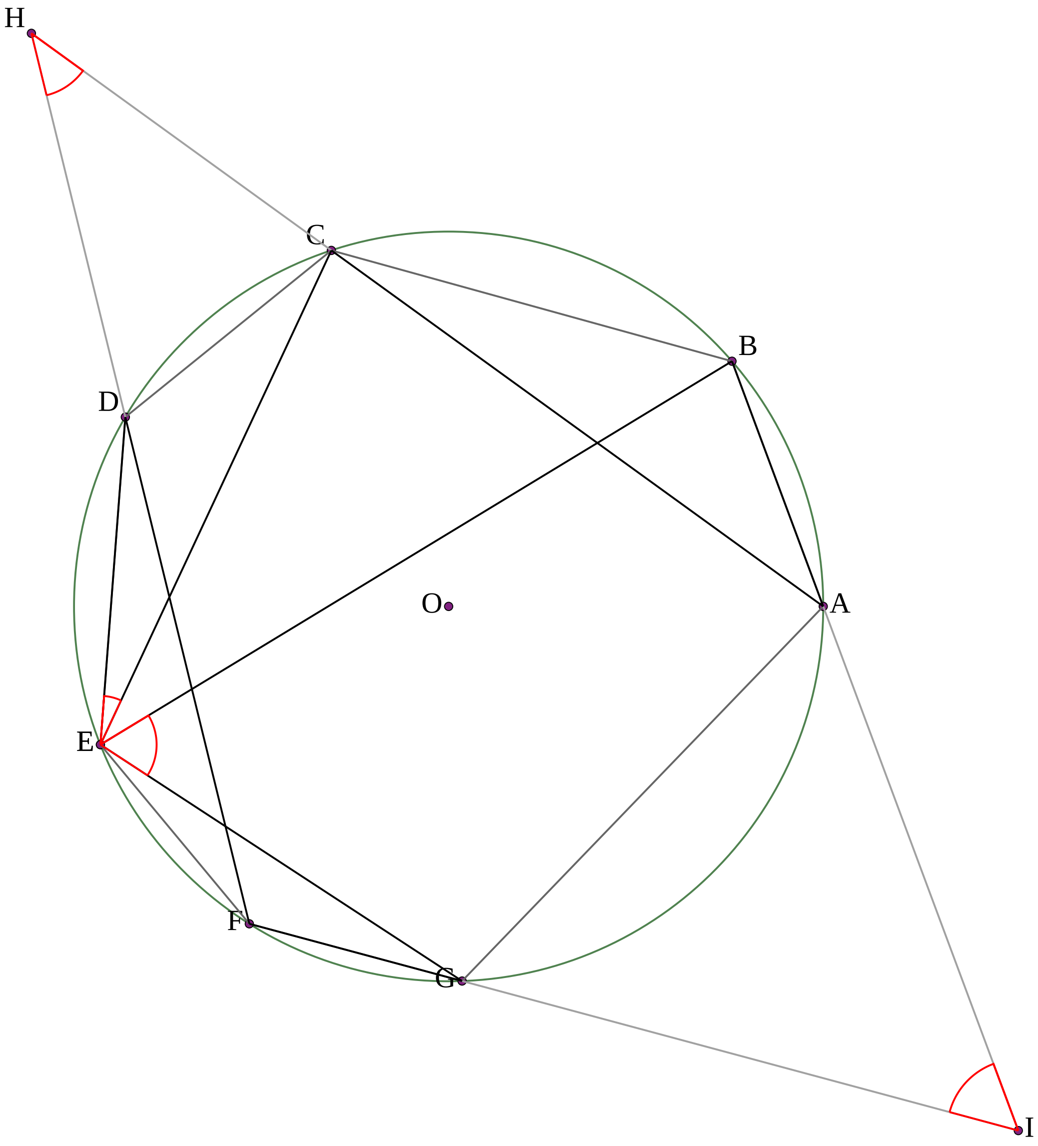
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FB and CA. Let H be the intersection of BD and EC.
Angle DEO = x. Angle BGA = y. Angle DHC = z.
Find angle AFO.

Example 97



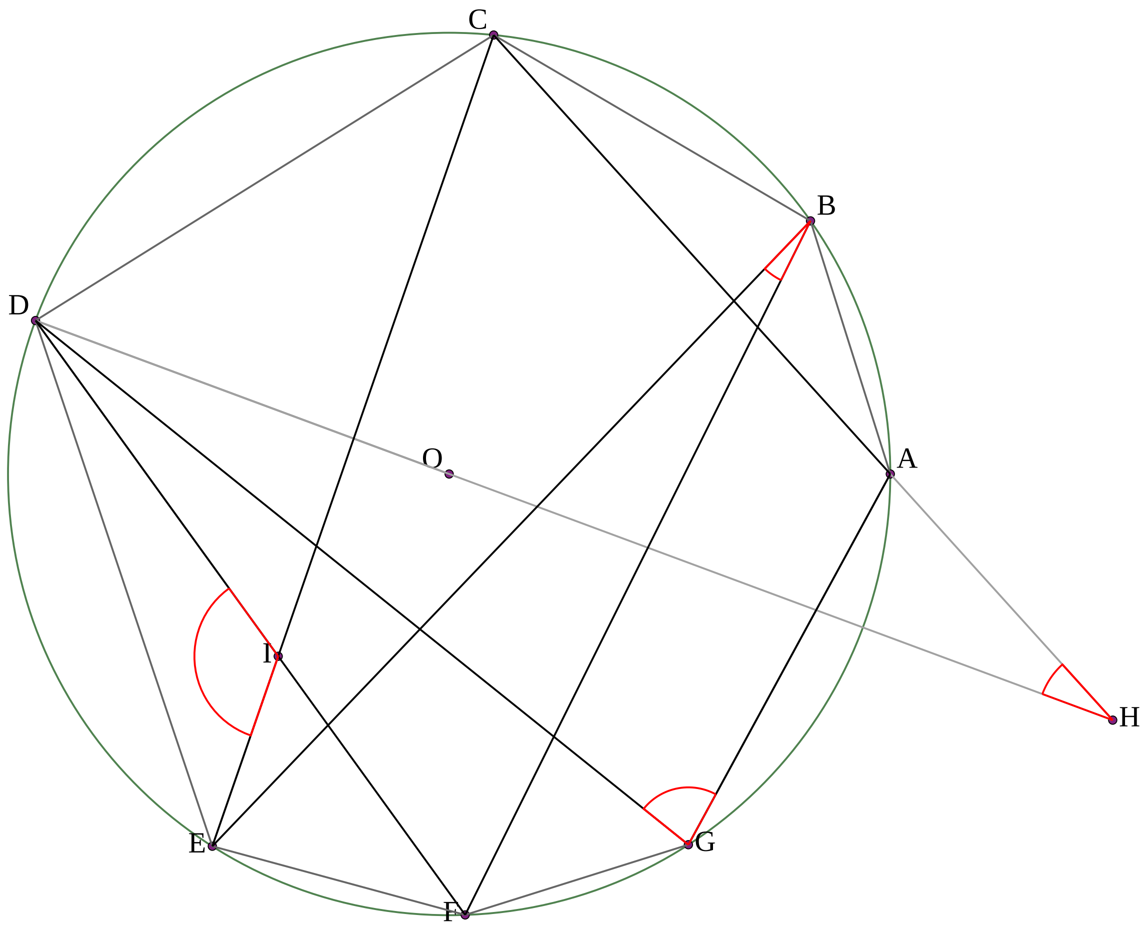
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and CA. Let H be the intersection of FB and AE.
Angle CBE = x. Angle BDE = y. Angle FGA = z.
Find angle BHE.

Example 98



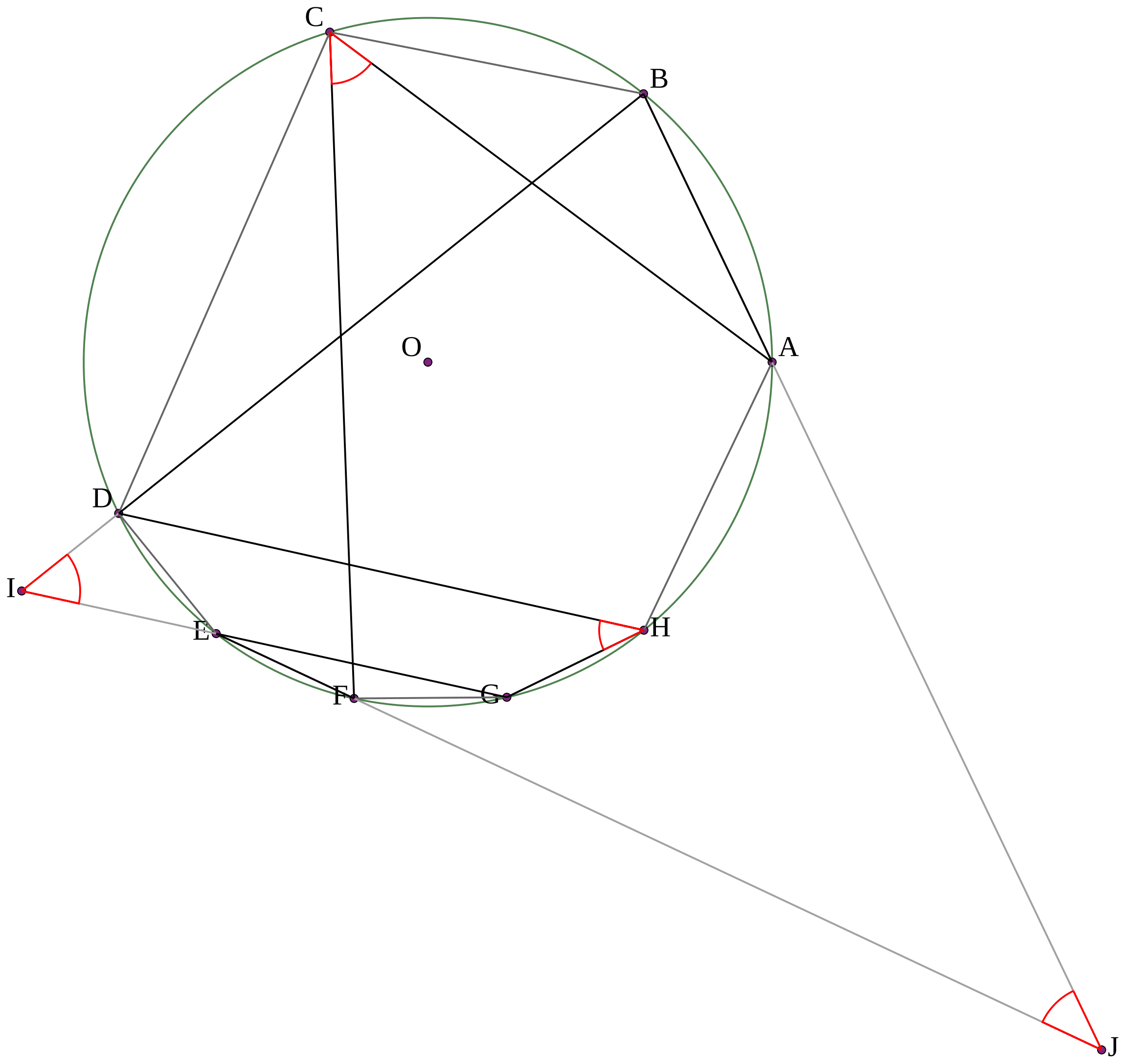
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CA and FD. Let I be the intersection of AB and GF.
Angle DEC = 21°. Angle CHD = 40°. Angle AIG = 55°.
Find angle BEG.

Example 99



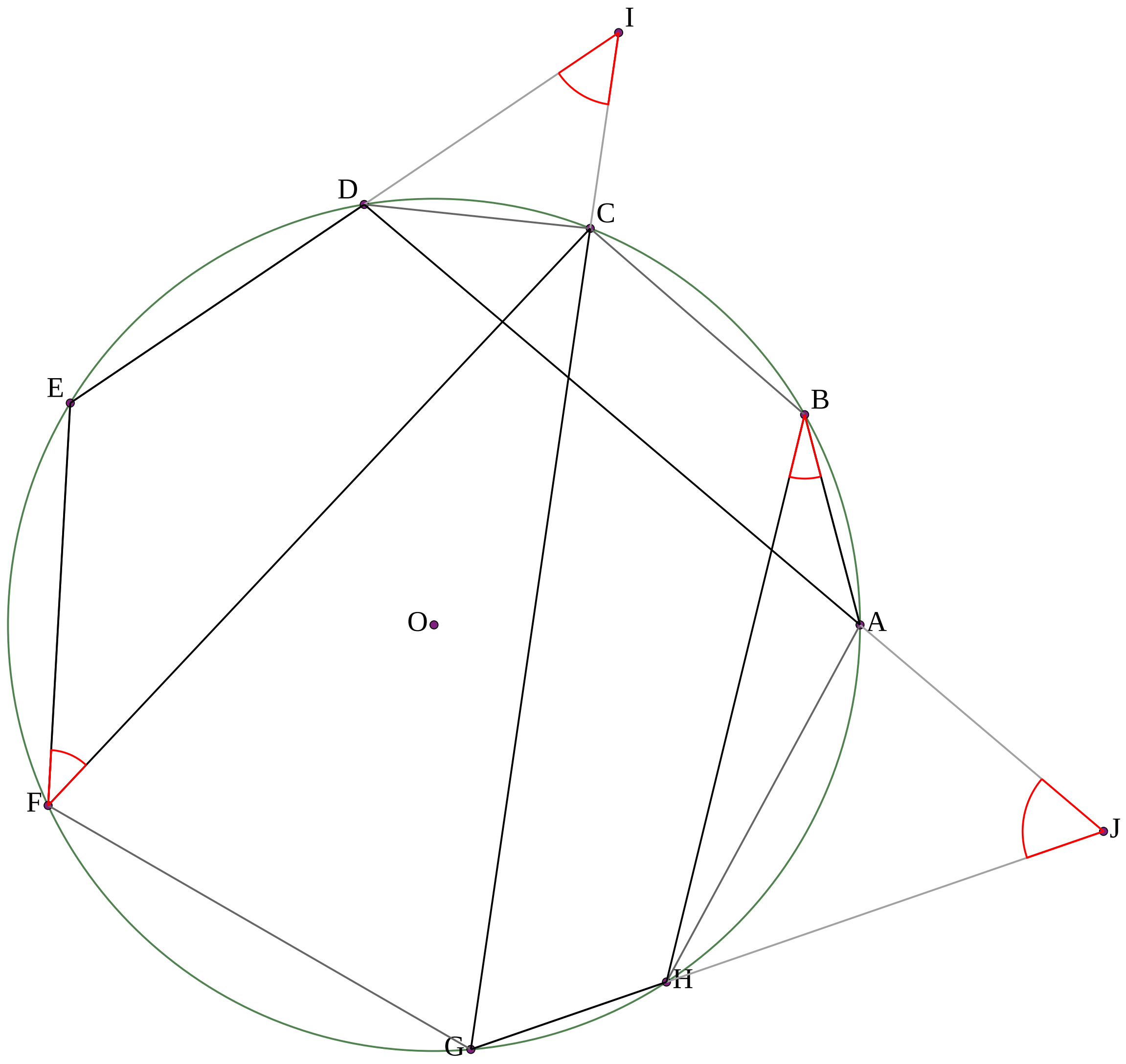
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AC and DO. Let I be the intersection of CE and FD.
Angle DGA = x. Angle AHD = y. Angle EID = z.
Find angle EBF.

Example 100



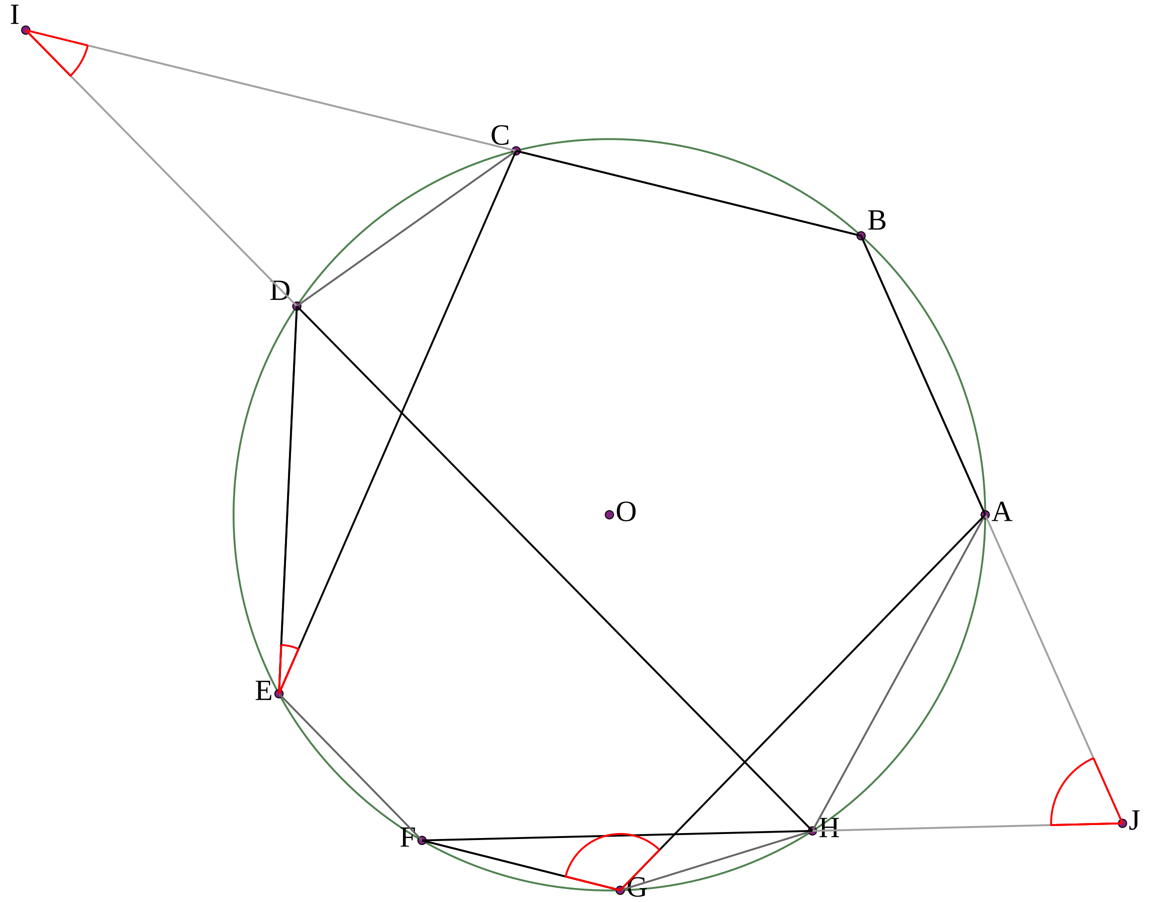
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GE and BD. Let J be the intersection of EF and AB.
Angle DHG = 39°. Angle EID = 51°. Angle FJA = 39°.
Find angle FCA.

Example 101



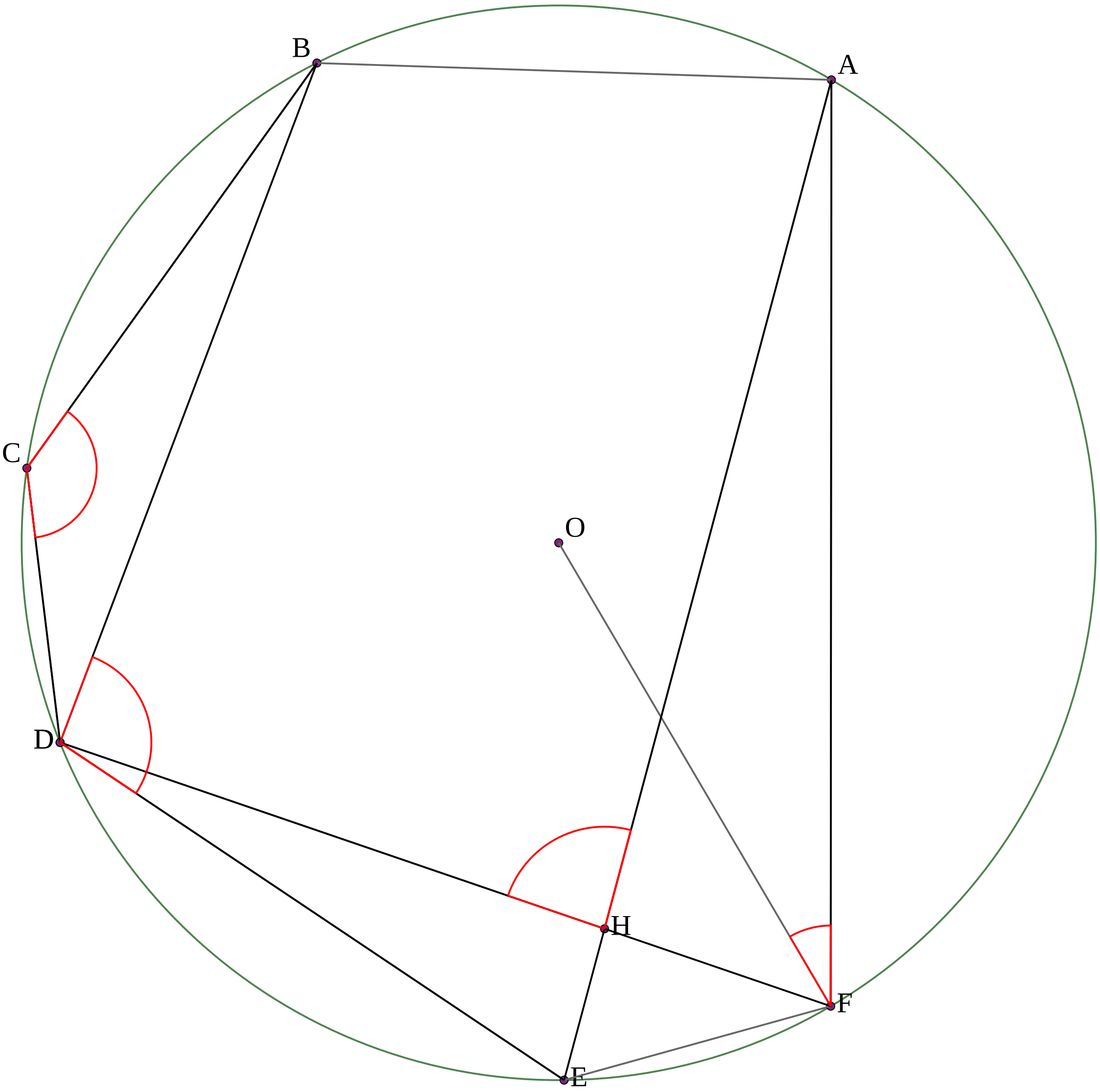
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CG and DE. Let J be the intersection of GH and AD.
Angle HBA = 28°. Angle HJA = 59°. Angle EFC = 40°.
Find angle CID.

Example 102



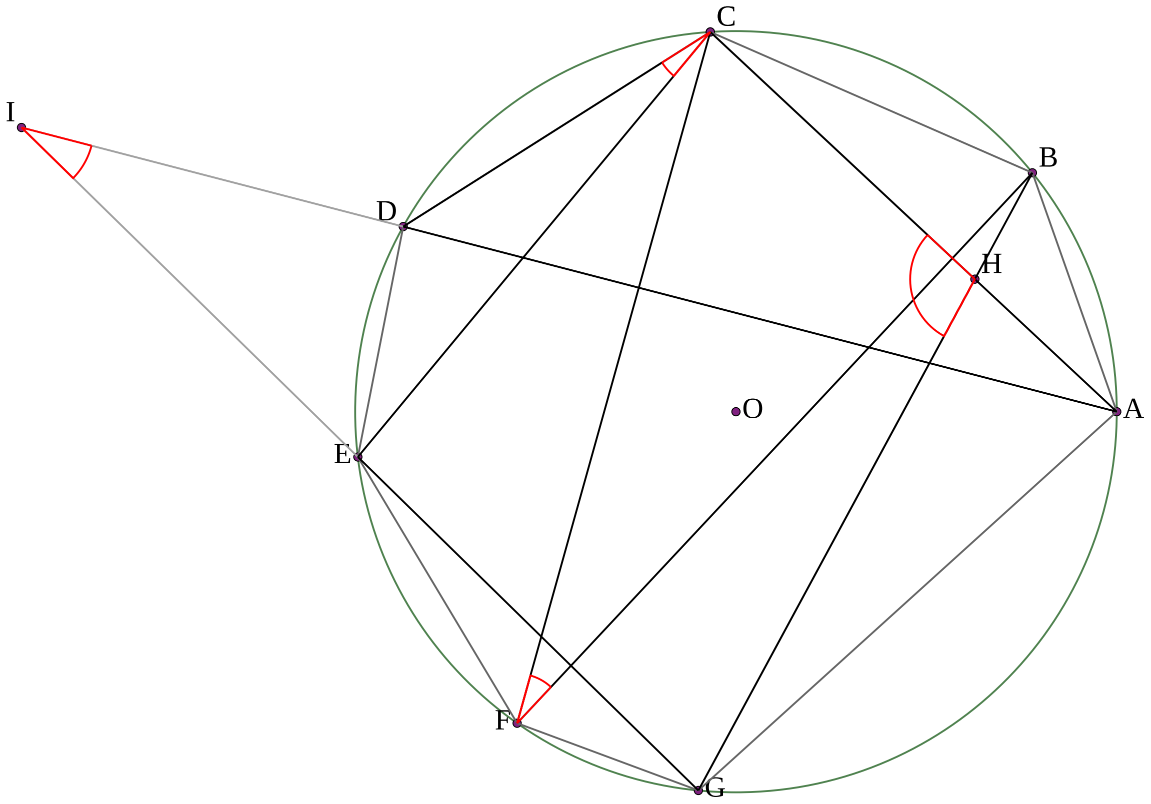
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DH and BC. Let J be the intersection of HF and AB.
Angle FGA = x. Angle DIC = y. Angle HJA = z.
Find angle CED.

Example 103



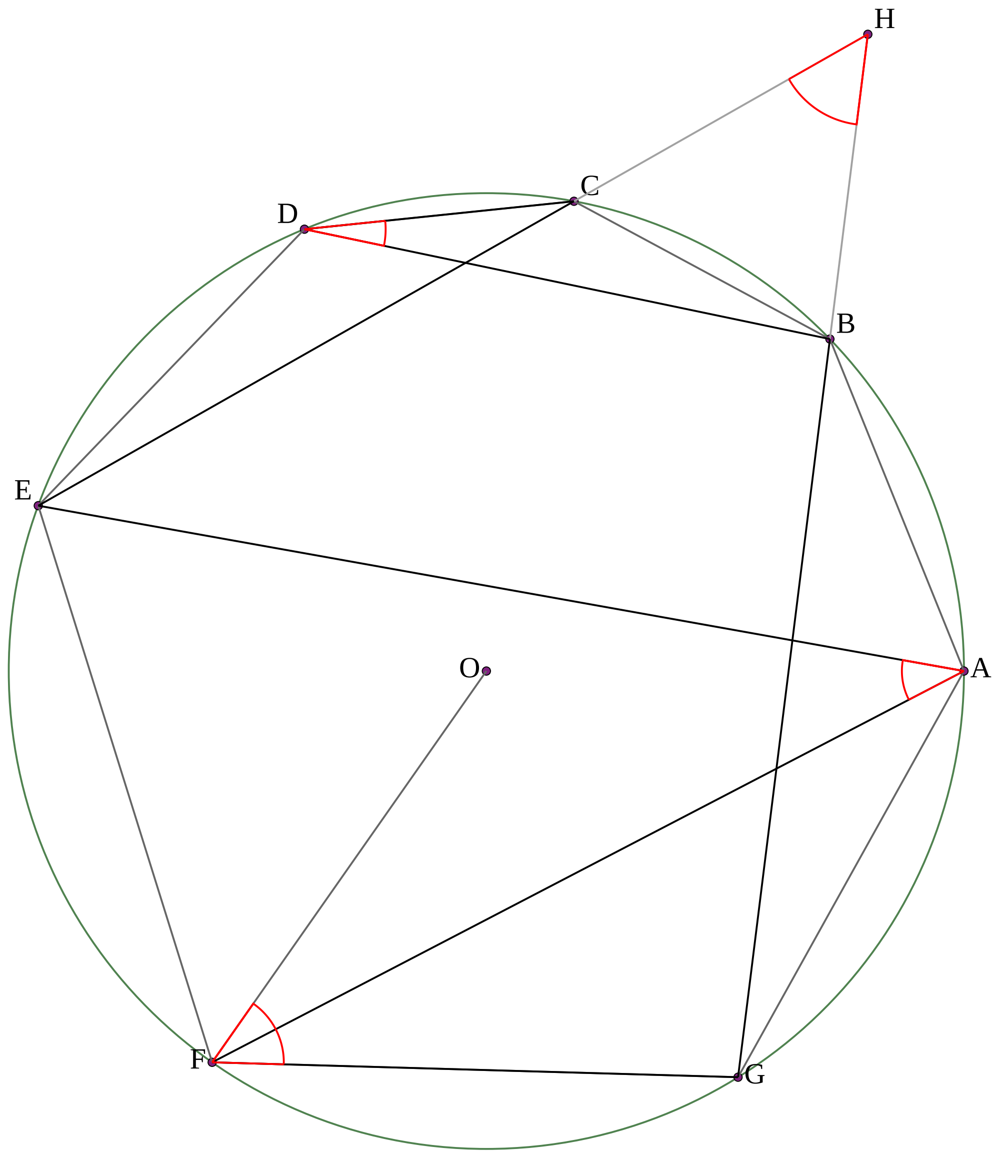
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of EA and FD.
Prove that BCD+AHD = AFO+BDE+90

Example 104



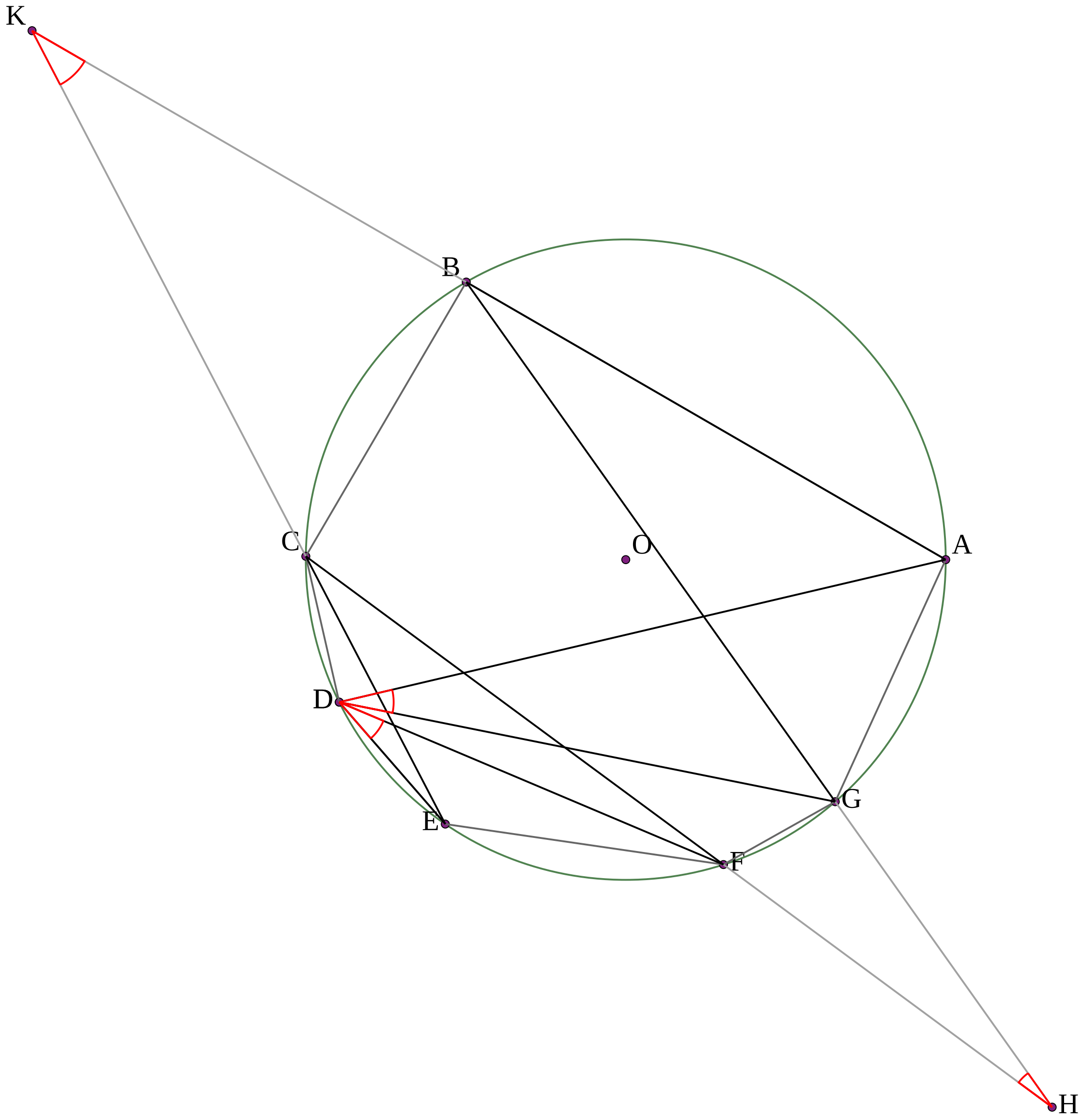
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of BG and AC. Let I be the intersection of GE and DA.
Angle CFB = x. Angle ECD = y. Angle EID = z.
Find angle GHC.

Example 105



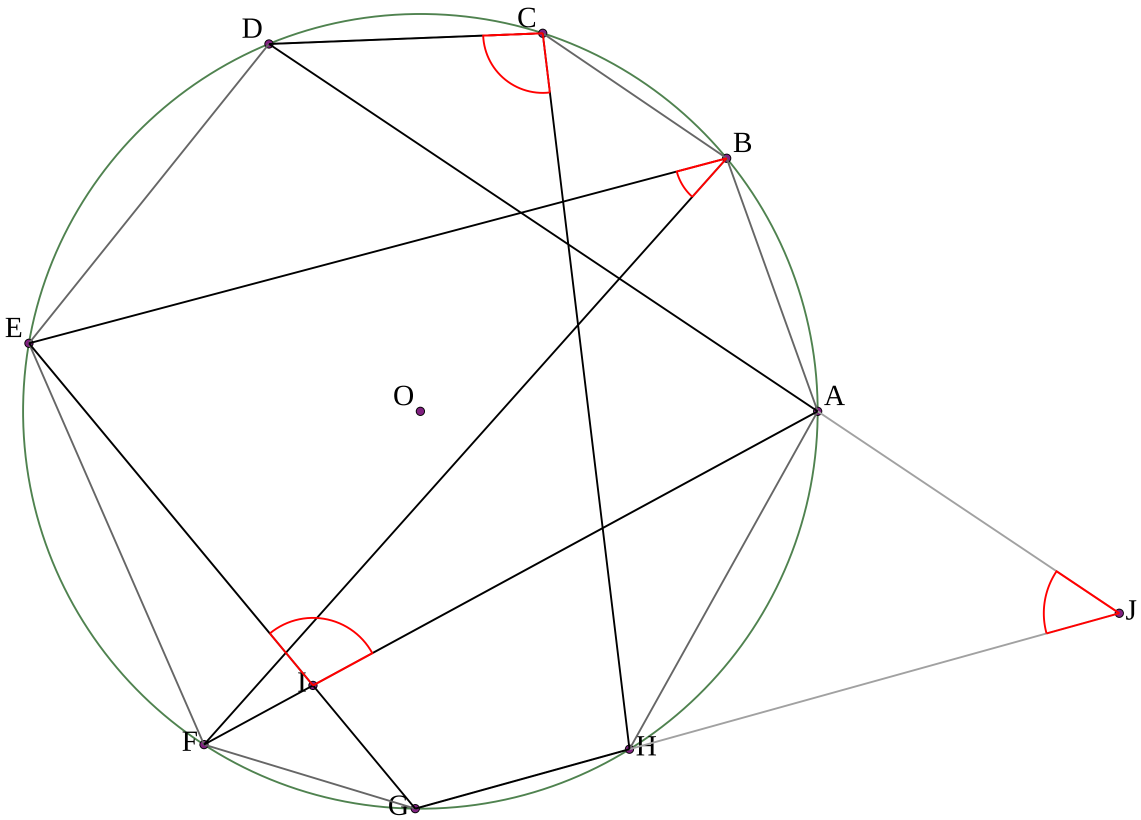
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GB and CE.
Prove that GFO+BDC+BHC = EAF+90

Example 106



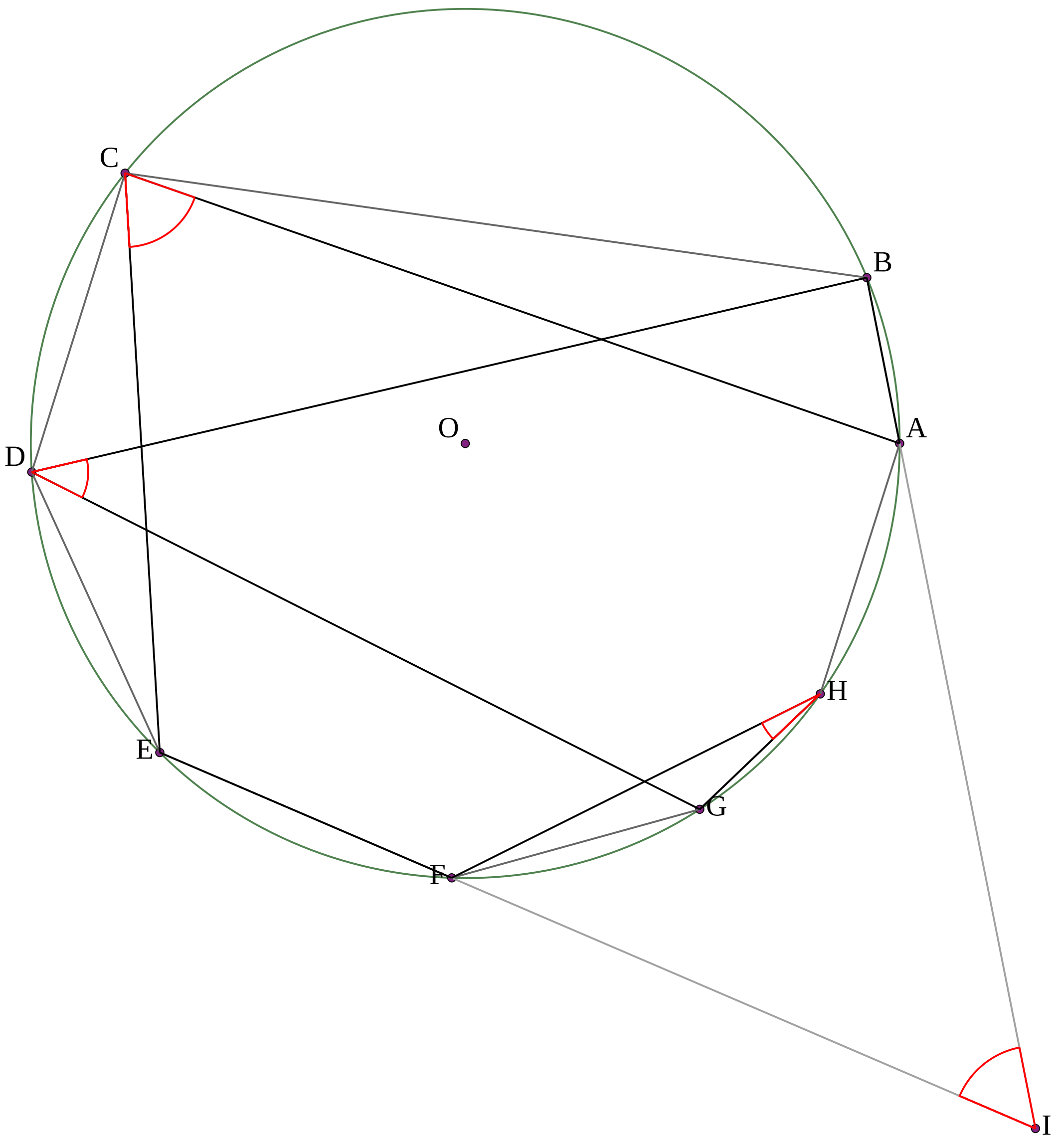
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CF and GB. Let K be the intersection of BA and EC.
Angle FDE = x. Angle GDA = y. Angle BKC = z.
Find angle FHG.

Example 107



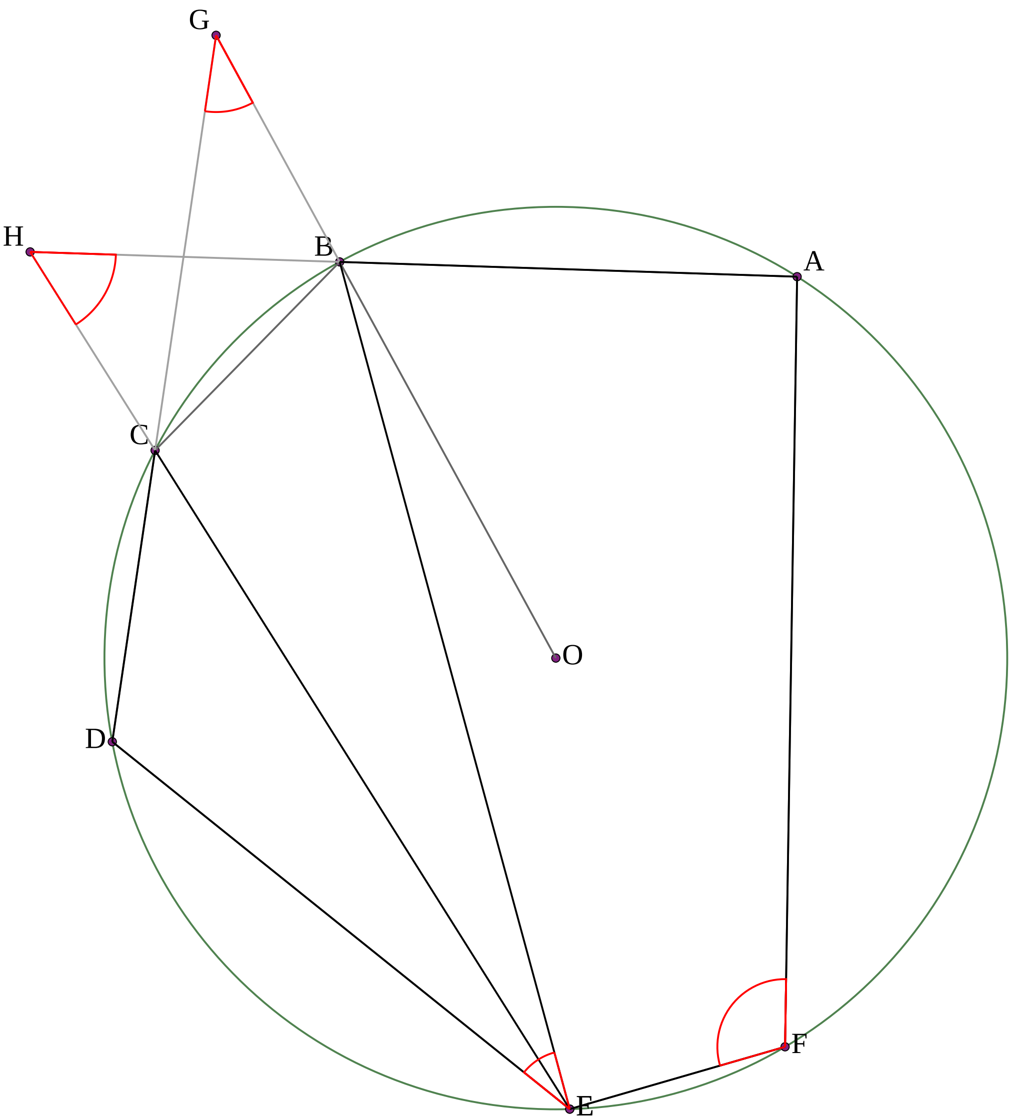
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FA and GE. Let J be the intersection of AD and HG.
Angle AJH = 49°. Angle EBF = 33°. Angle AIE = 101°.
Find angle DCH.

Example 108



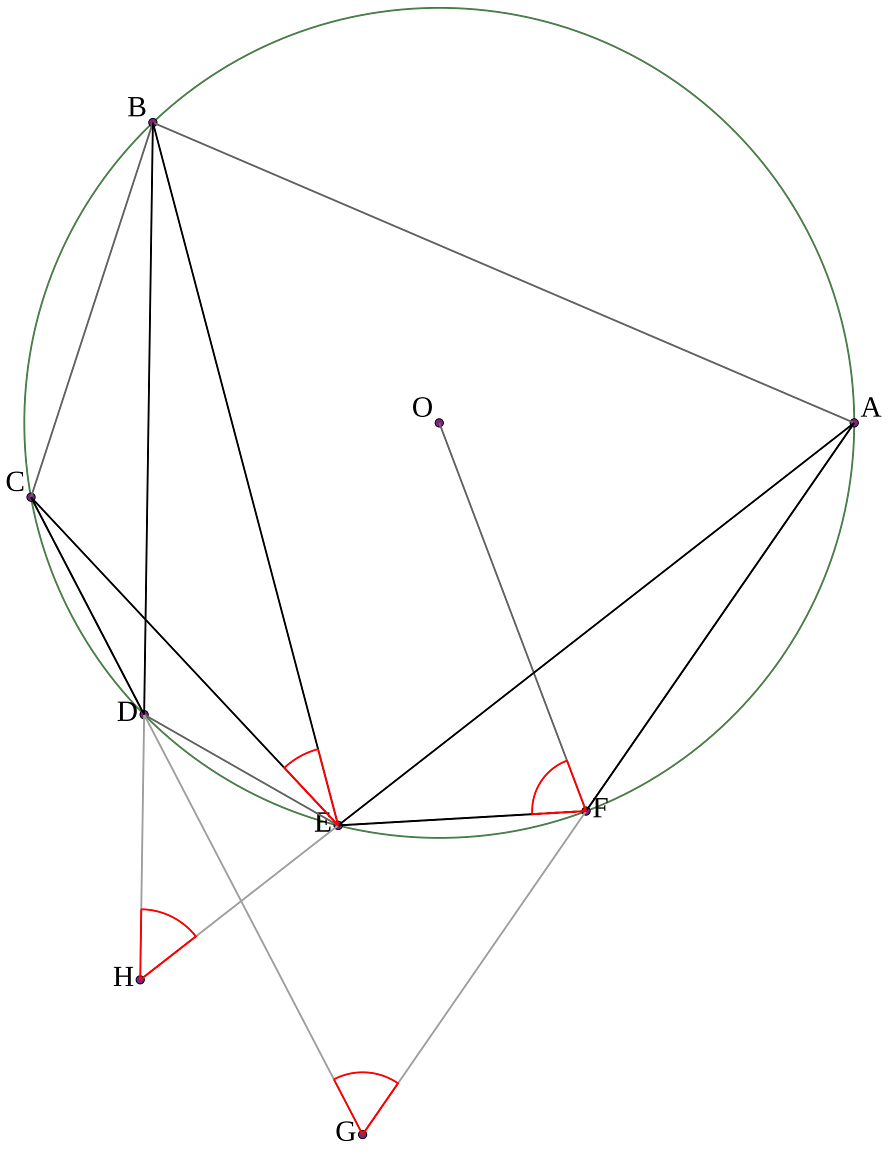
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FE and AB.
Angle BDG = 40°. Angle GHF = 17°. Angle ECA = 67°.
Find angle FIA.

Example 109



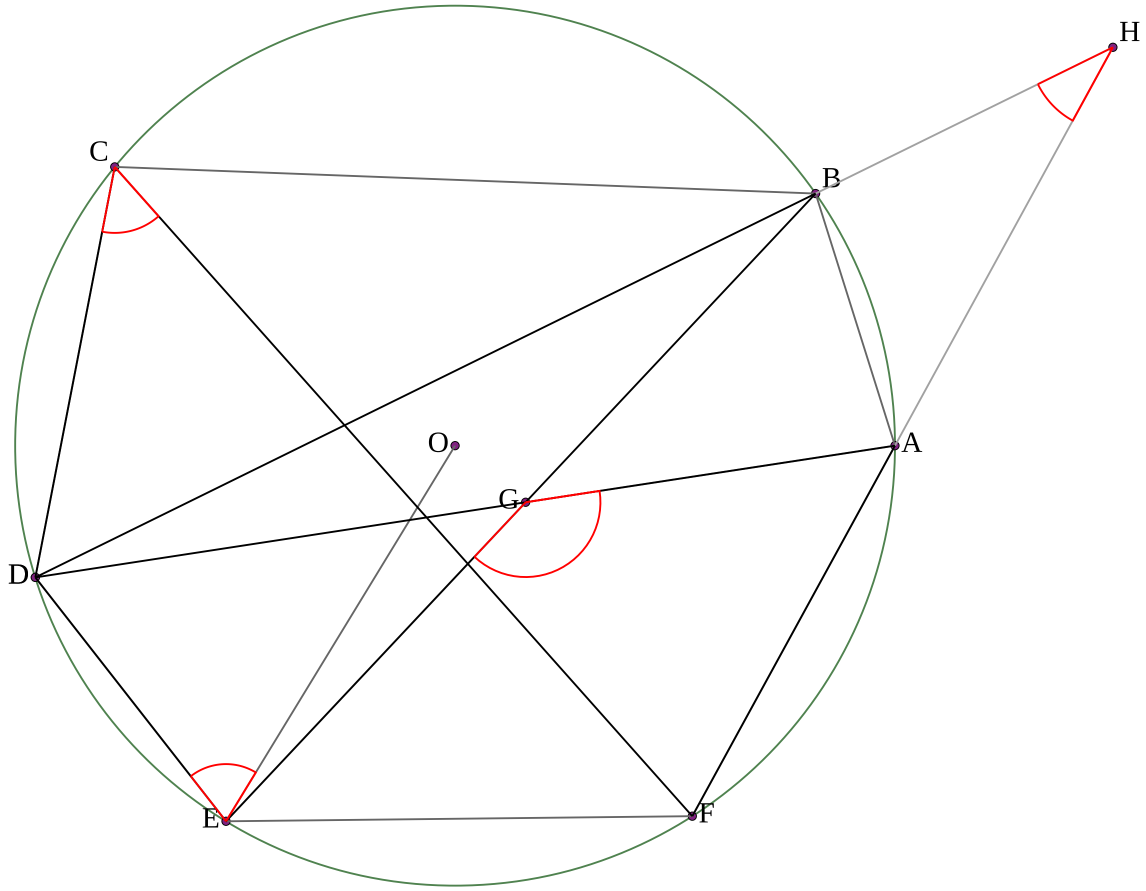
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of OB and CD. Let H be the intersection of BA and EC.
Prove that AFE+BHC = BED+BGC+90

Example 110



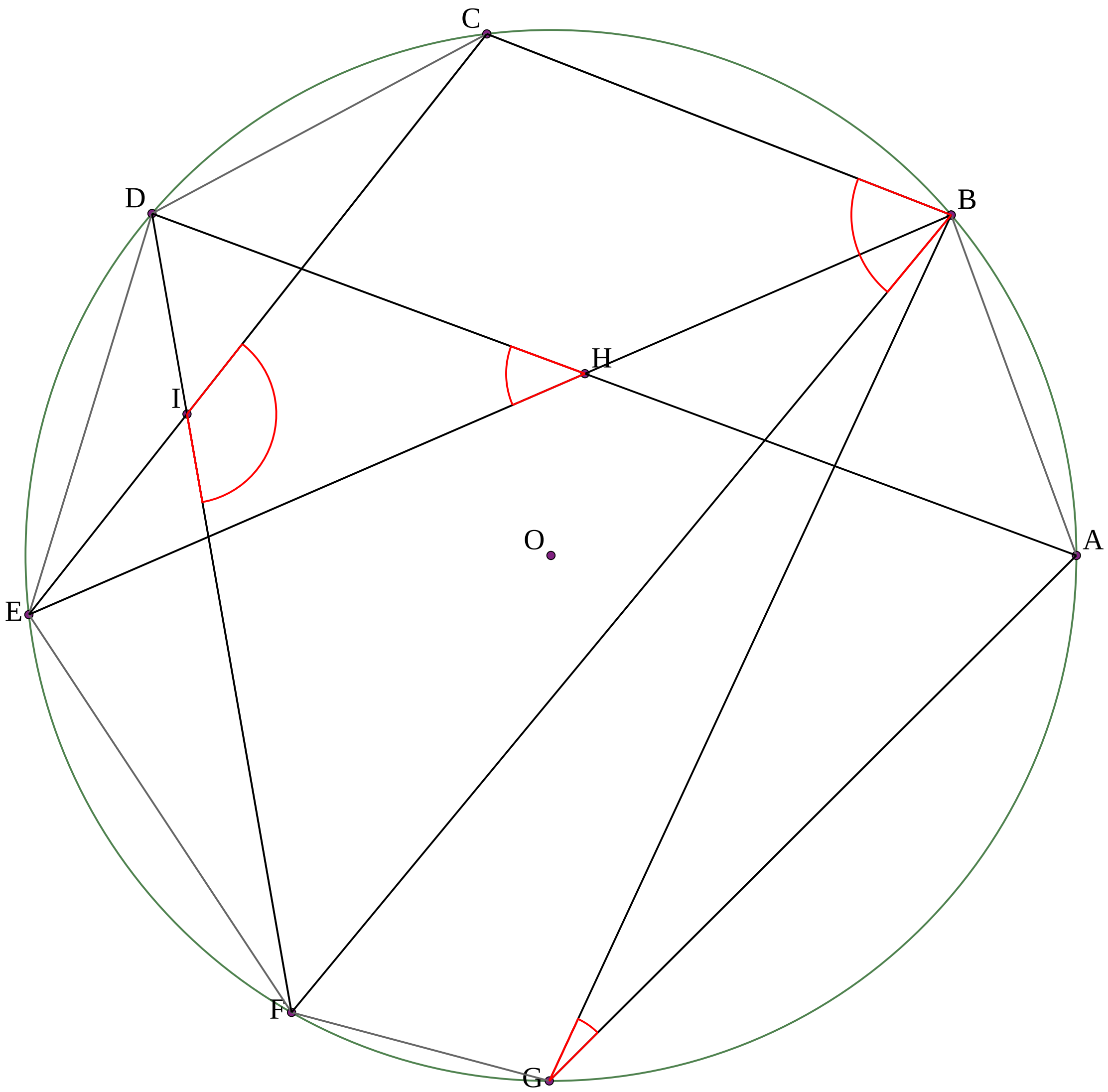
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FA and CD. Let H be the intersection of AE and DB.
Angle FGD = 62°. Angle EFO = 73°. Angle CEB = 28°.
Find angle EHD.

Example 111



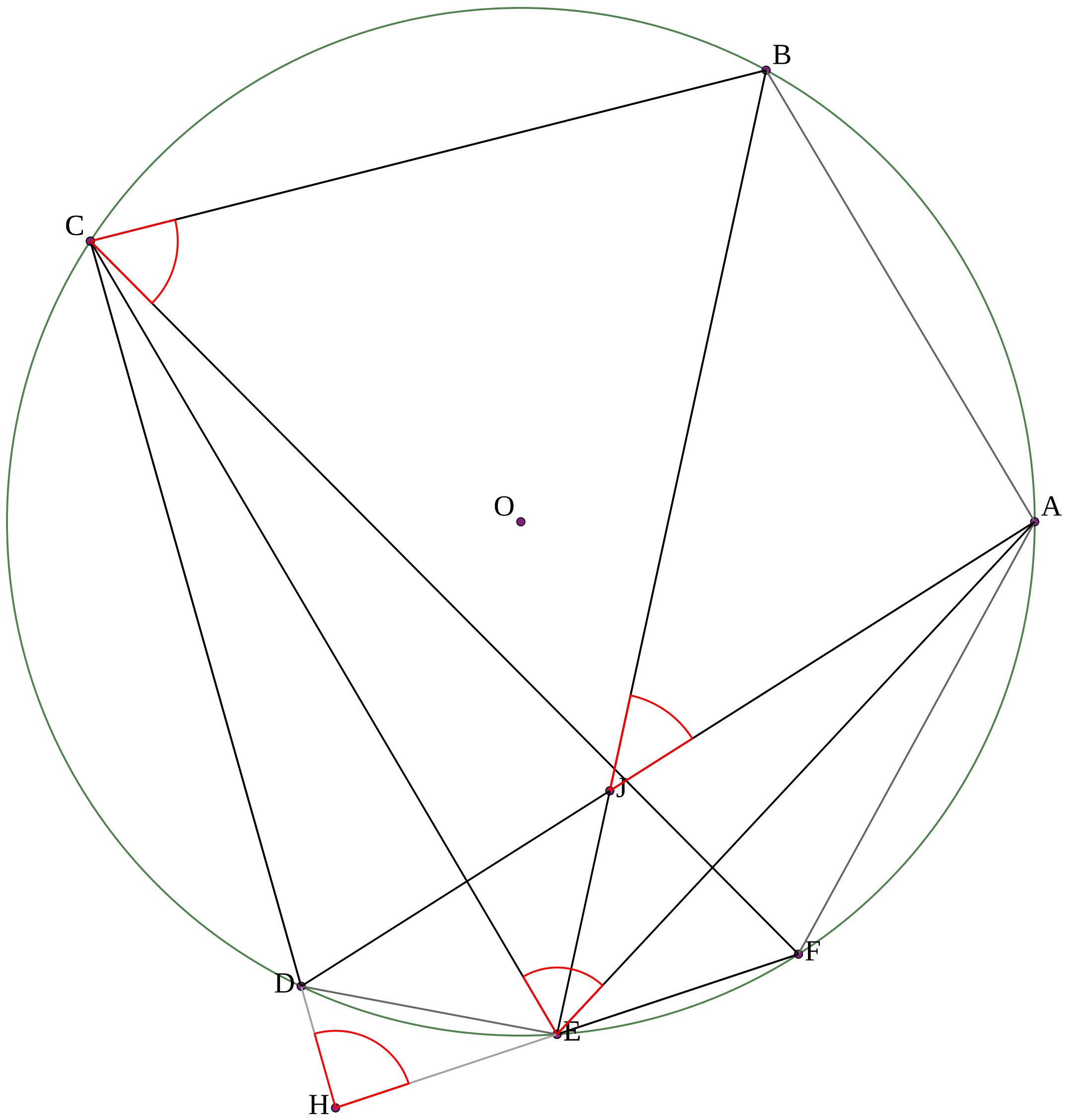
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DA and BE. Let H be the intersection of AF and DB.
Angle OED = 69°. Angle AHB = 35°. Angle FCD = 53°.
Find angle AGE.

Example 112



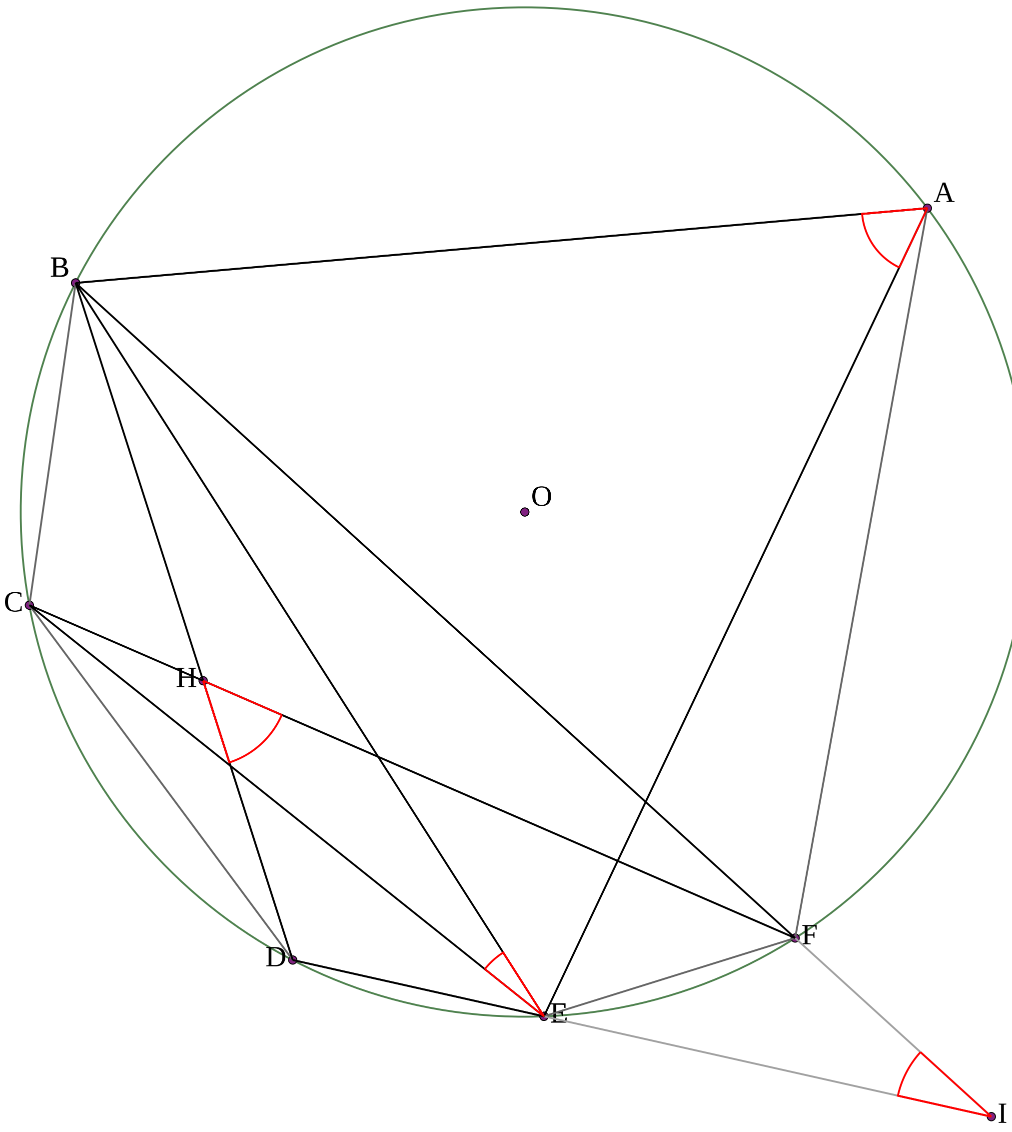
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AD and BE. Let I be the intersection of DF and EC.
Angle DHE = 44°. Angle BGA = 20°. Angle FIC = 132°.
Find angle FBC.

Example 113



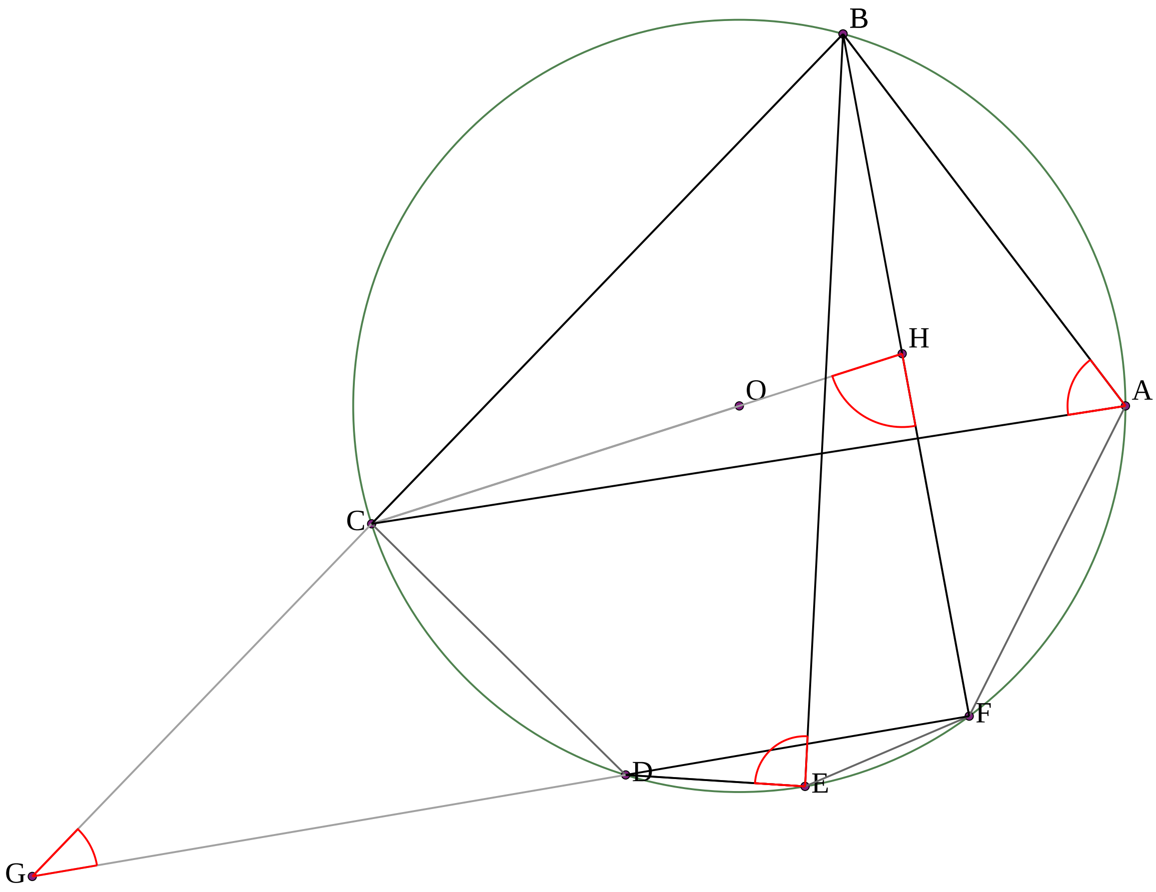
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of EF and CD. Let J be the intersection of EB and DA.
Prove that DHE+AJB = AEC+BCF

Example 114



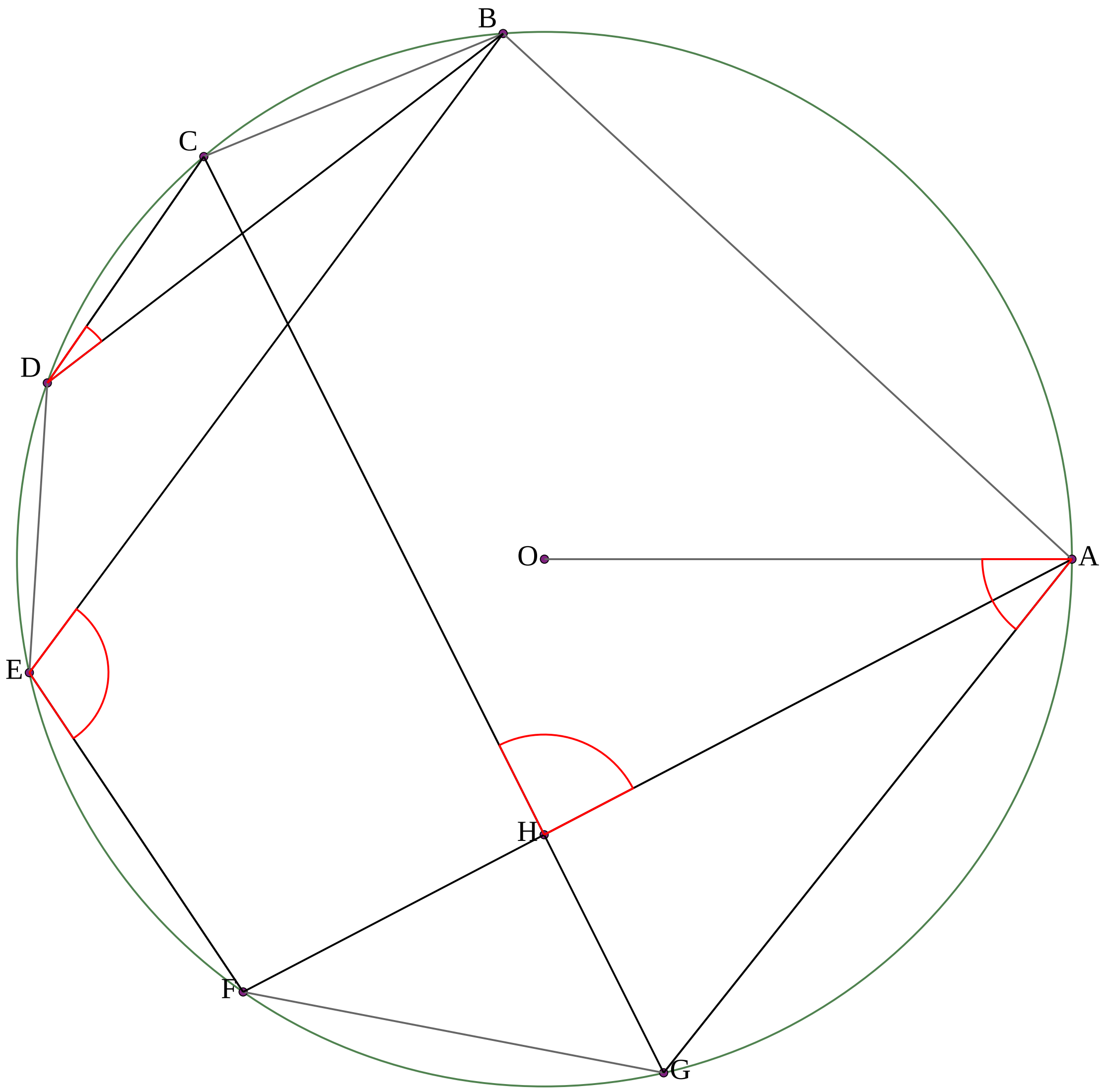
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of BD and CF. Let I be the intersection of DE and FB.
Prove that DHF+EIF = BAE+BEC

Example 115



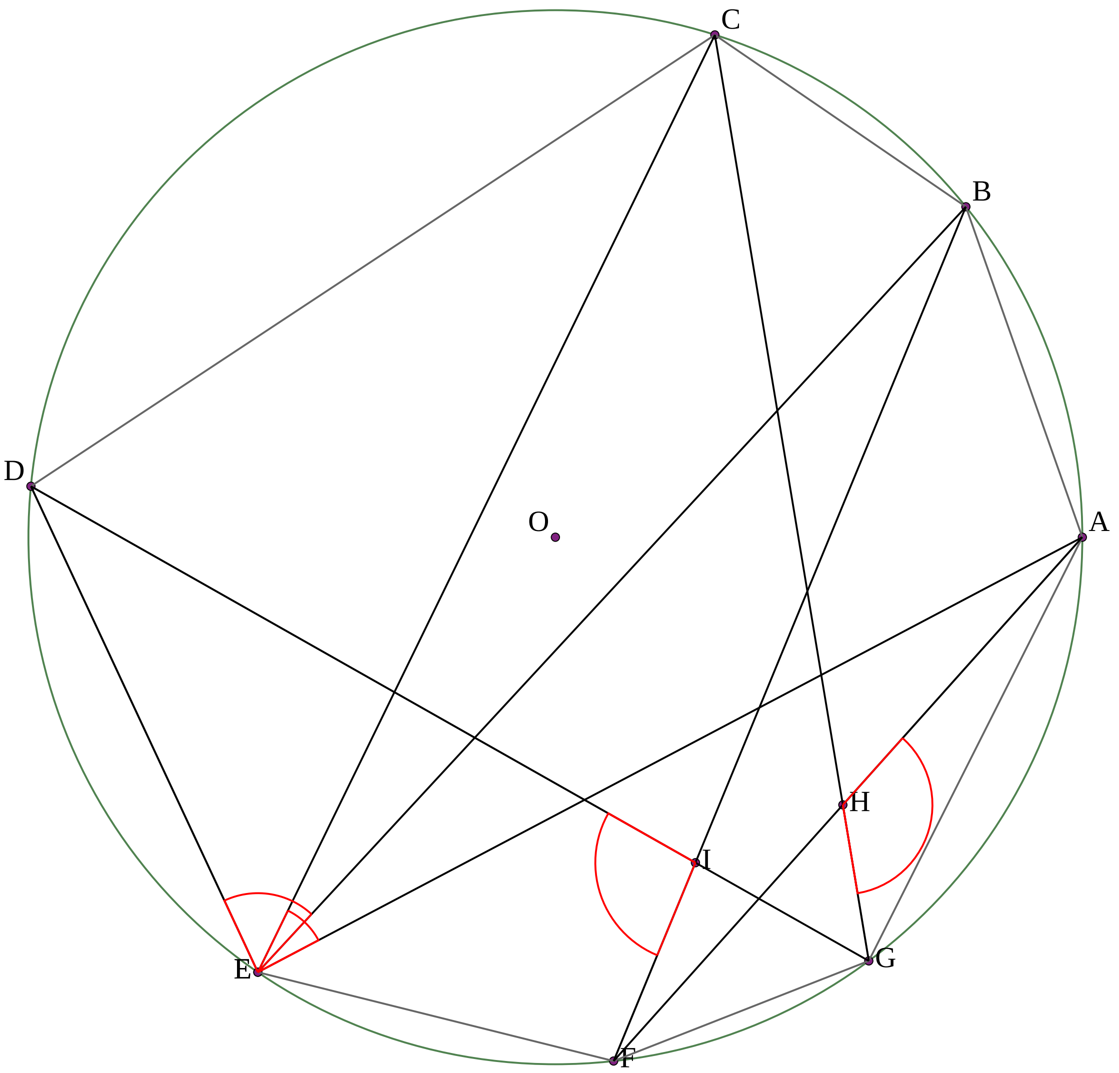
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BC and FD. Let H be the intersection of OC and BF.
Angle CAB = x. Angle CGD = y. Angle CHF = z.
Find angle DEB.

Example 116



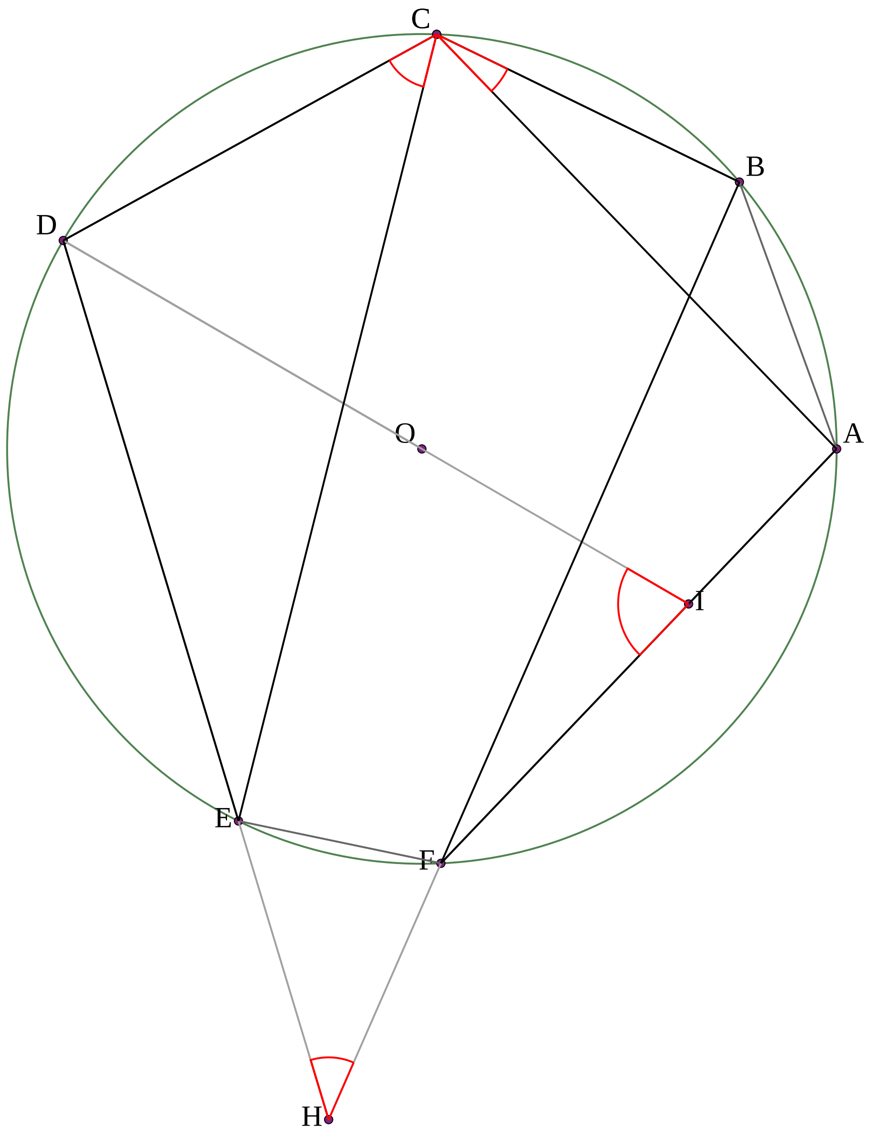
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FA and GC.
Prove that BDC+BEF+GAO = AHC+90

Example 117



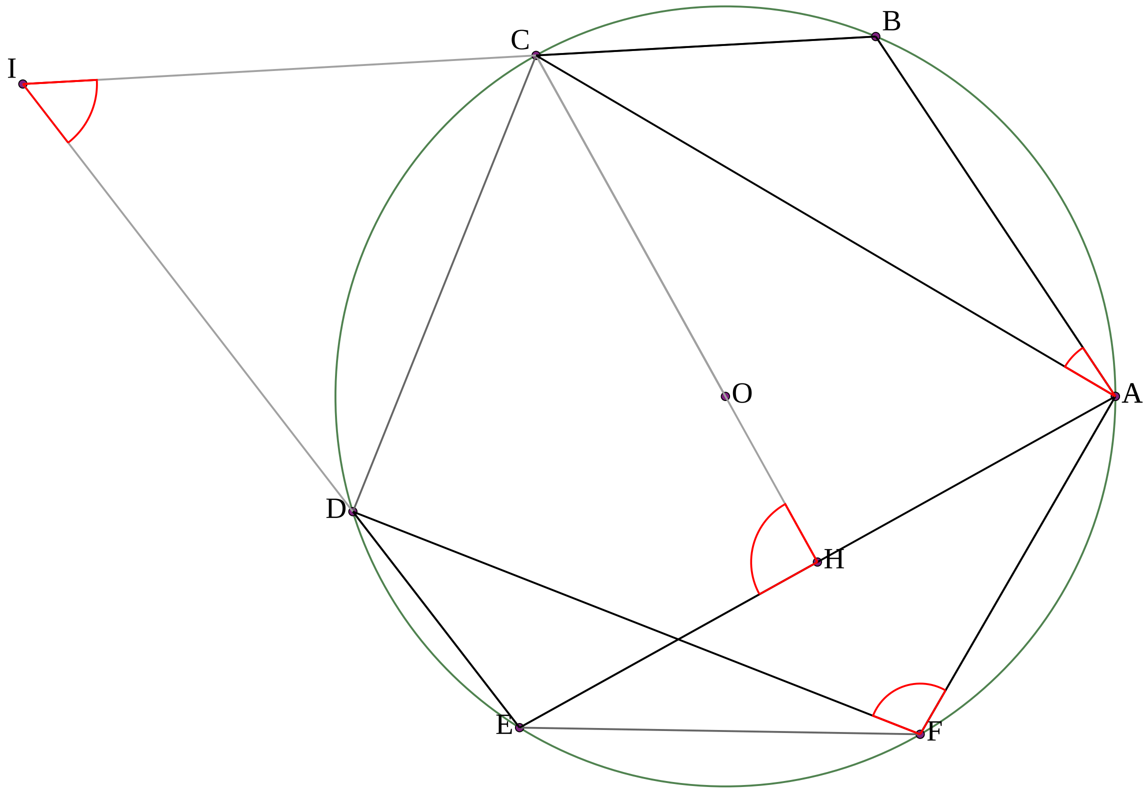
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CG and FA. Let I be the intersection of GD and BF.
Angle DEB = 68°. Angle DIF = 97°. Angle GHA = 129°.
Find angle AEC.

Example 118



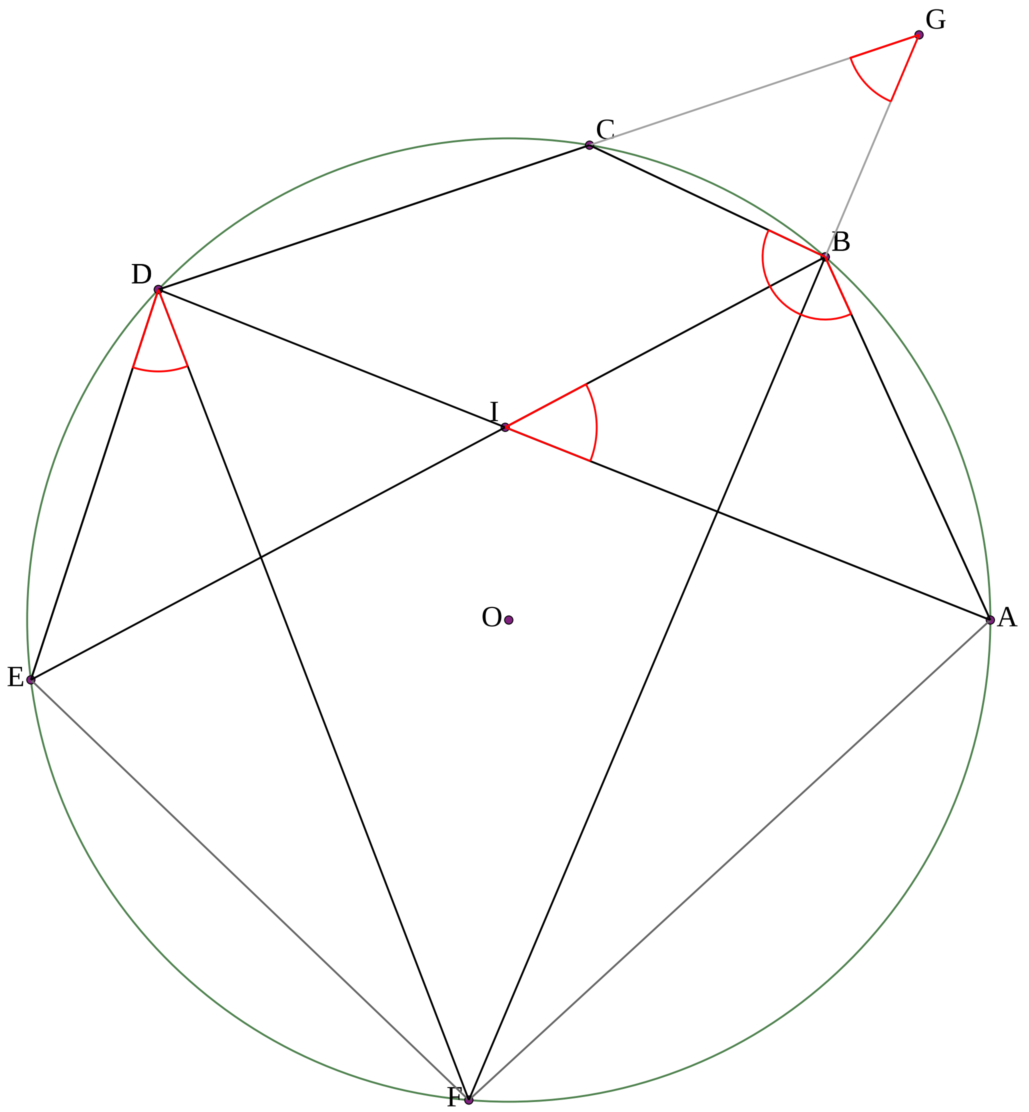
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of ED and FB. Let I be the intersection of OD and AF.
Angle EHF = x. Angle DIF = y. Angle ACB = z.
Find angle ECD.

Example 119



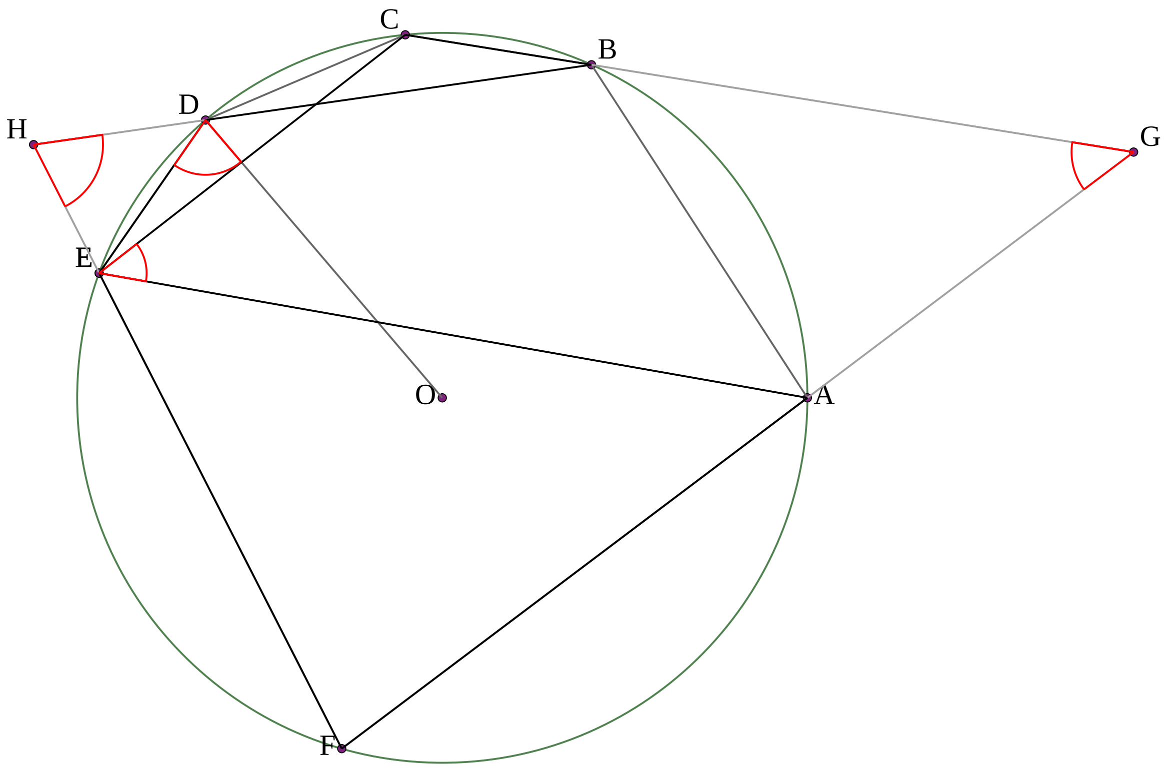
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of OC and AE. Let I be the intersection of CB and ED.
Angle DFA = 99°. Angle CAB = 26°. Angle CID = 56°.
Find angle CHE.

Example 120



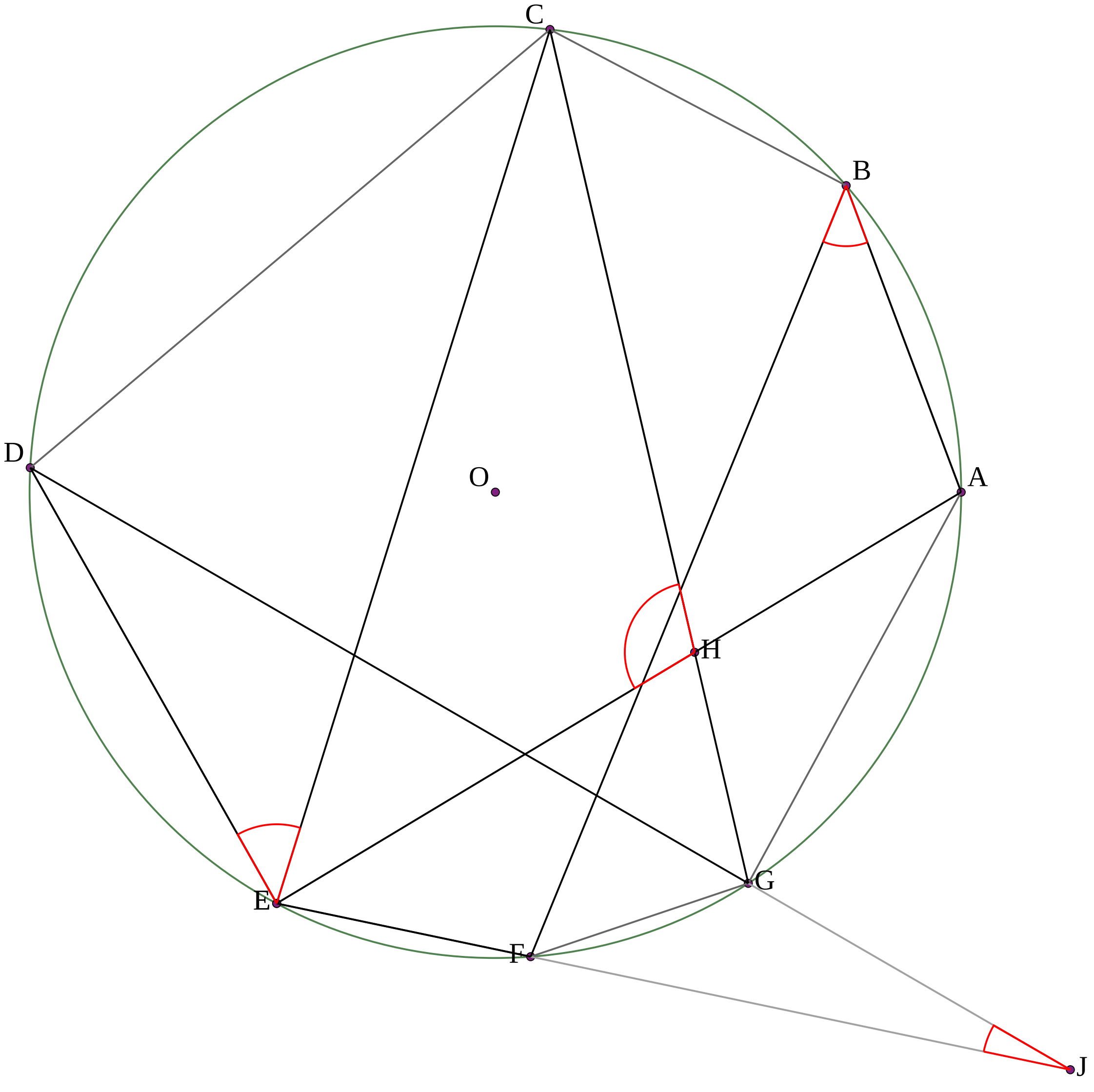
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CD and BF. Let I be the intersection of EB and DA.
Angle CGB = x. Angle ABC = y. Angle BIA = z.
Find angle EDF.

Example 121



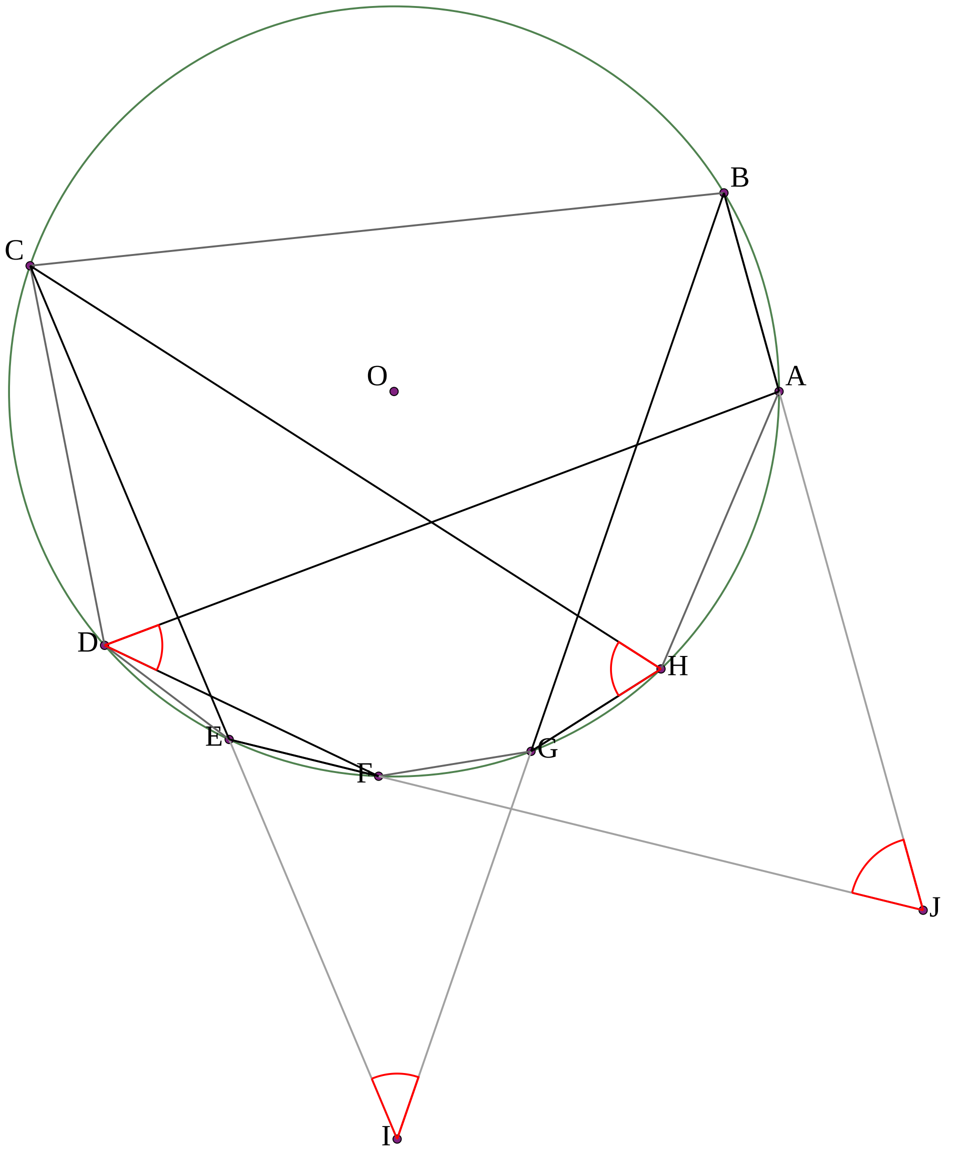
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CB and FA. Let H be the intersection of BD and EF.
Angle AEC = 48°. Angle BGA = 46°. Angle DHE = 71°.
Find angle ODE.

Example 122



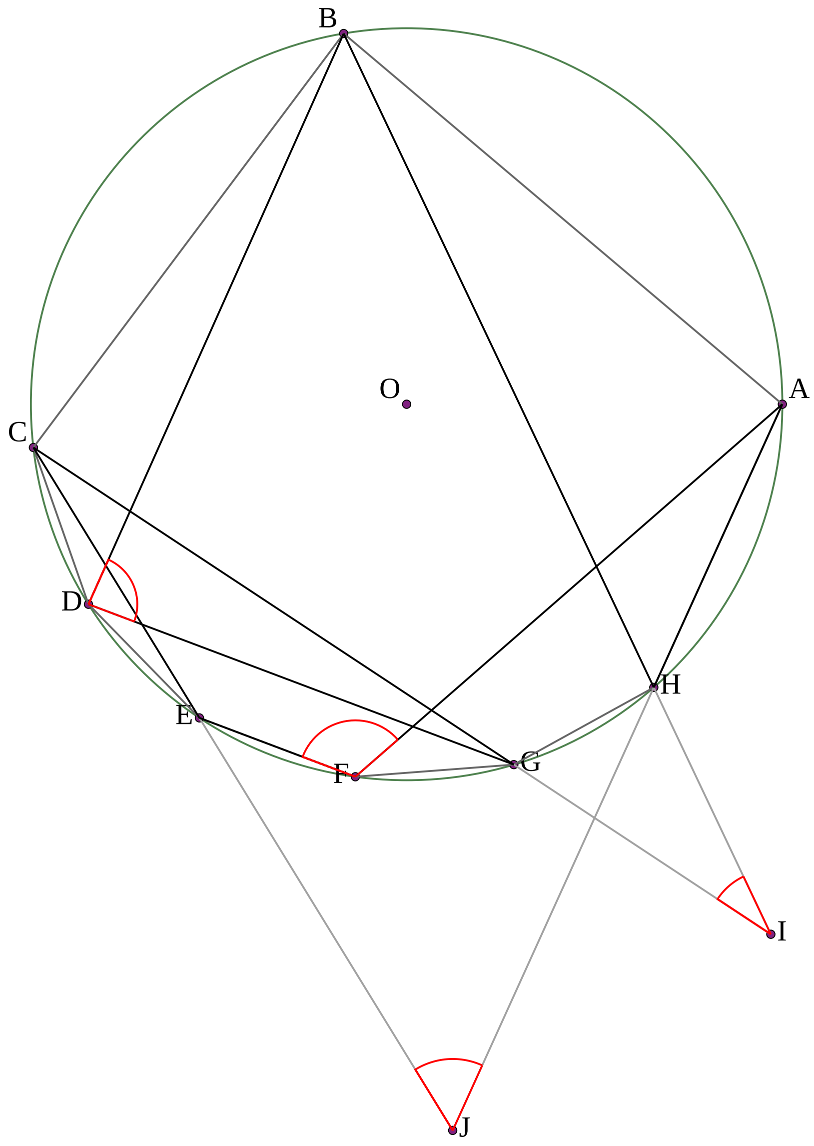
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AE and GC. Let J be the intersection of DG and EF.
Angle EHC = 108°. Angle DEC = 47°. Angle GJF = 18°.
Find angle FBA.

Example 123



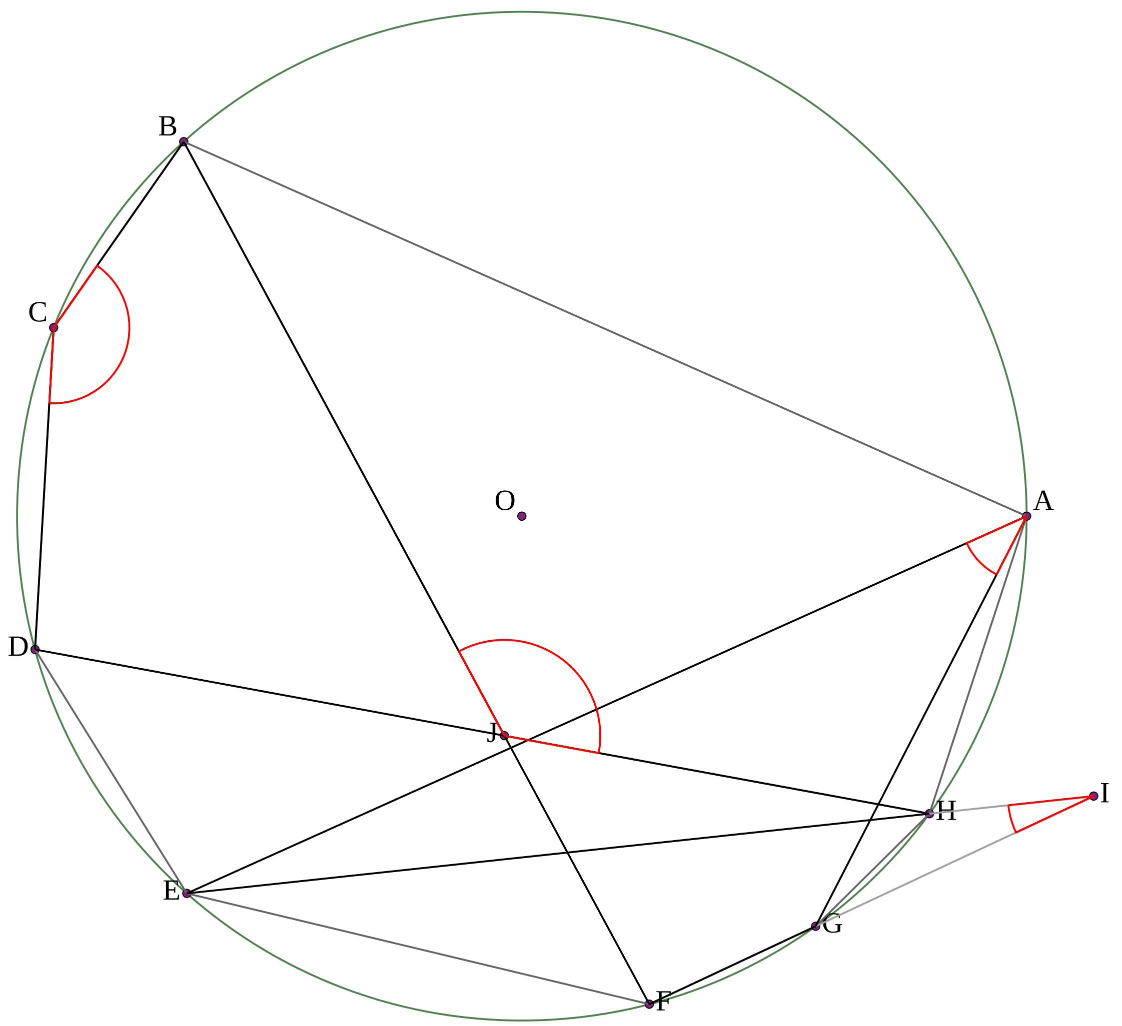
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GB and EC. Let J be the intersection of BA and FE.
Angle CHG = 65°. Angle GIE = 42°. Angle ADF = 46°.
Find angle AJF.

Example 124



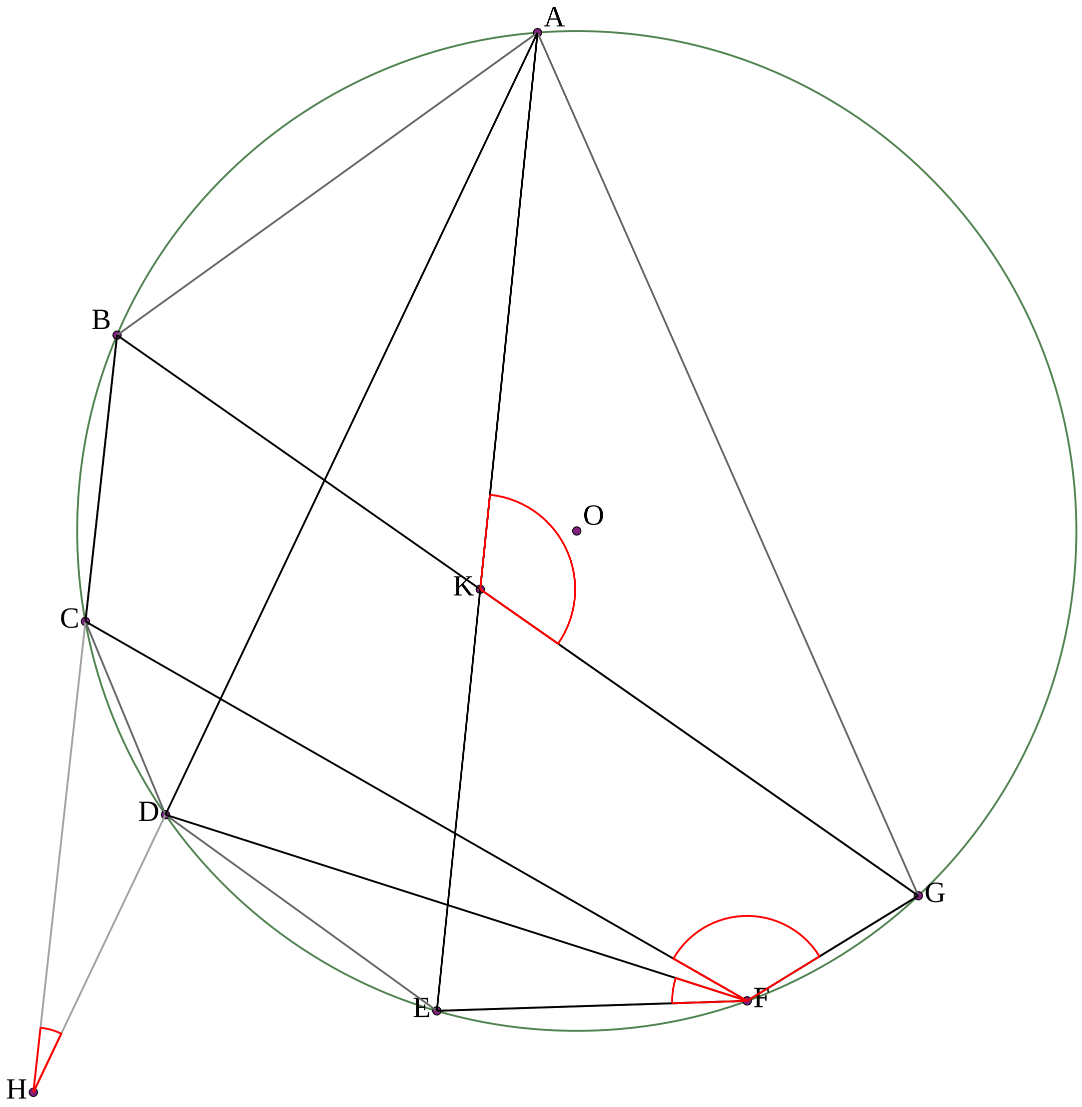
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GC and HB. Let J be the intersection of CE and AH.
Prove that BDG+AFE+GIH = EJH+180

Example 125



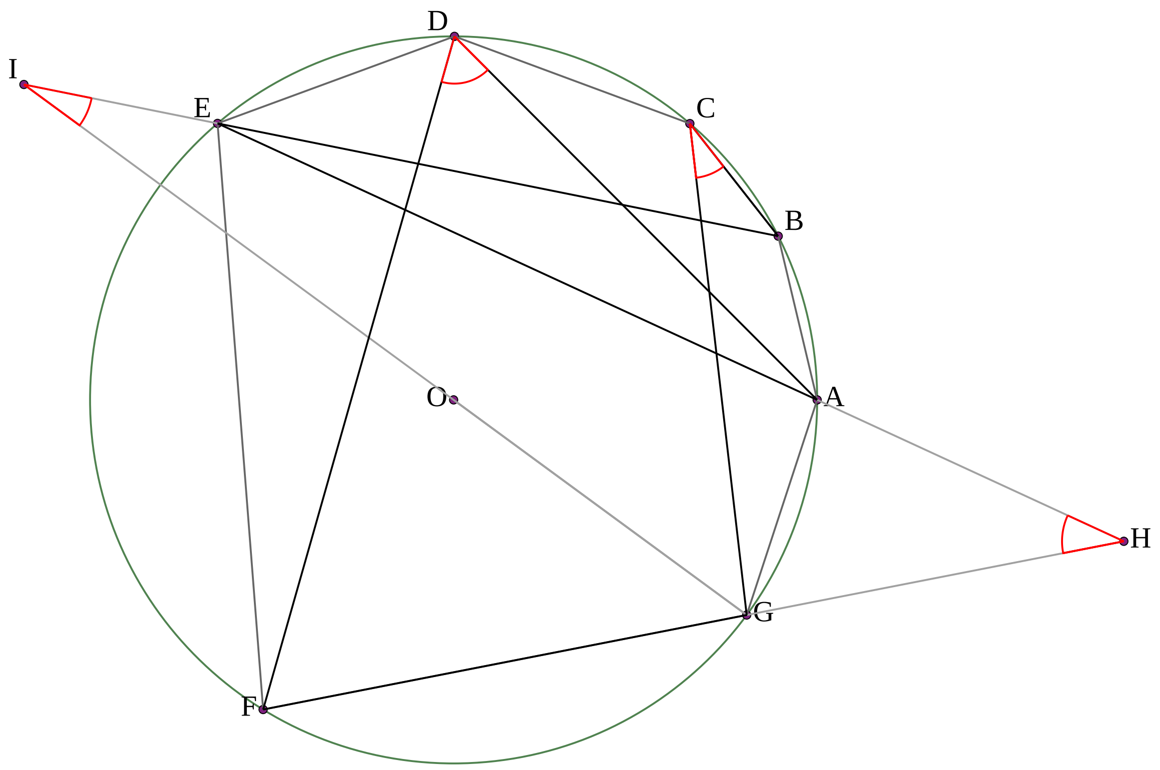
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GF and HE. Let J be the intersection of FB and DH.
Prove that BCD+GIH = EAG+BJH

Example 126



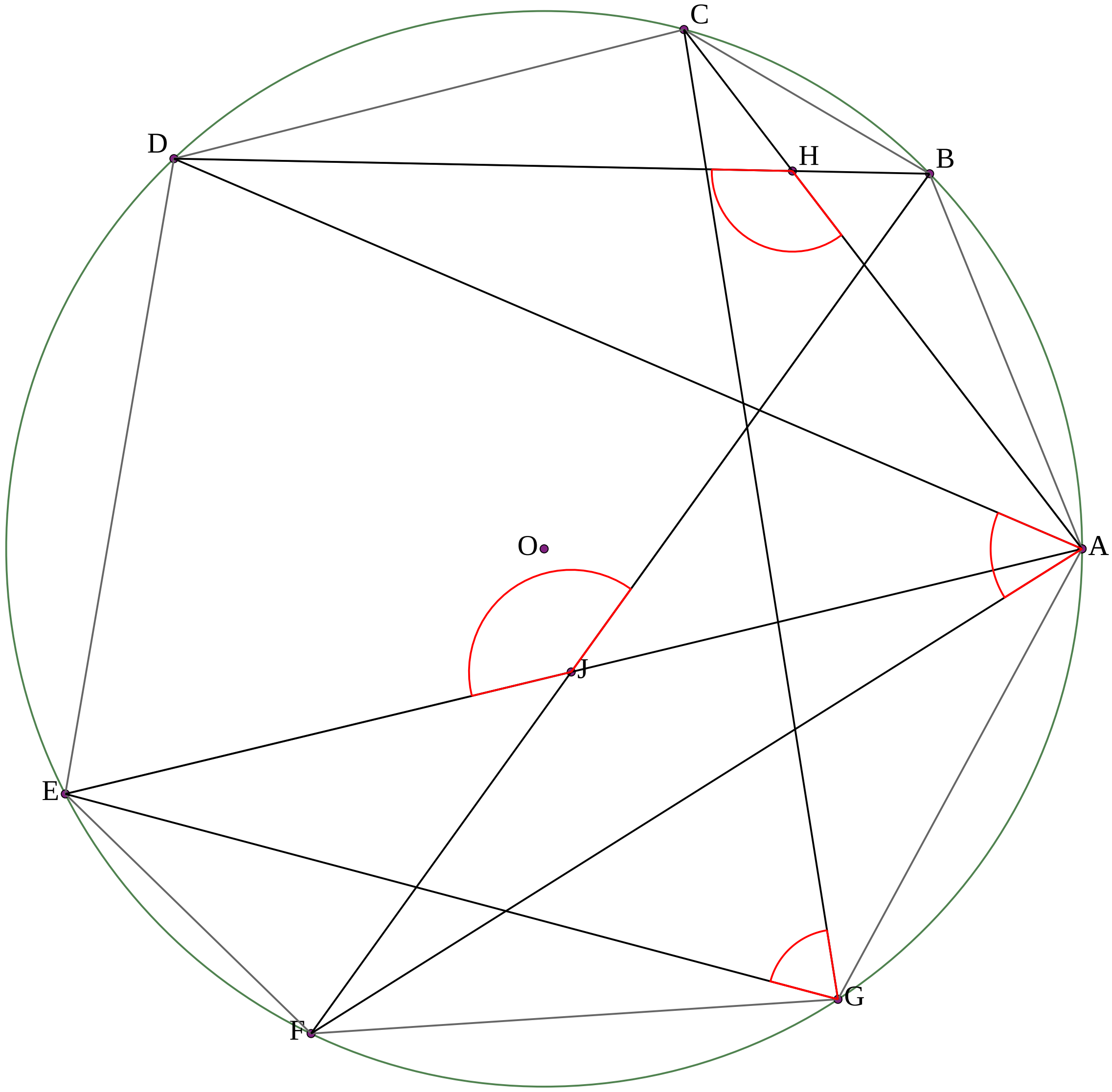
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AD and CB. Let K be the intersection of BG and EA.
Angle DHC = x. Angle GKA = y. Angle CFG = z.
Find angle DFE.

Example 127



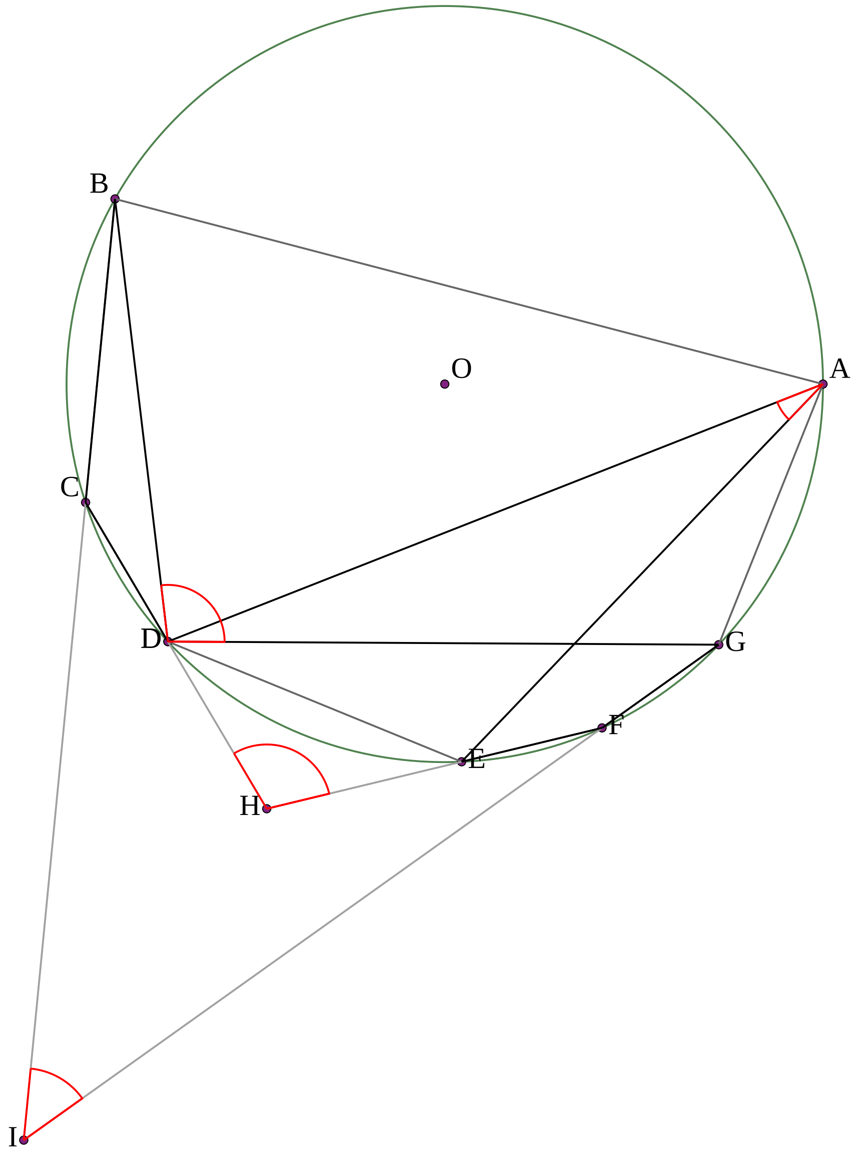
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and EA. Let I be the intersection of OG and BE.
Prove that ADF+AHG+EIG = BCG+90

Example 128



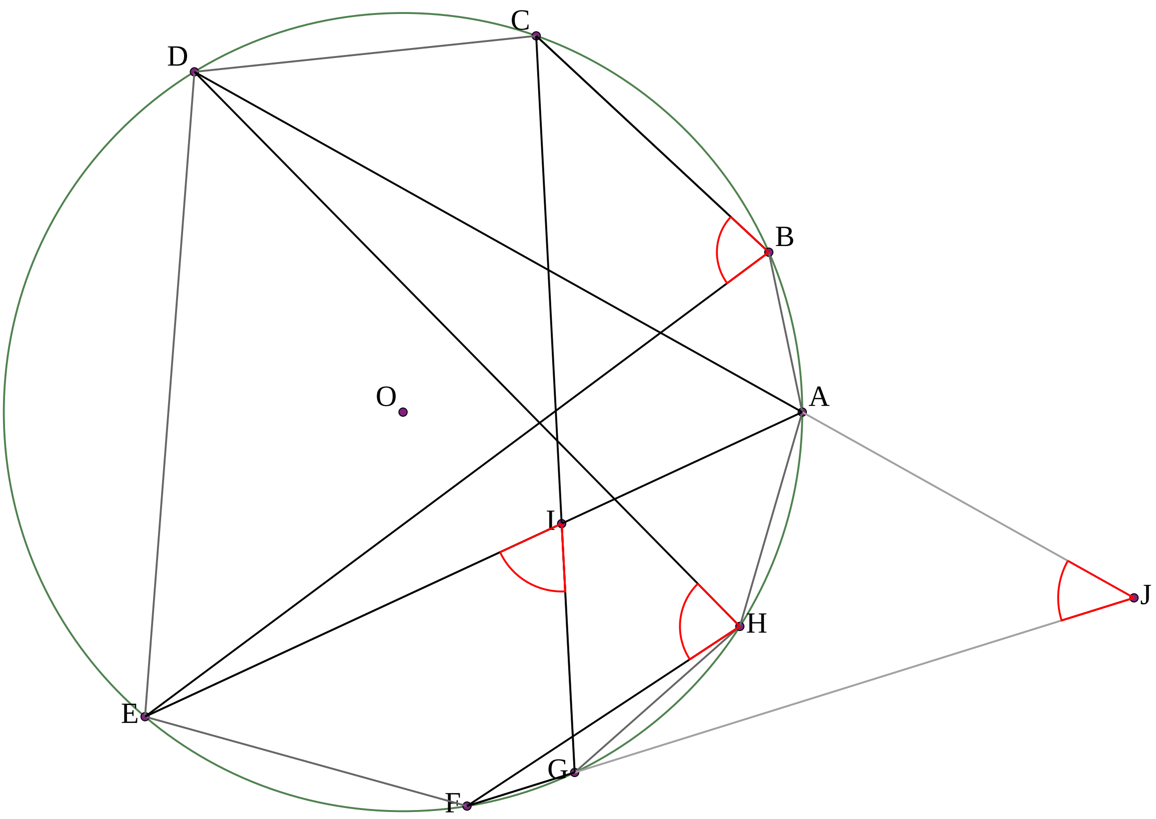
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CA and BD. Let J be the intersection of FB and AE.
Prove that CGE+AHD = DAF+BJE

Example 129



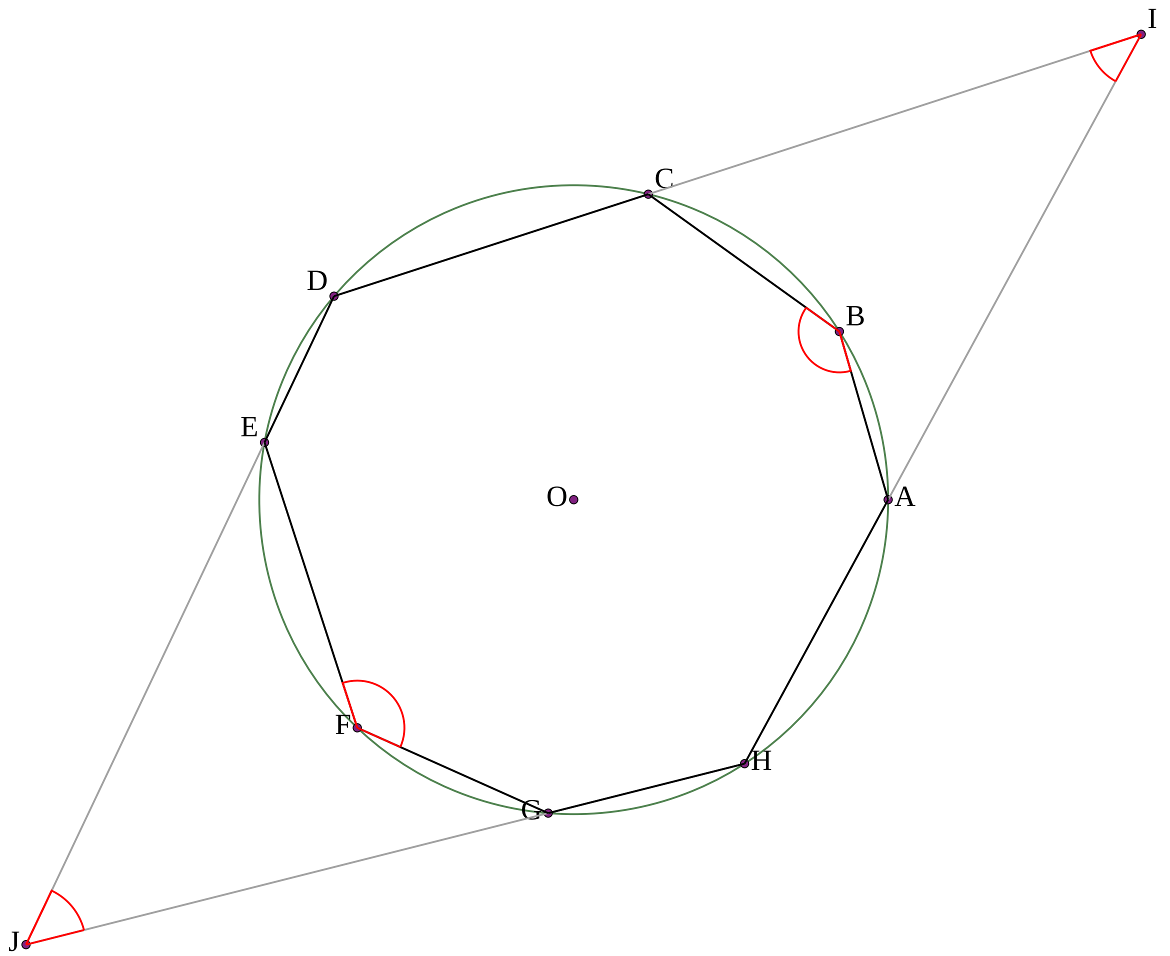
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and CD. Let I be the intersection of FG and BC.
Prove that DAE+BDG+DHE = CIF+180

Example 130



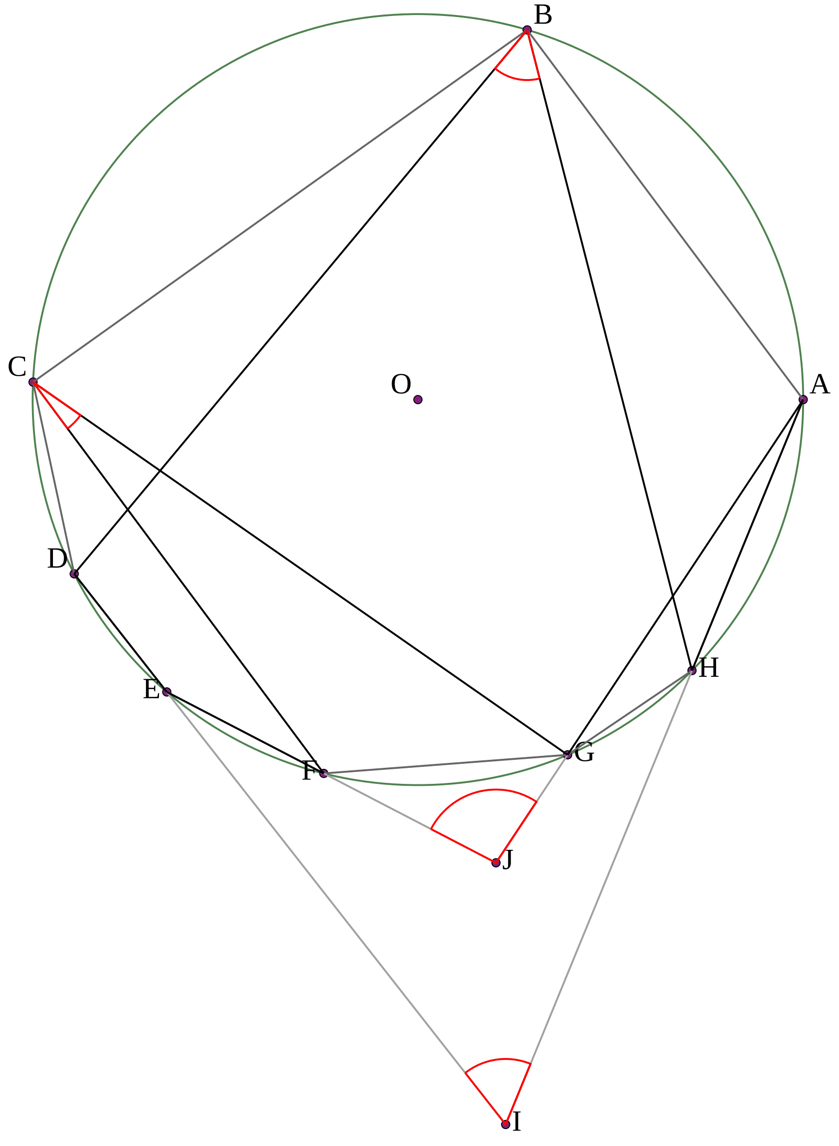
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CG and AE. Let J be the intersection of GF and DA.
Angle GJA = x. Angle EBC = y. Angle GIE = z.
Find angle FHD.

Example 131



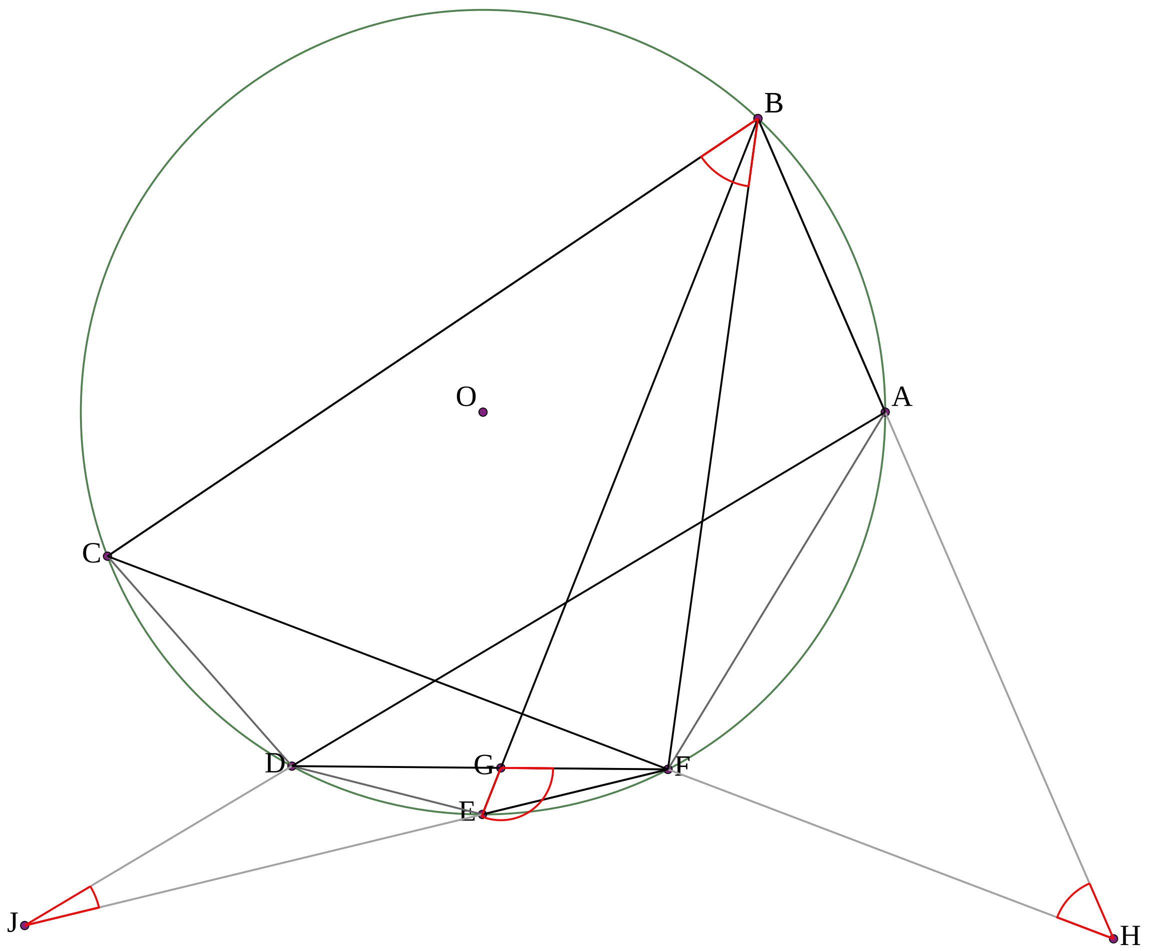
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CD and HA. Let J be the intersection of DE and GH.
Prove that ABC+EFG = AIC+EJG+180

Example 132



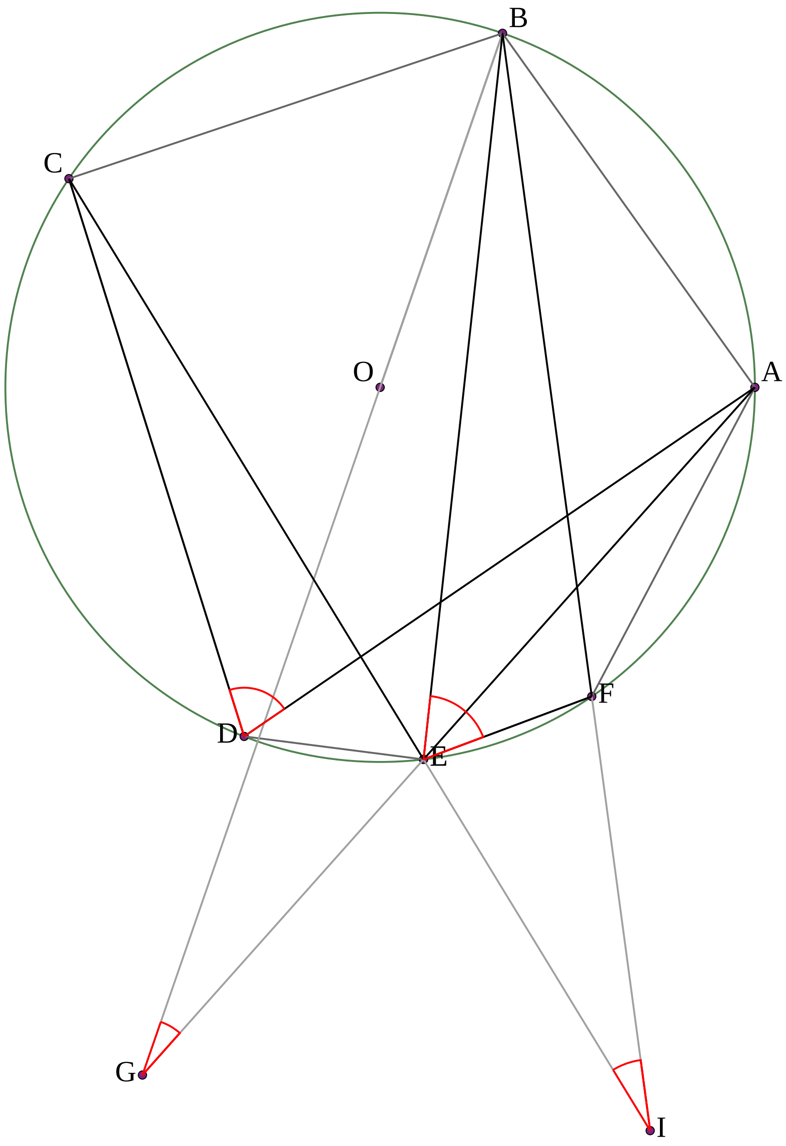
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of HA and ED. Let J be the intersection of AG and FE.
Angle DBH = x. Angle HIE = y. Angle GCF = z.
Find angle GJF.

Example 133



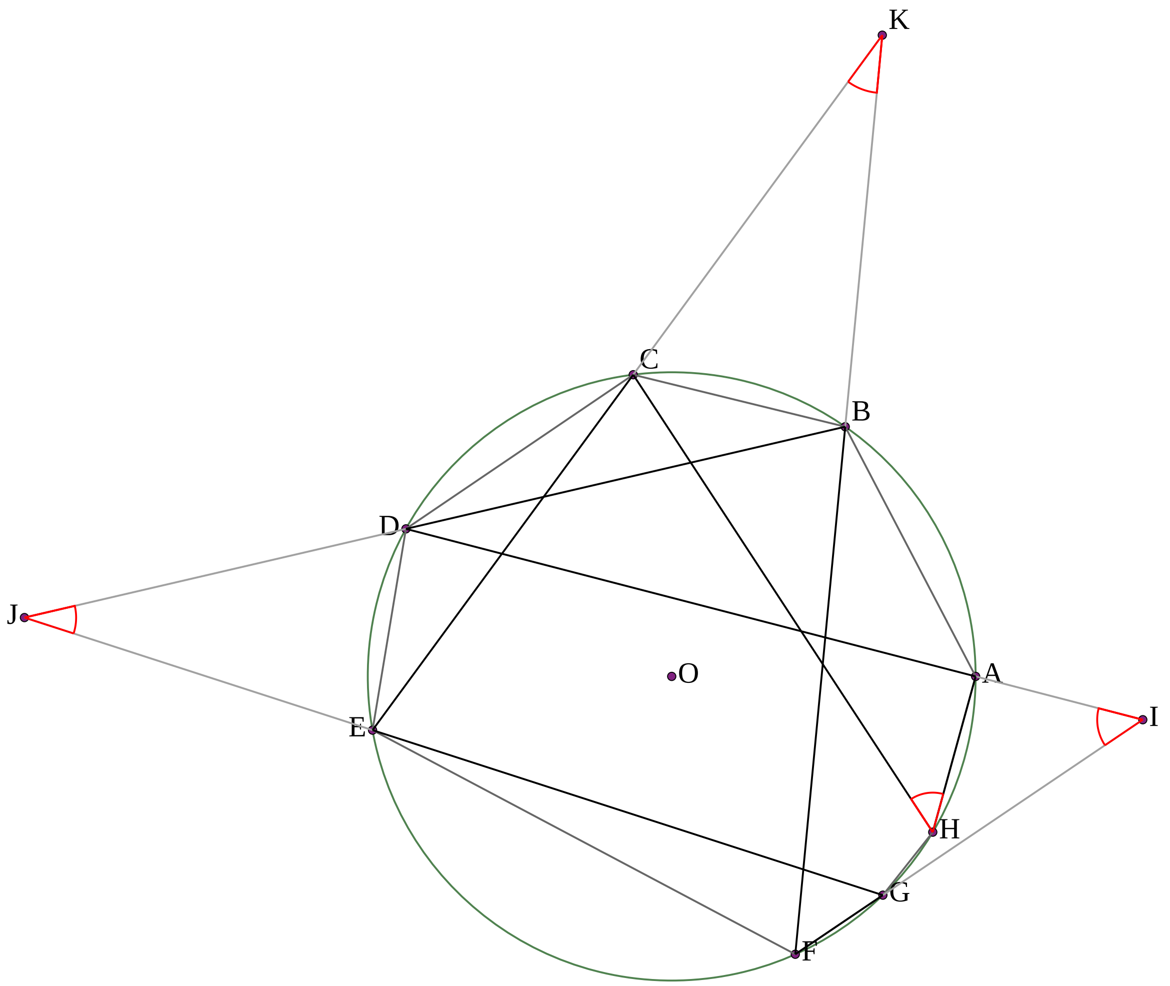
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DF and BE. Let H be the intersection of FC and BA. Let J be the intersection of EF and AD.
Angle FGE = x. Angle CBF = y. Angle EJD = z.
Find angle FHA.

Example 134



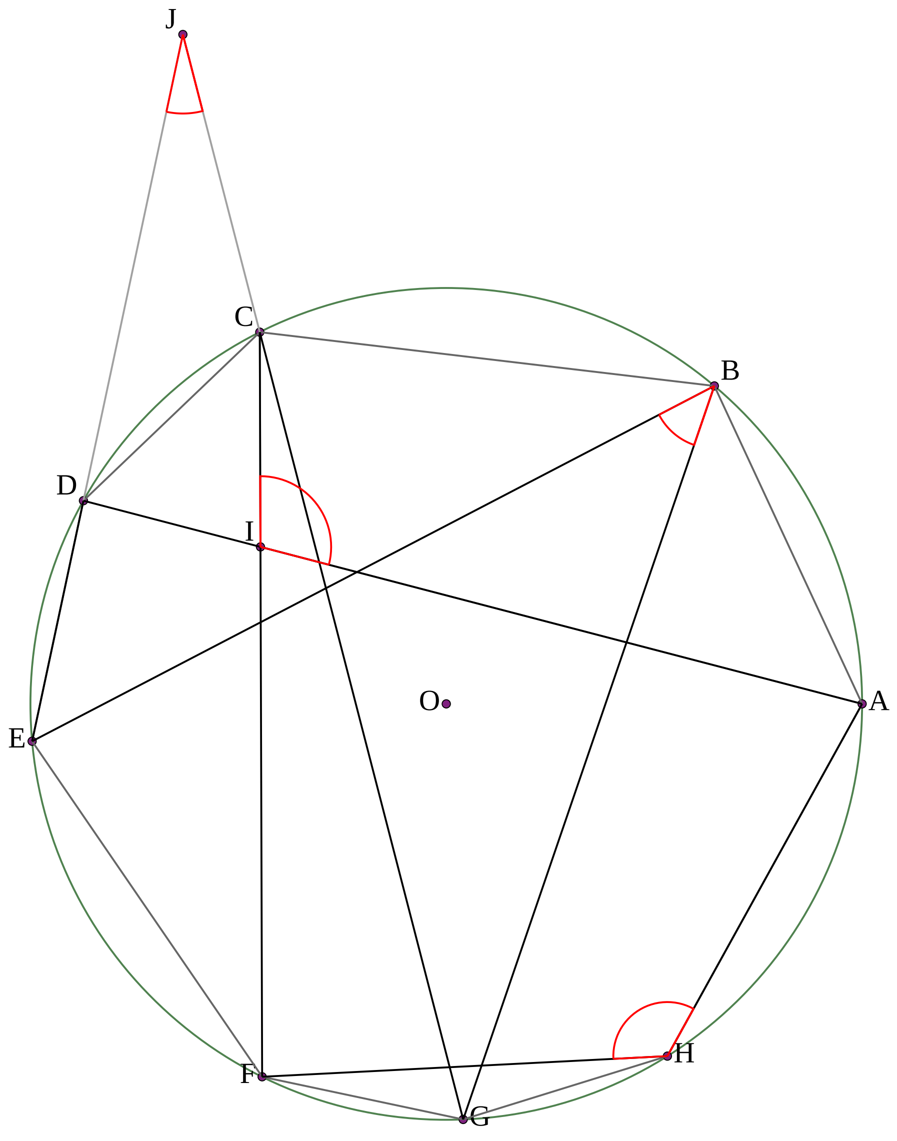
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AE and BO. Let I be the intersection of FB and EC.
Prove that ADC+BEF = BGE+EIF+90

Example 135



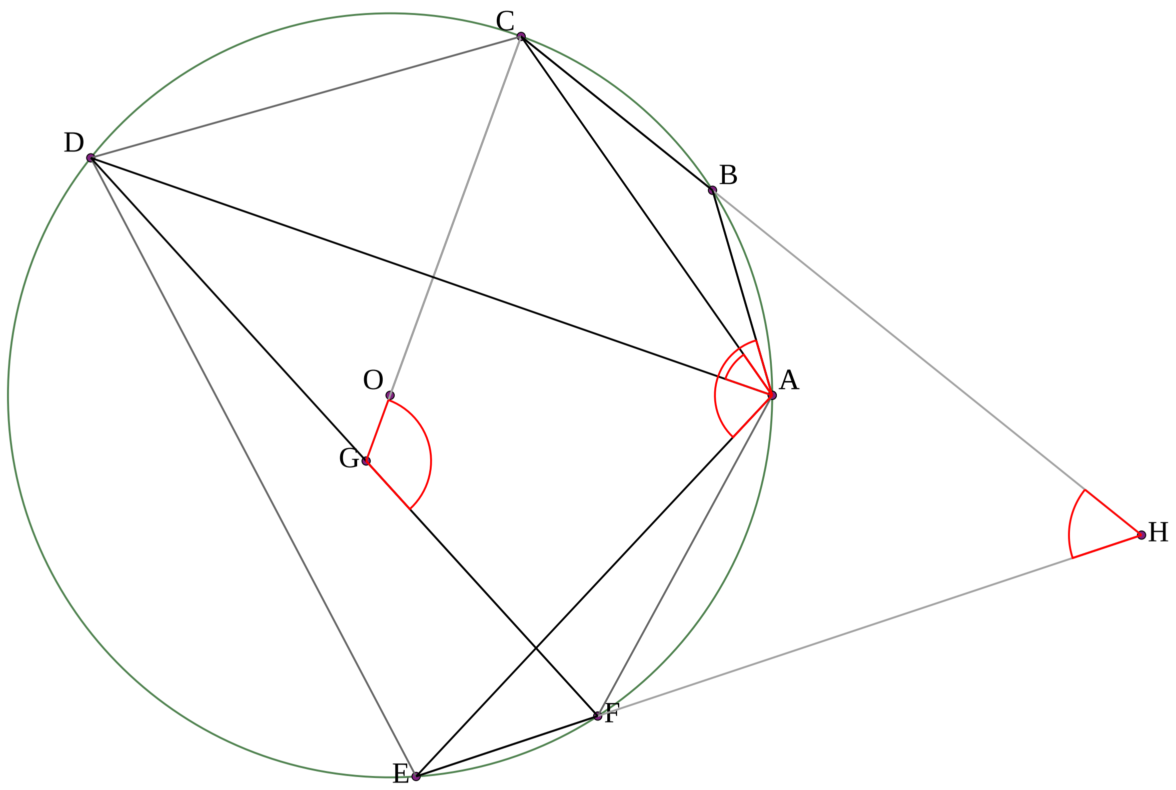
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of AD and FG. Let J be the intersection of DB and GE. Let K be the intersection of BF and EC.
Angle CHA = x. Angle BKC = y. Angle DJE = z.
Find angle AIG.

Example 136



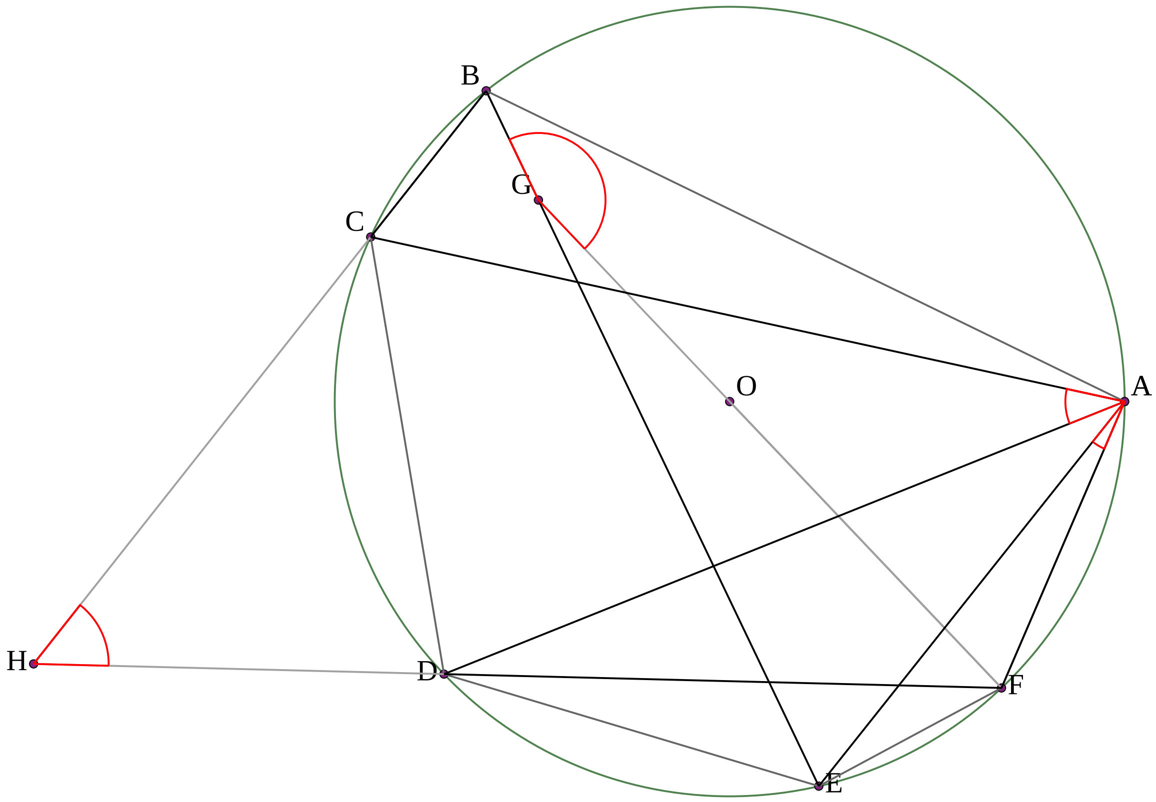
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FC and DA. Let J be the intersection of CG and ED.
Angle AHF = x. Angle CJD = y. Angle CIA = z.
Find angle GBE.

Example 137



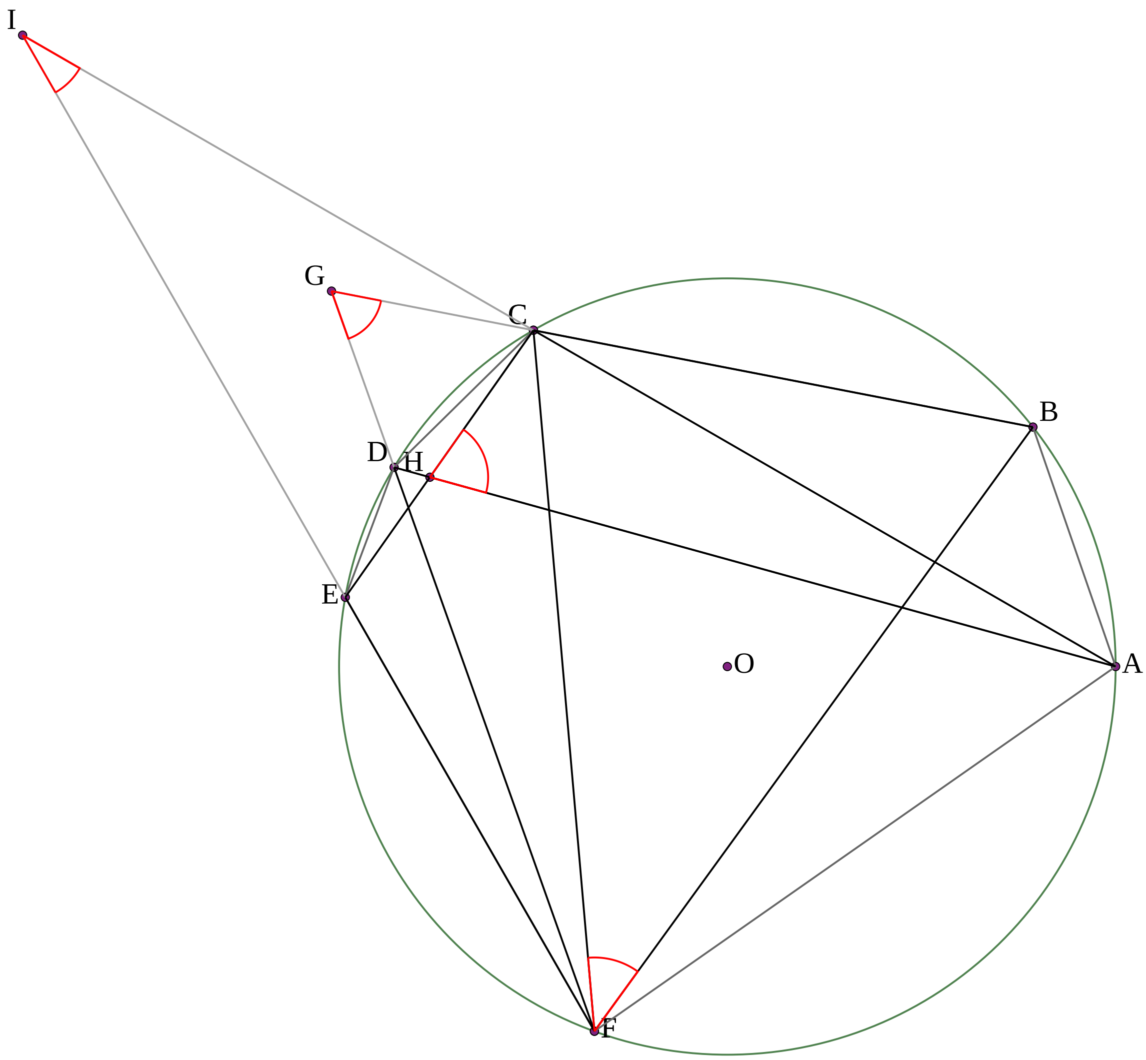
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DF and CO. Let H be the intersection of FE and BC.
Angle EAB = x. Angle CAD = y. Angle FGC = z.
Find angle FHB.

Example 138



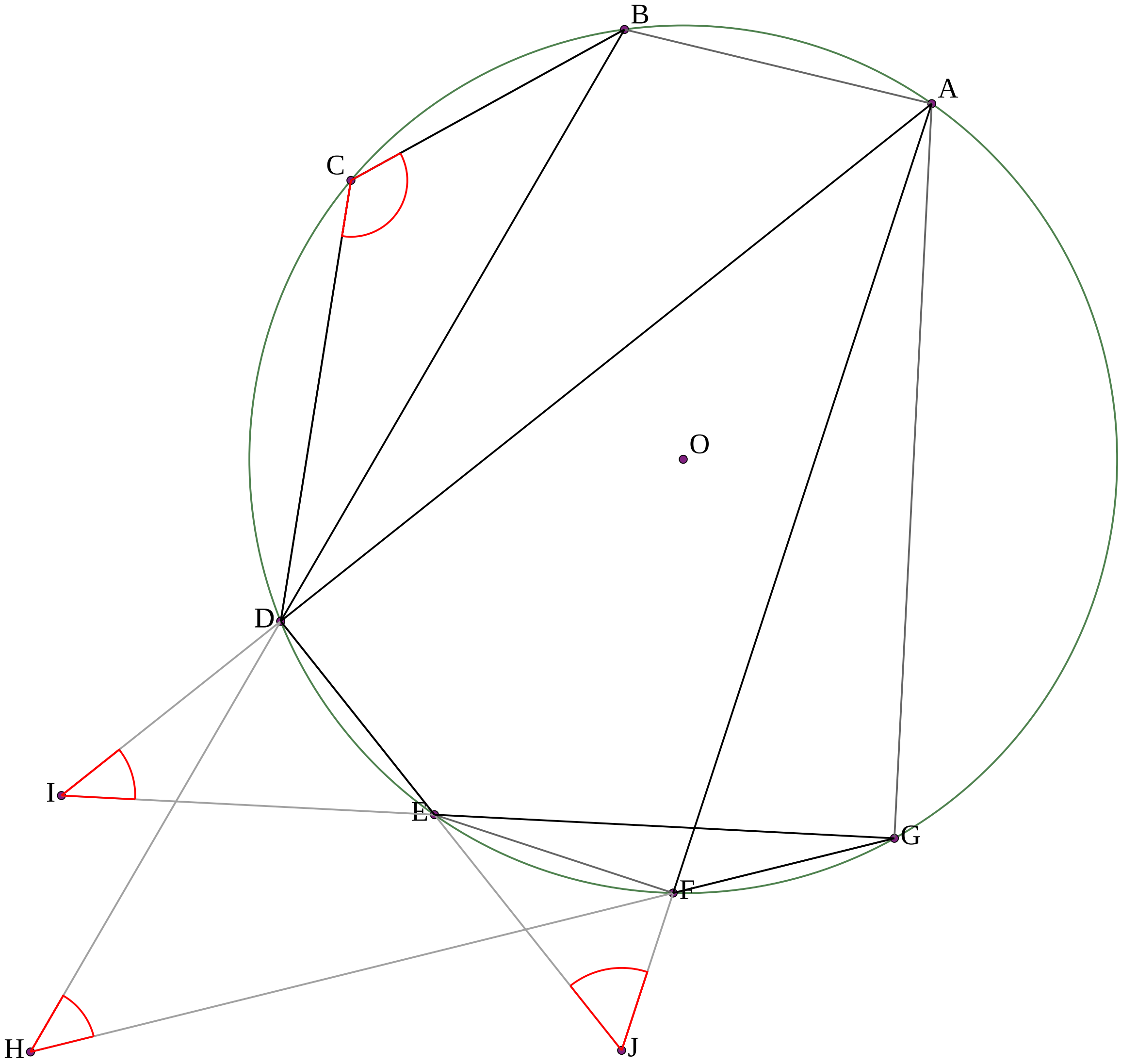
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EB and FO. Let H be the intersection of BC and DF.
Prove that EAF+BGF = CAD+CHD+90

Example 139



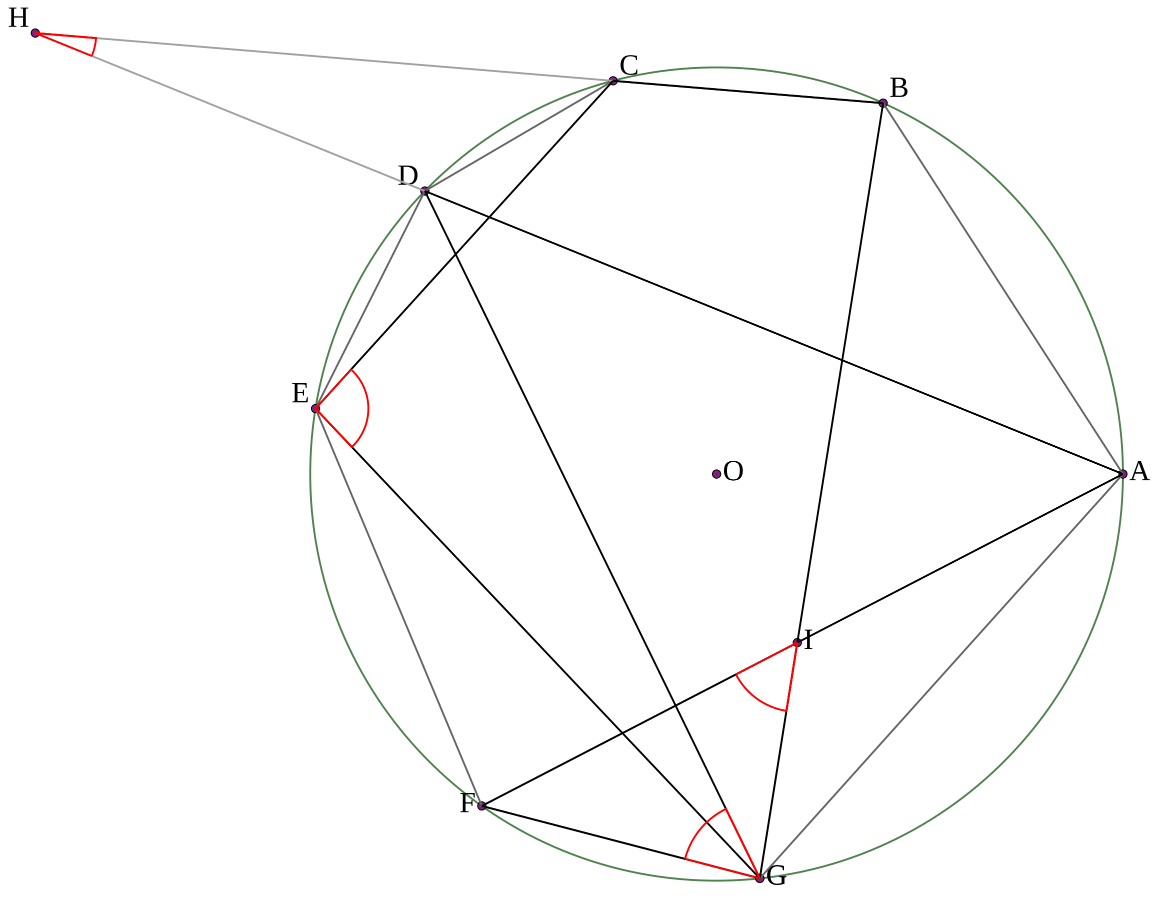
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and CB. Let H be the intersection of DA and EC. Let I be the intersection of AC and FE.
Prove that AHC+CIE = CGD+BFC

Example 140



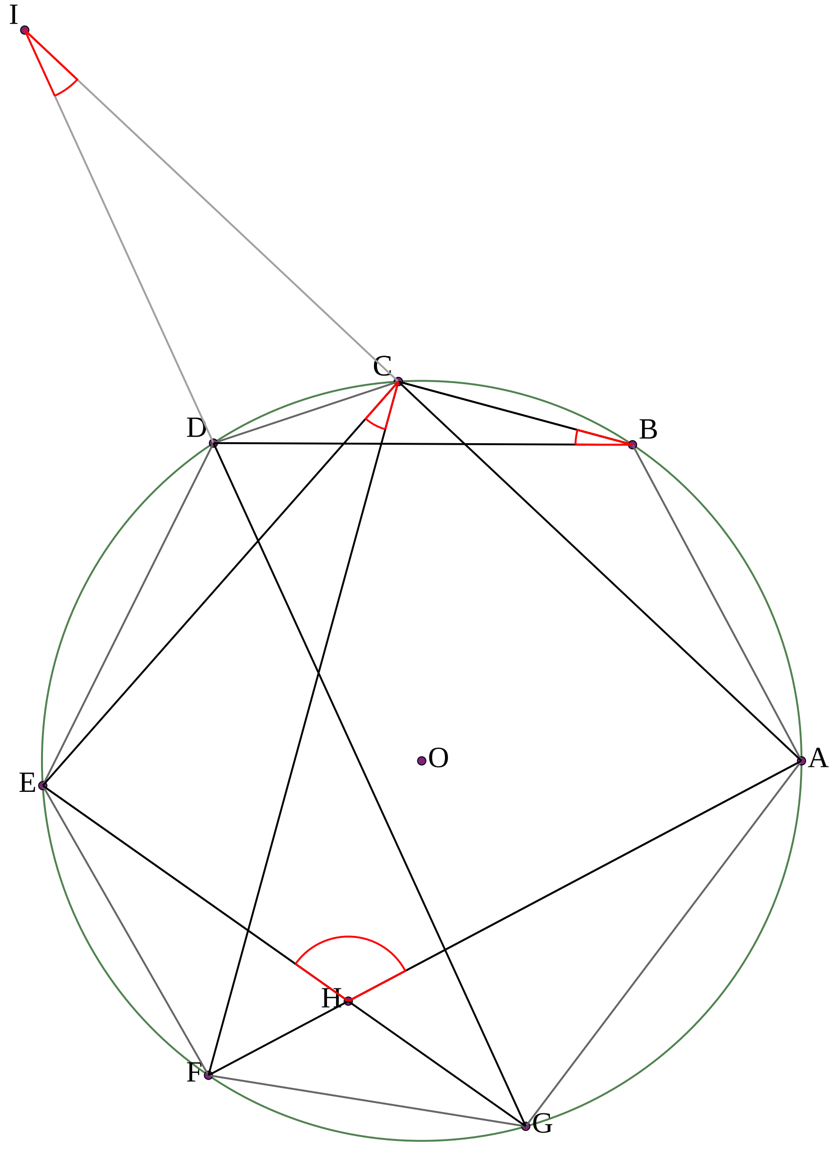
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of BD and FG. Let I be the intersection of DA and GE. Let J be the intersection of AF and ED.
Angle DCB = 128°. Angle DIE = 41°. Angle FJE = 57°.
Find angle DHF.

Example 141



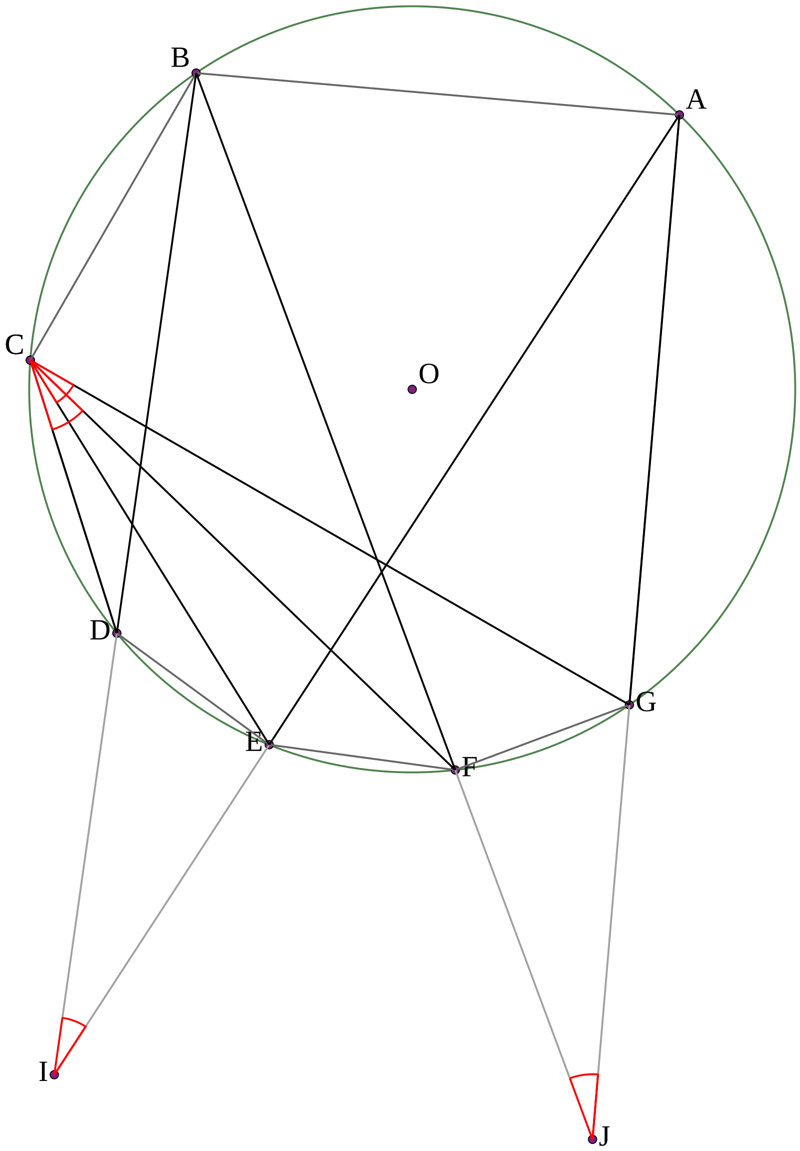
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CB and DA. Let I be the intersection of BG and AF.
Prove that CEG+FIG+DGF = CHD+180

Example 142



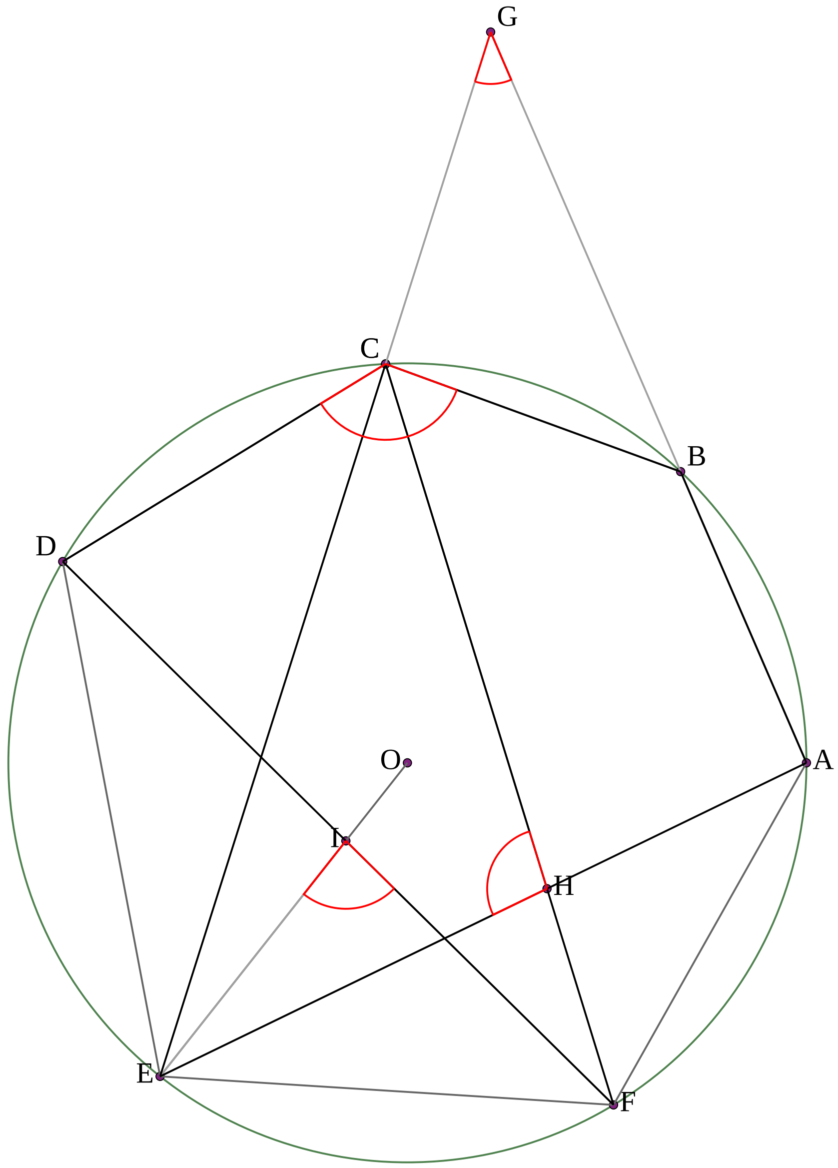
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FA and GE. Let I be the intersection of AC and DG.
Prove that ECF+CBD+AHE+CID = 180

Example 143



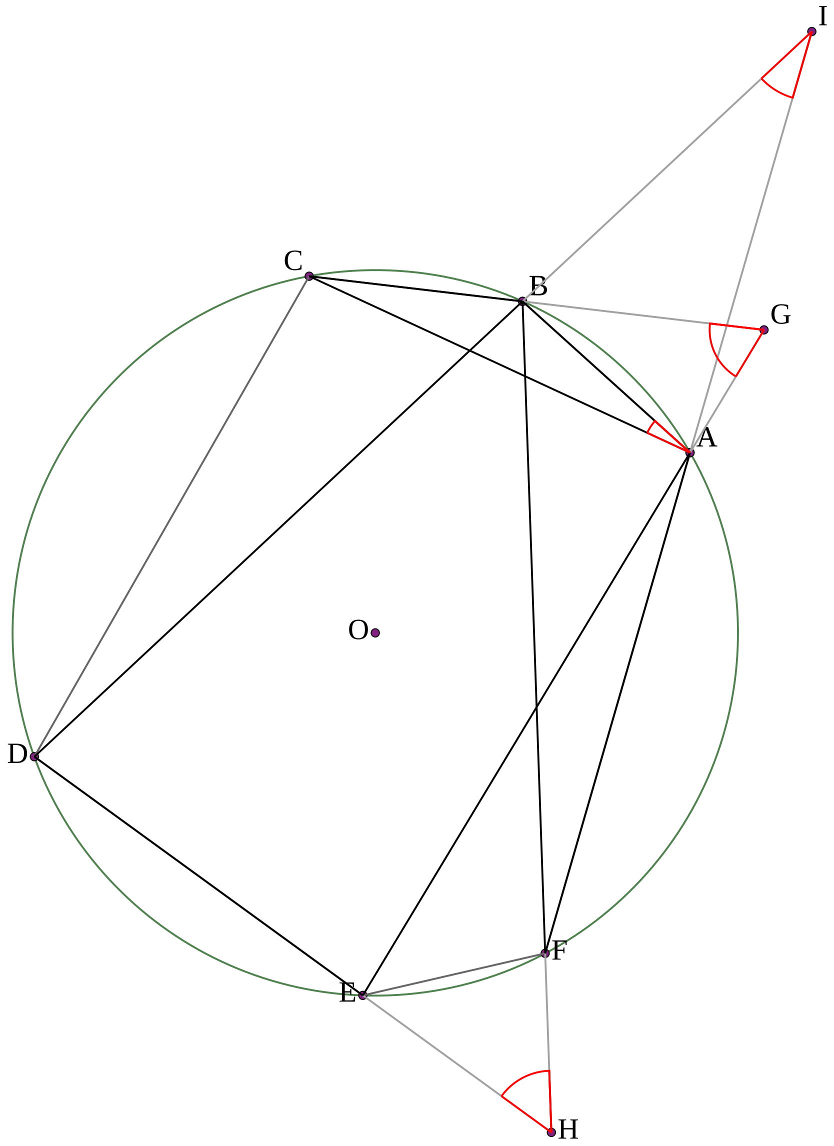
Let ABCDEFG be a cyclic heptagon with center O. Let I be the intersection of EA and BD. Let J be the intersection of AG and FB.
Prove that DIE+DCF = ECG+FJG

Example 144



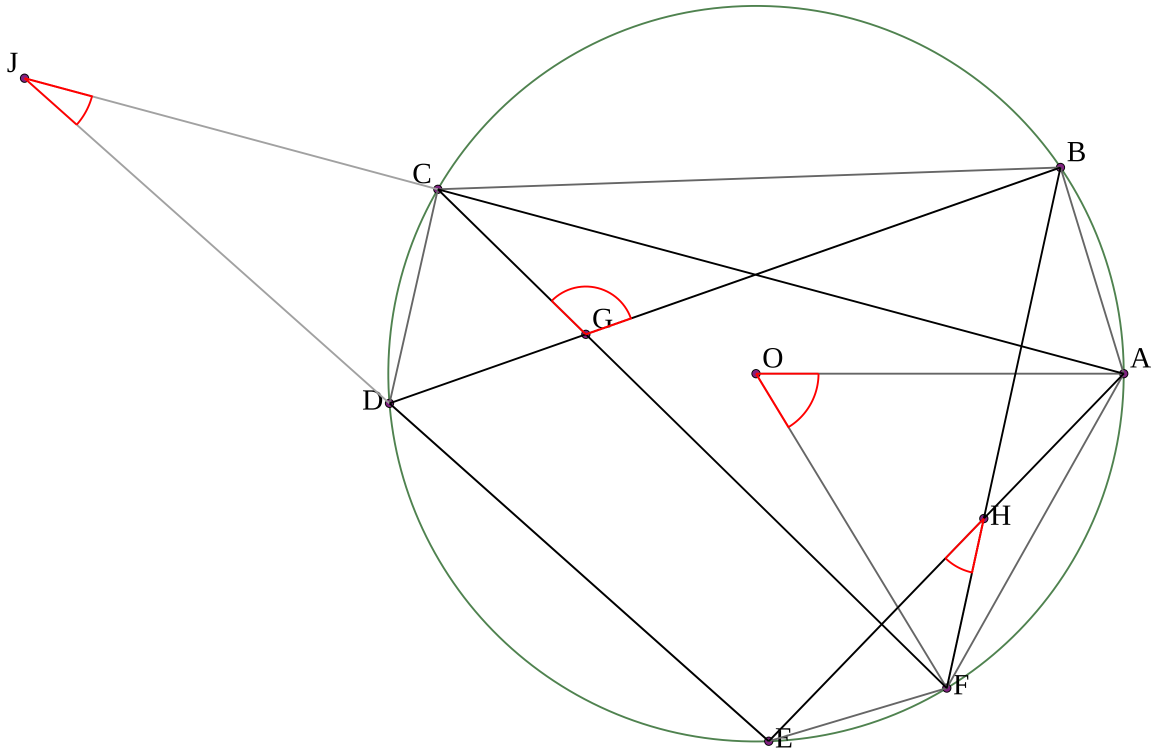
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BA and EC. Let H be the intersection of AE and FC. Let I be the intersection of OE and DF.
Angle BGC = 41°. Angle EHC = 99°. Angle DCB = 129°.
Find angle EIF.

Example 145



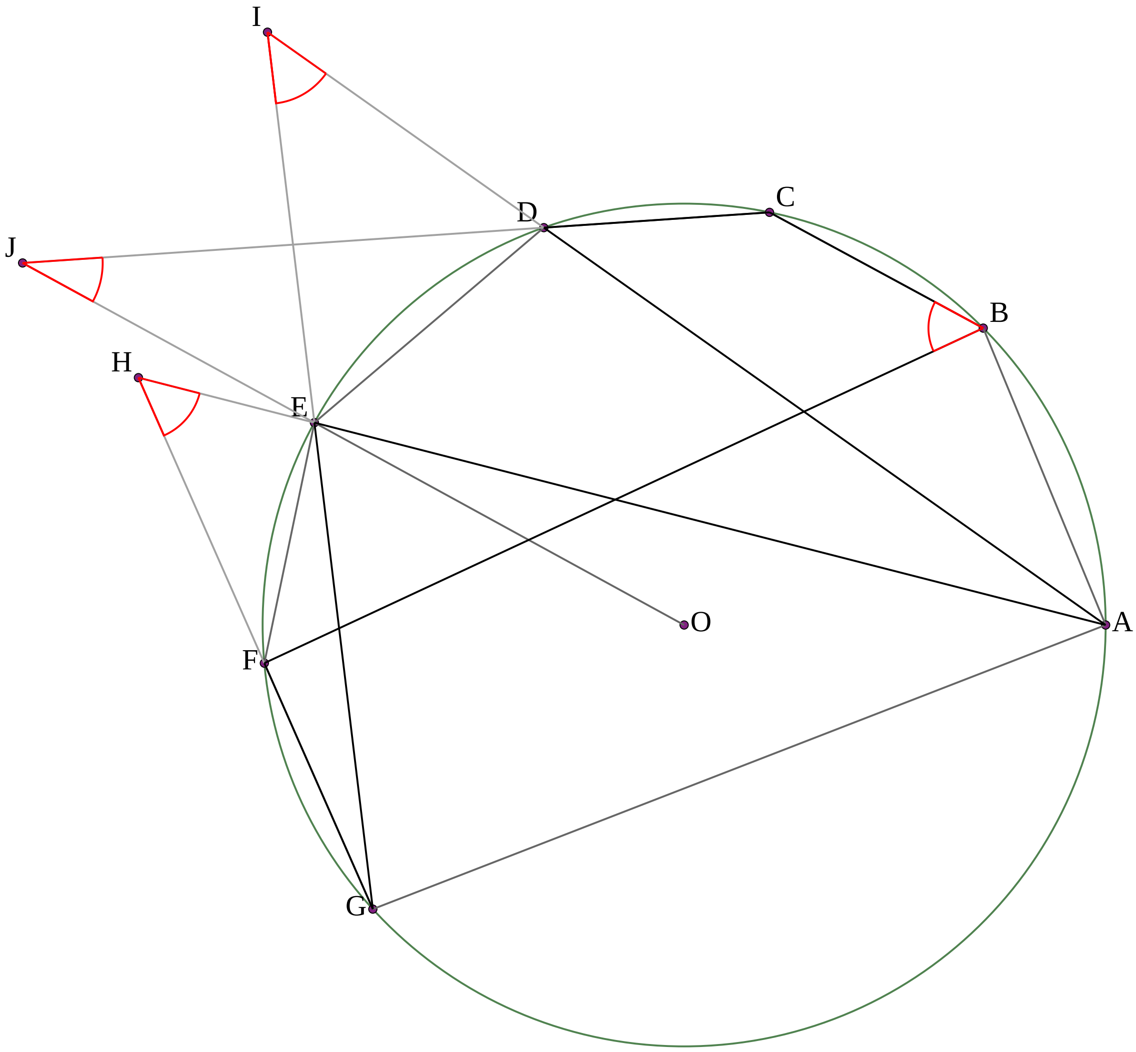
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CB and AE. Let H be the intersection of BF and ED. Let I be the intersection of FA and DB.
Prove that EHF+AIB = BAC+AGB

Example 146



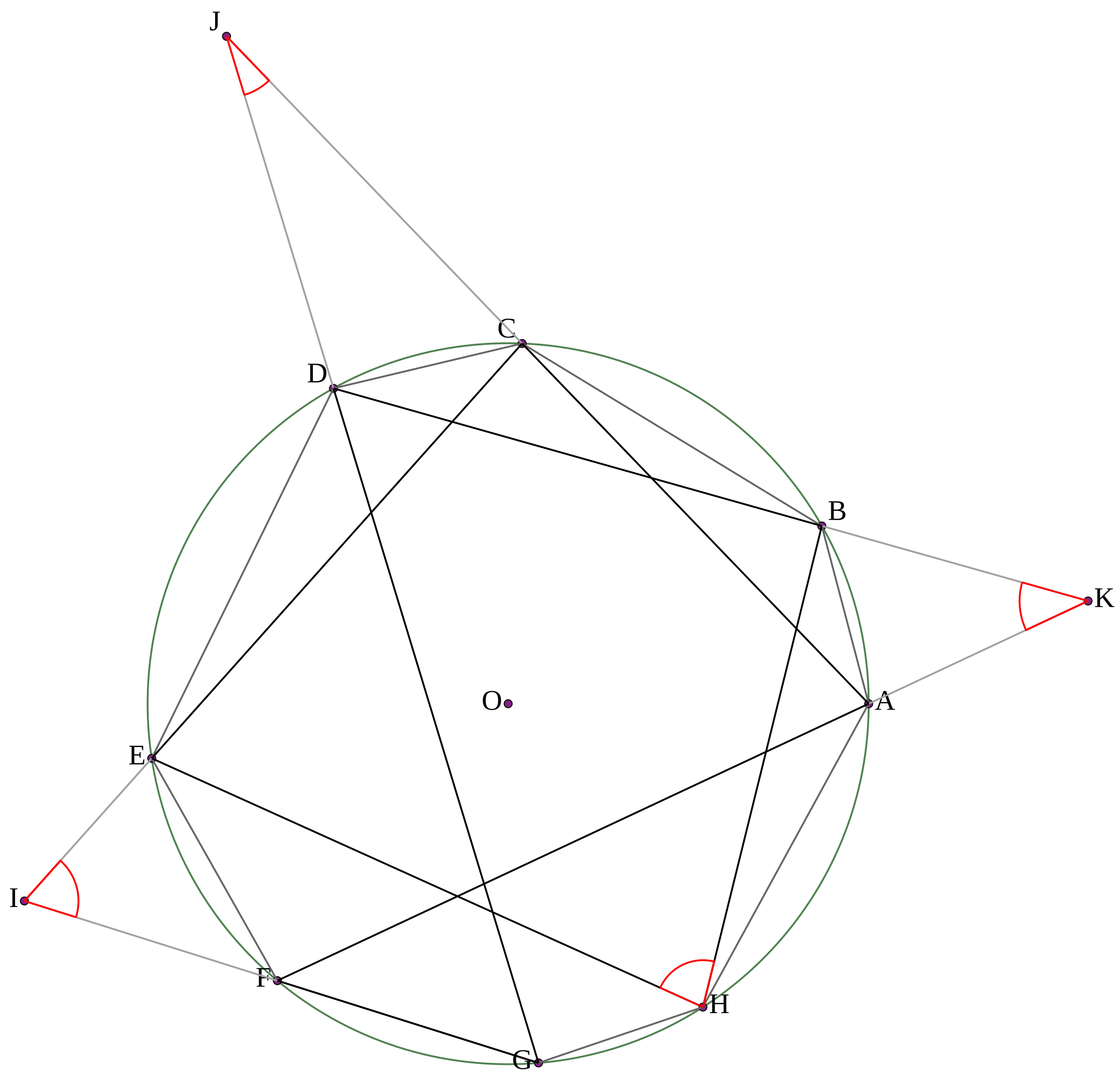
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DB and FC. Let H be the intersection of BF and AE. Let J be the intersection of CA and ED.
Angle CJD = x. Angle BGC = y. Angle FOA = z.
Find angle FHE.

Example 147



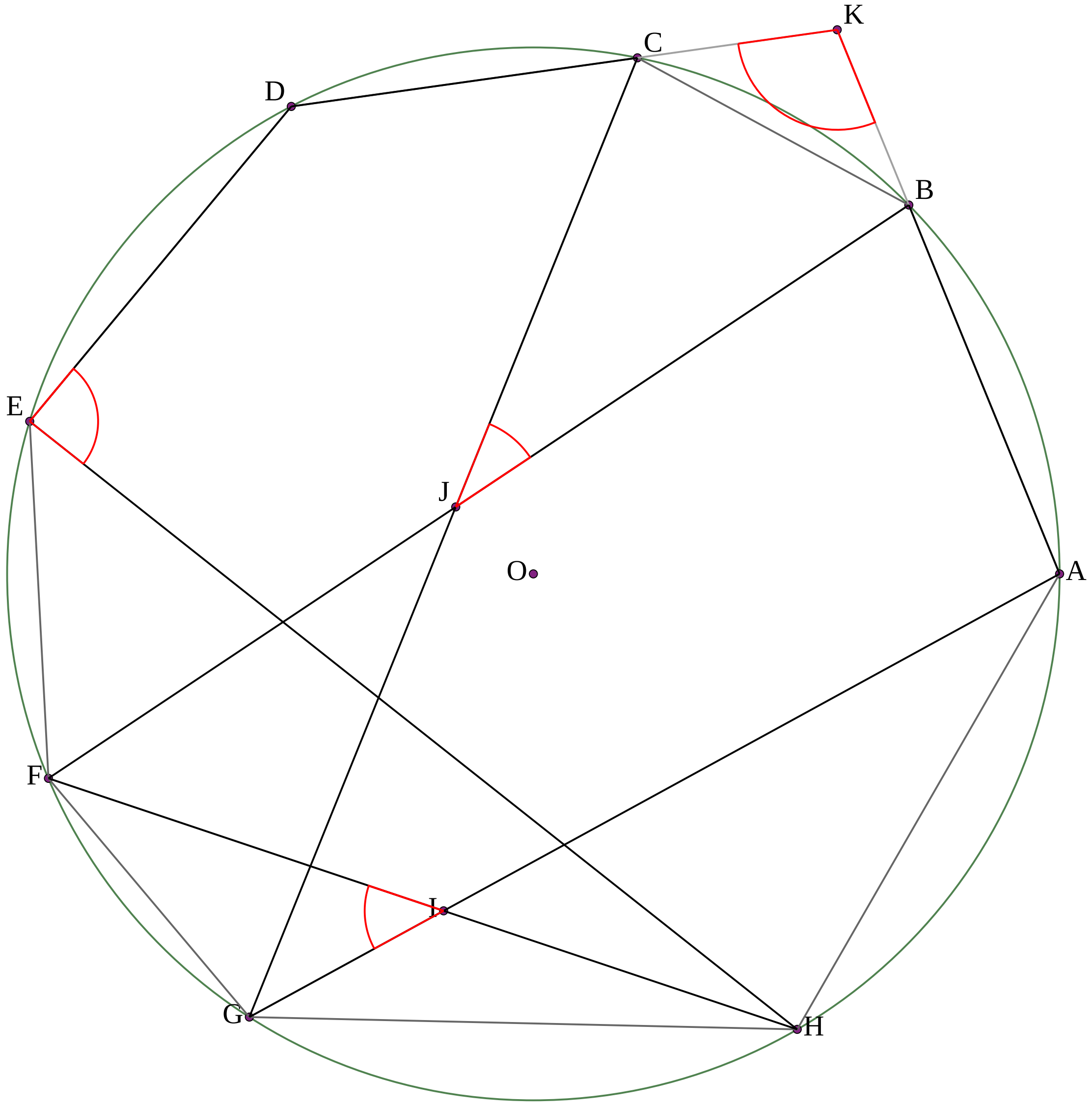
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and EA. Let I be the intersection of GE and AD. Let J be the intersection of OE and DC.
Angle CBF = x. Angle FHE = y. Angle EJD = z.
Find angle EID.

Example 148



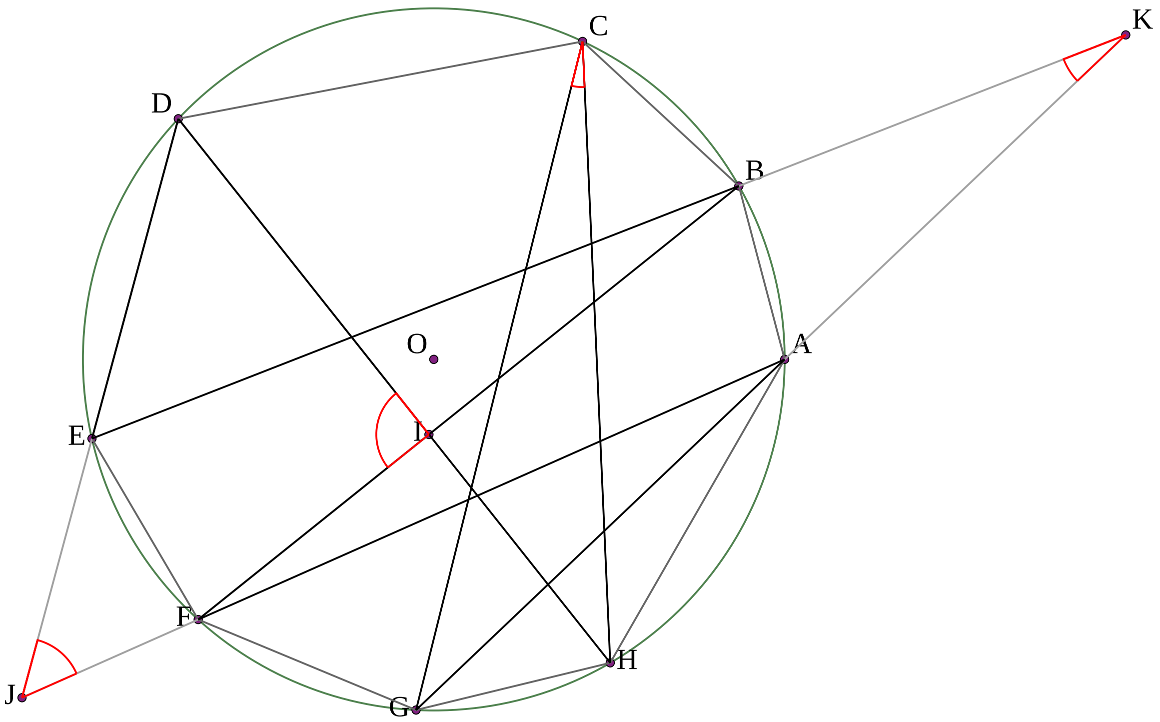
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of EC and FG. Let J be the intersection of CA and GD. Let K be the intersection of AF and DB.
Angle BHE = 80°. Angle CJD = 27°. Angle AKB = 41°.
Find angle EIF.

Example 149



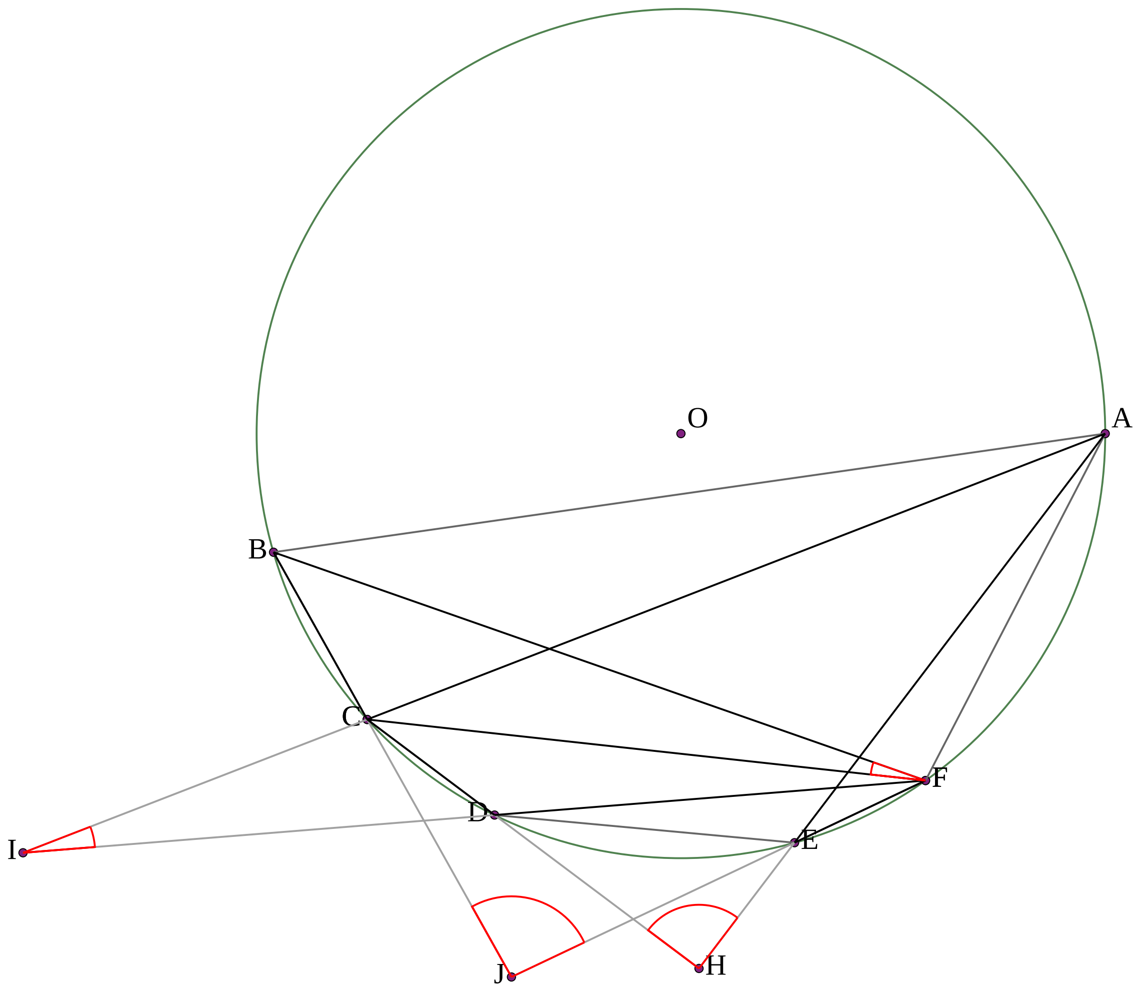
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of HF and AG. Let J be the intersection of FB and GC. Let K be the intersection of BA and CD.
Angle DEH = x. Angle BJC = y. Angle BKC = z.
Find angle FIG.

Example 150



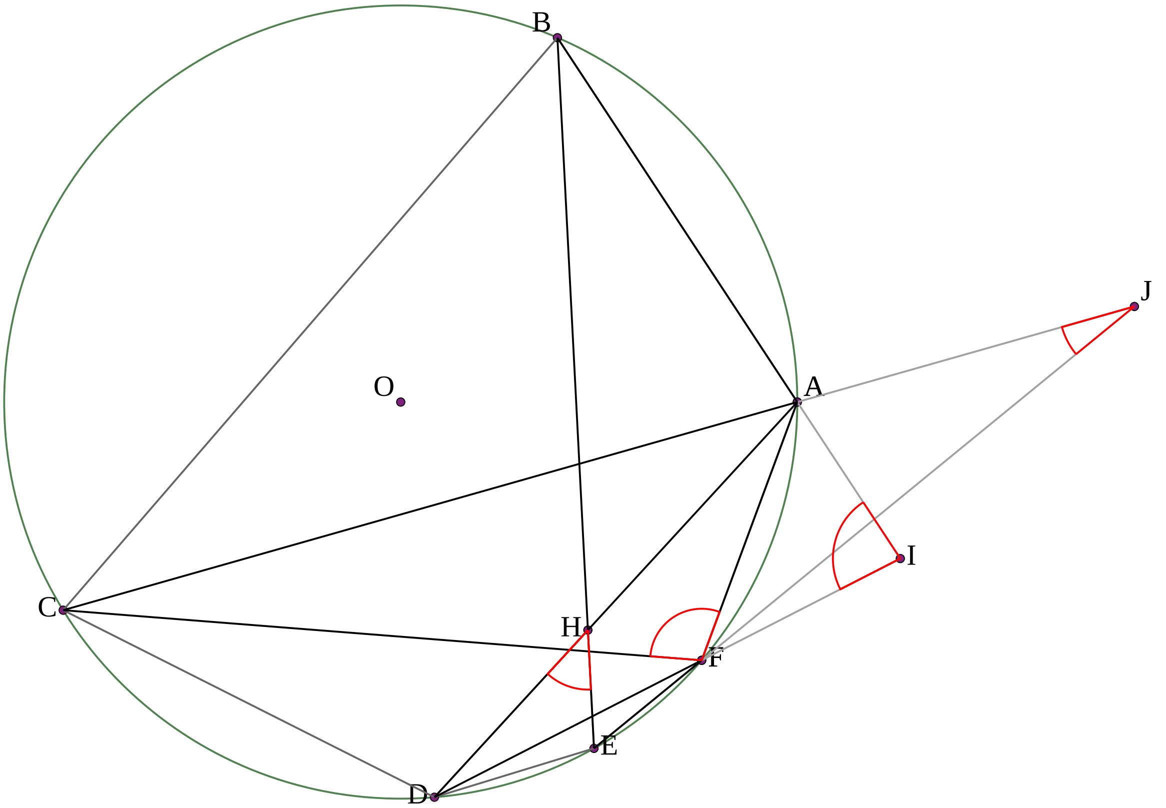
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of HD and BF. Let J be the intersection of DE and FA. Let K be the intersection of EB and AG.
Angle GCH = x. Angle EJF = y. Angle BKA = z.
Find angle DIF.

Example 151



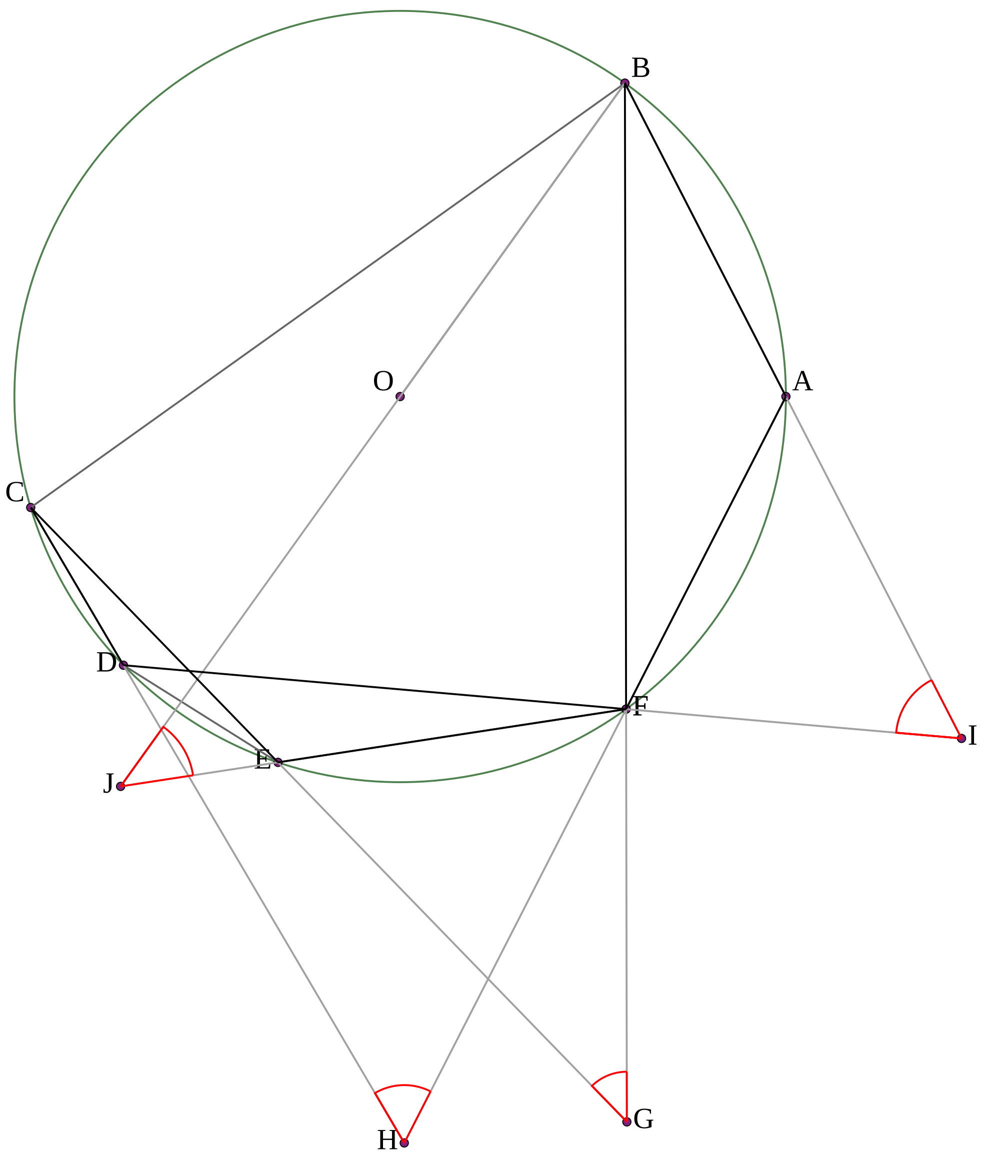
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of CD and AE. Let I be the intersection of DF and CA. Let J be the intersection of BC and EF.
Prove that BFC+CJE = DHE+CID

Example 152



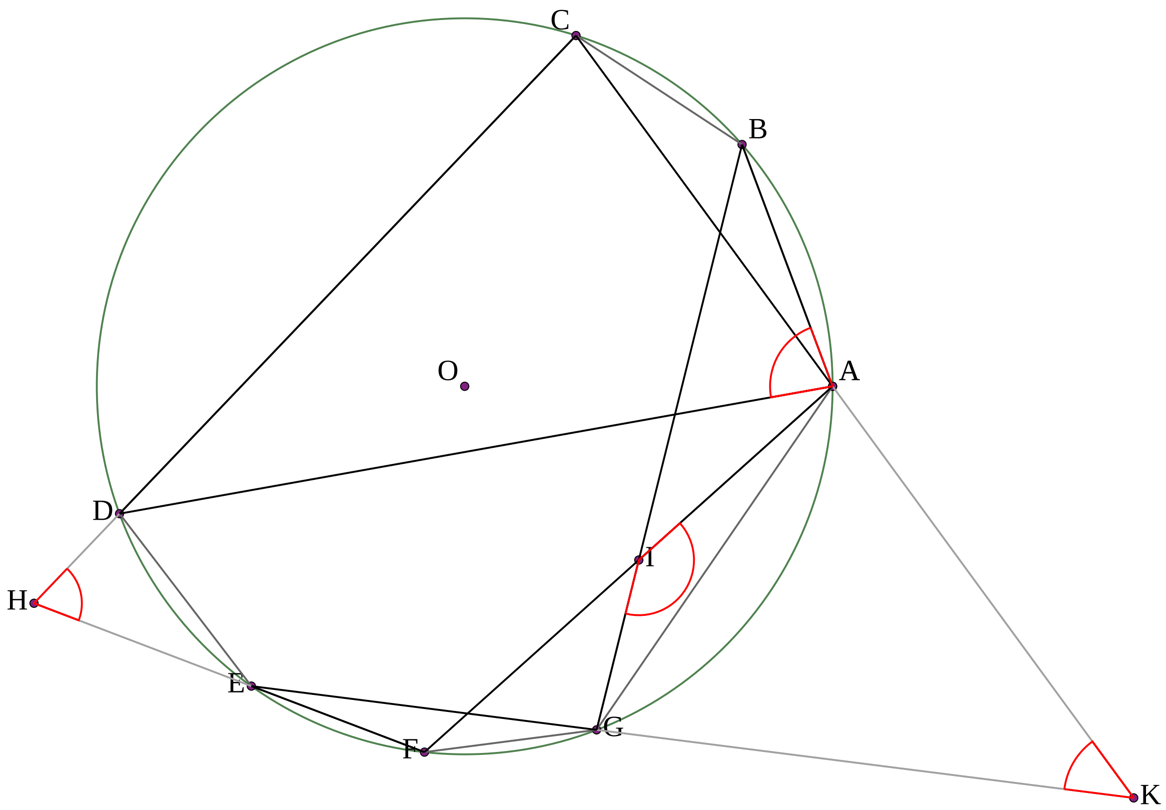
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of AD and BE. Let I be the intersection of DF and AB. Let J be the intersection of CA and EF.
Angle DHE = x. Angle FIA = y. Angle AFC = z.
Find angle AJF.

Example 153



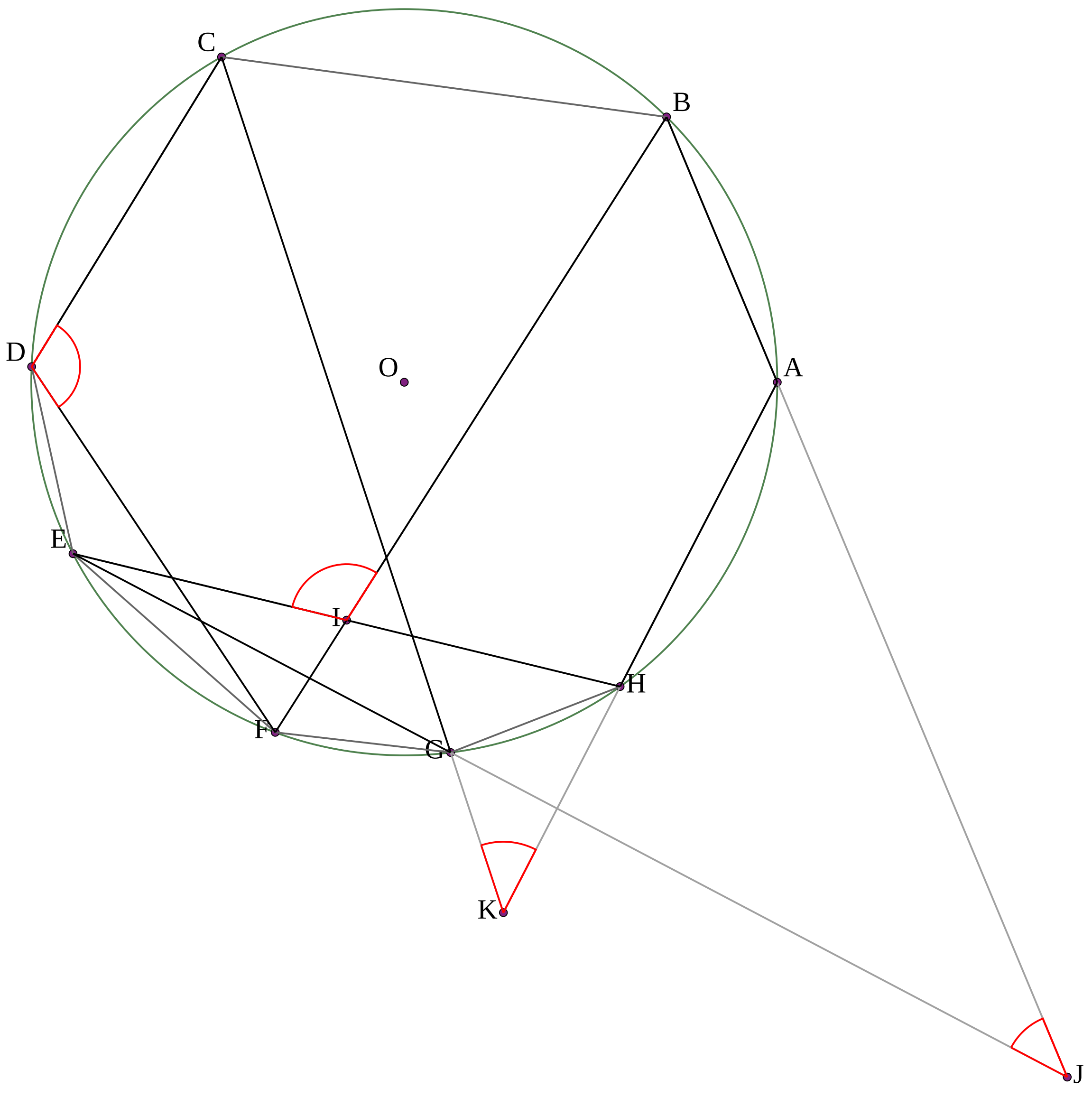
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and FB. Let H be the intersection of CD and AF. Let I be the intersection of DF and BA. Let J be the intersection of OB and FE.
Angle DHF = x. Angle EGF = y. Angle FIA = z.
Find angle BJE.

Example 154



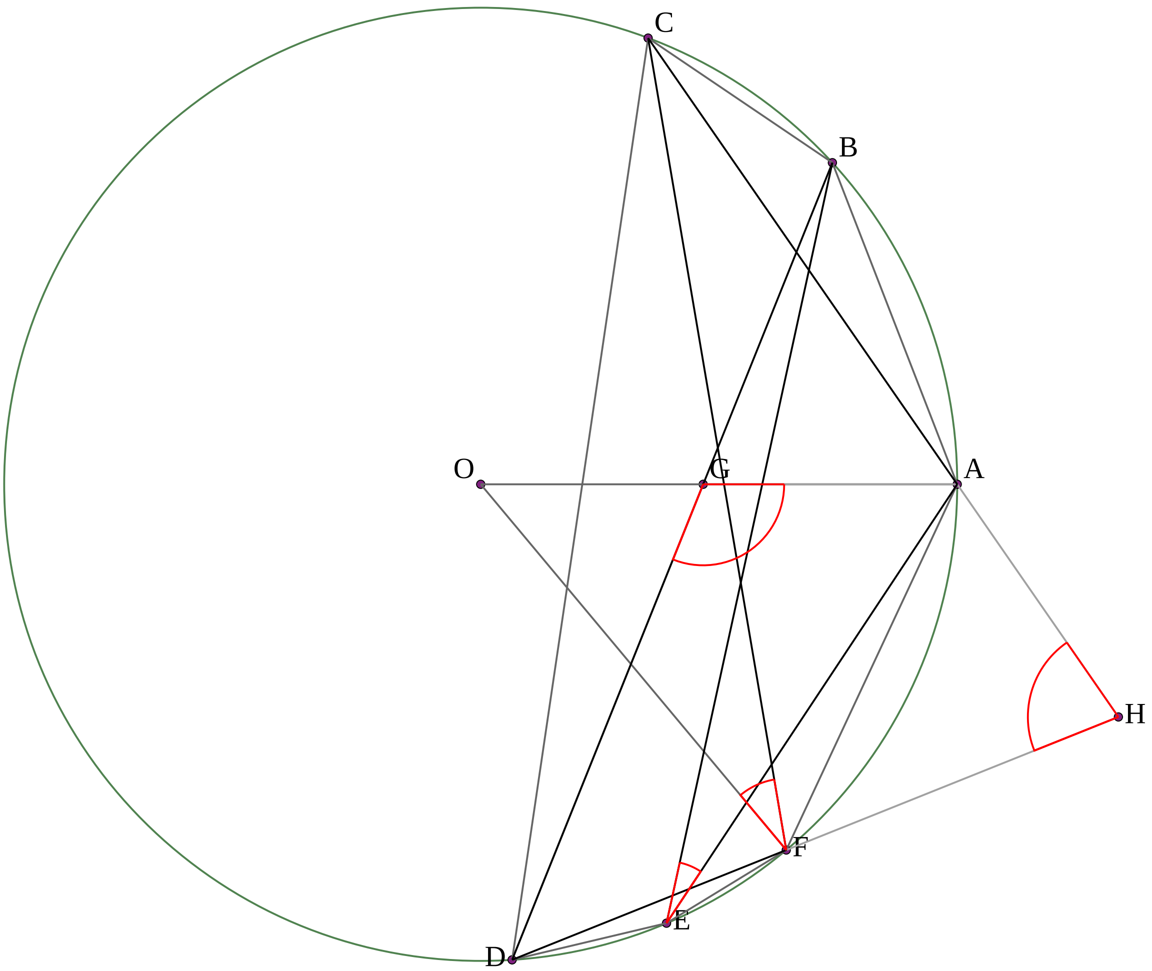
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and DC. Let I be the intersection of FA and BG. Let K be the intersection of CA and GE.
Angle DAB = 80°. Angle EHD = 67°. Angle AKG = 47°.
Find angle AIG.

Example 155



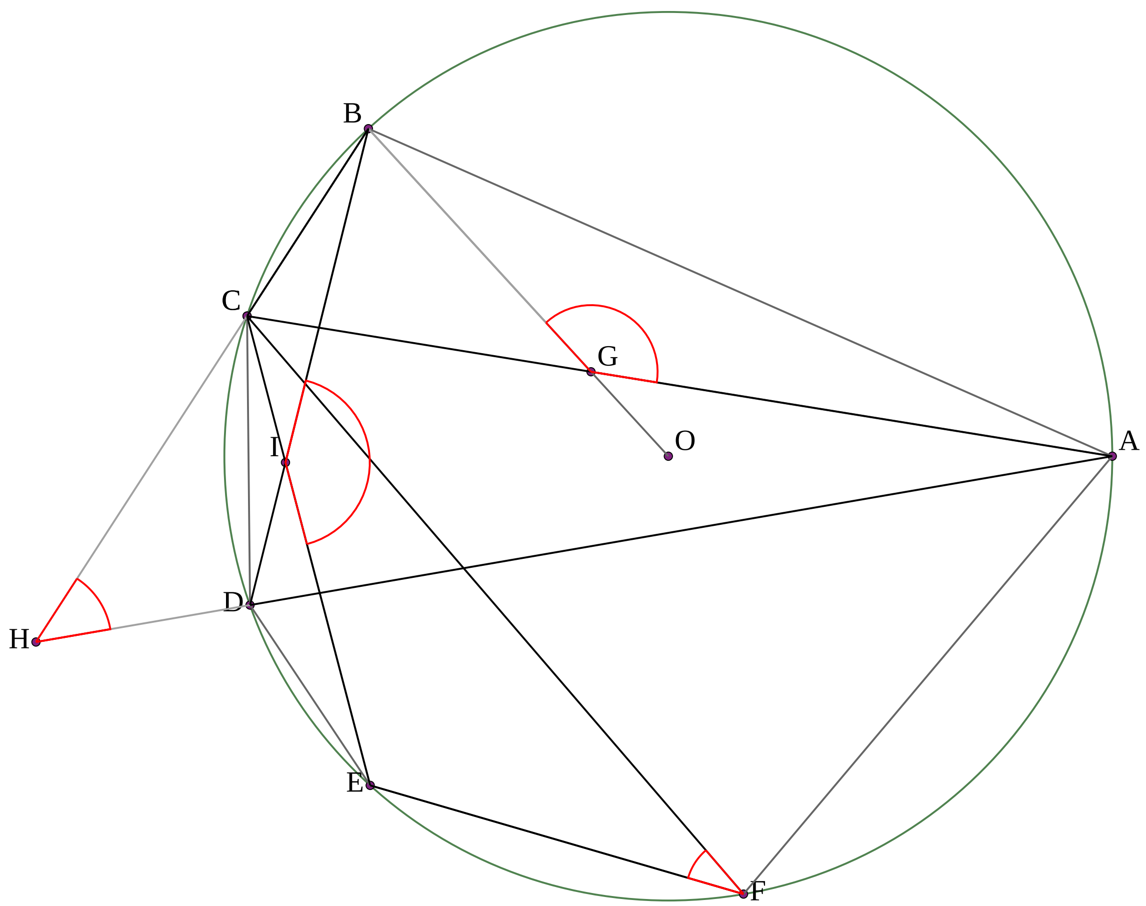
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FB and HE. Let J be the intersection of BA and EG. Let K be the intersection of AH and GC.
Angle BIE = 109°. Angle HKG = 46°. Angle CDF = 115°.
Find angle AJG.

Example 156



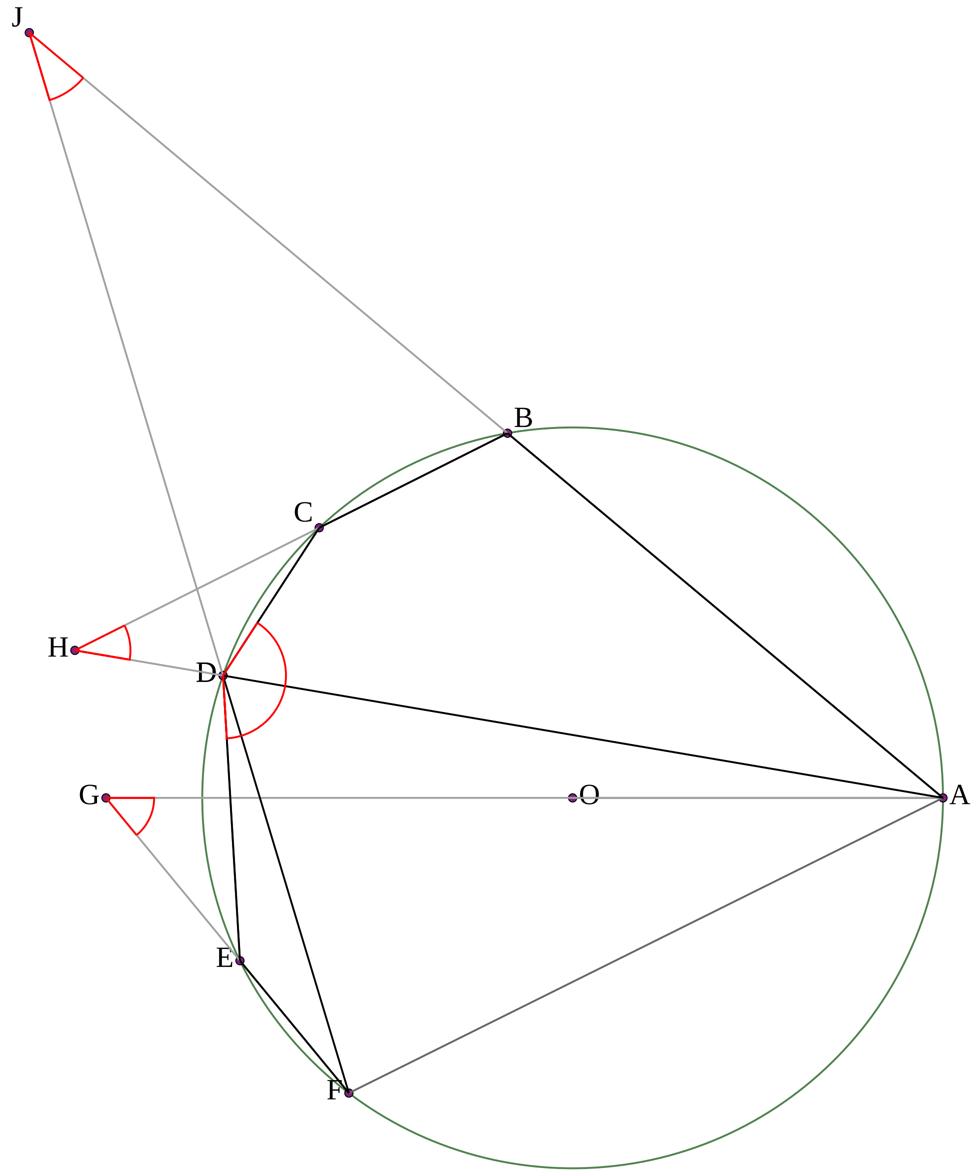
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BD and AO. Let H be the intersection of DF and CA.
Angle AEB = 21°. Angle DGA = 112°. Angle FHA = 77°.
Find angle OFC.

Example 157



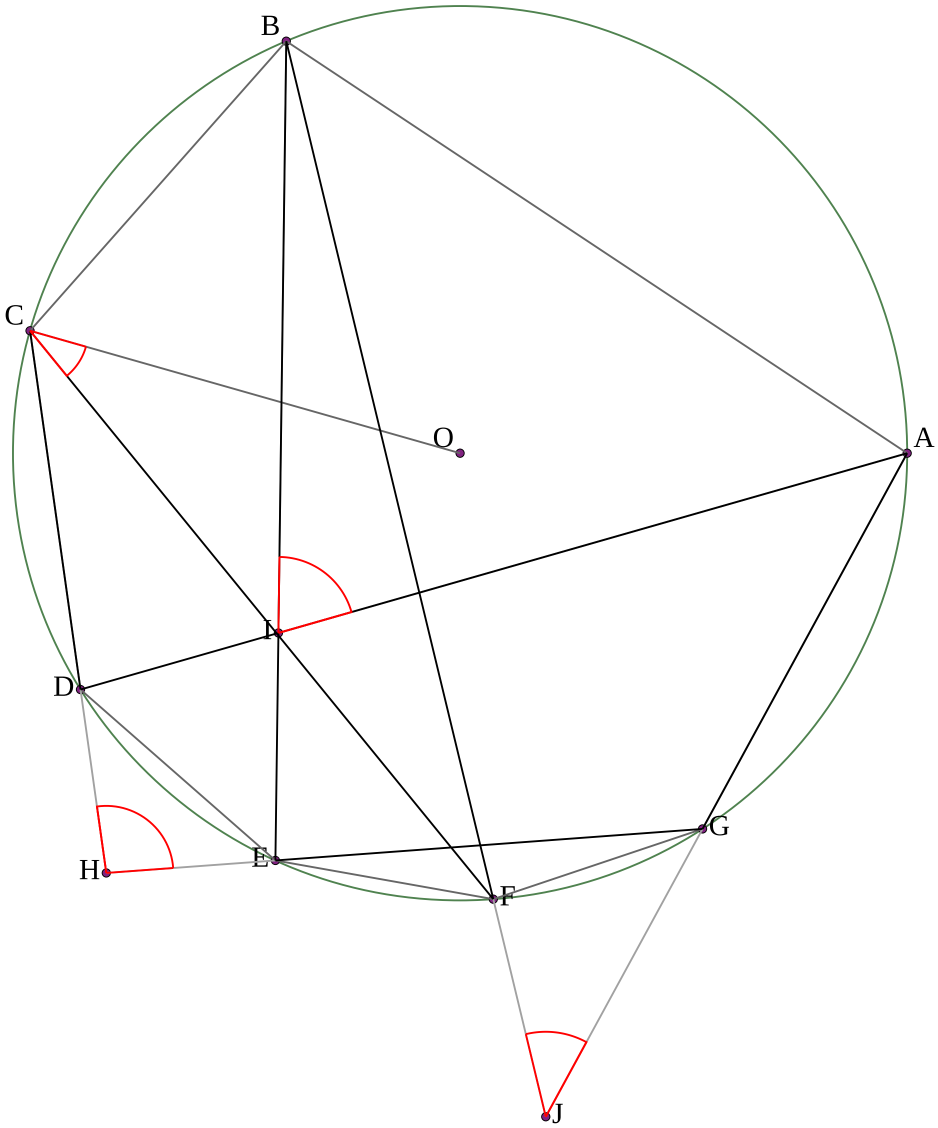
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CA and BO. Let H be the intersection of AD and BC. Let I be the intersection of DB and CE.
Angle EFC = 33°. Angle DHC = 47°. Angle BIE = 151°.
Find angle AGB.

Example 158



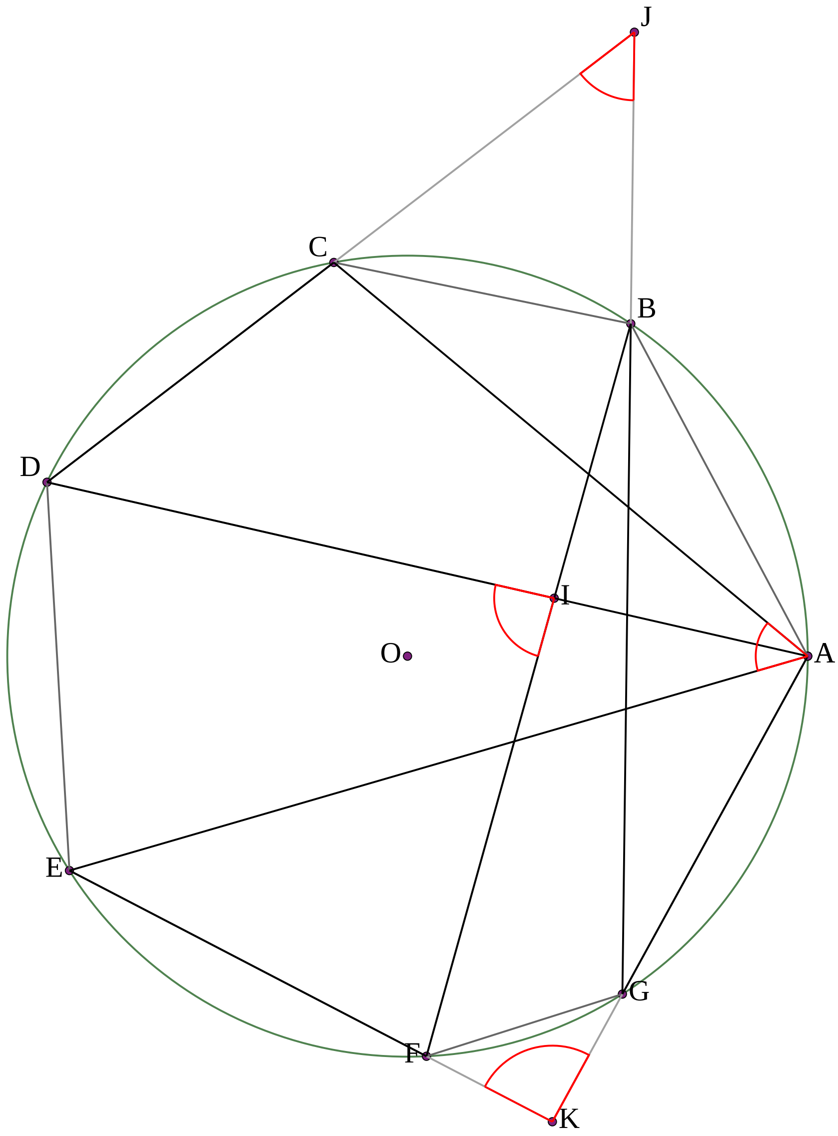
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of OA and EF. Let H be the intersection of AD and CB. Let J be the intersection of FD and BA.
Prove that CDE+BJD = AGE+CHD+90

Example 159



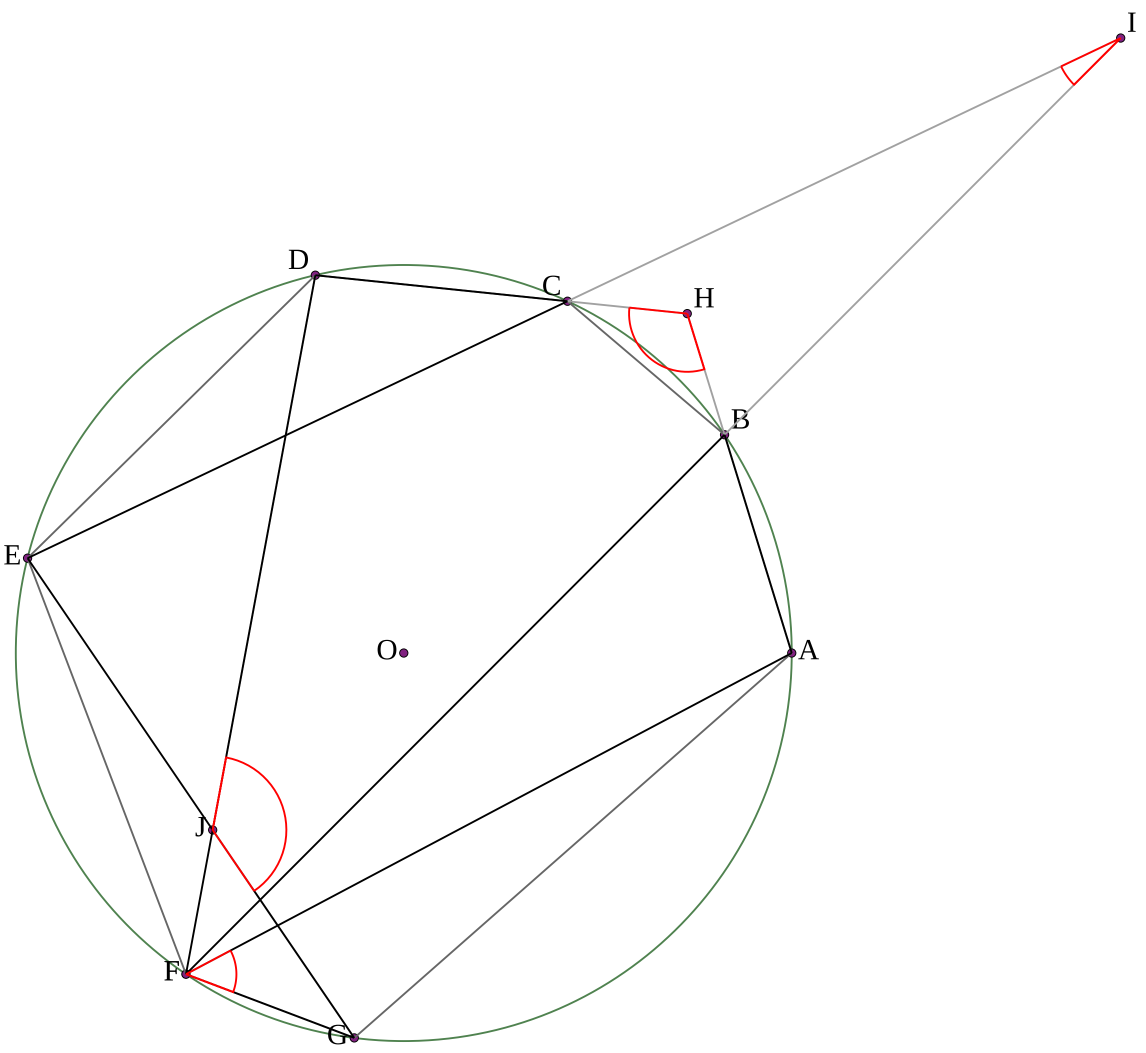
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CD and GE. Let I be the intersection of DA and EB. Let J be the intersection of AG and BF.
Angle DHE = x. Angle AIB = y. Angle FCO = z.
Find angle GJF.

Example 160



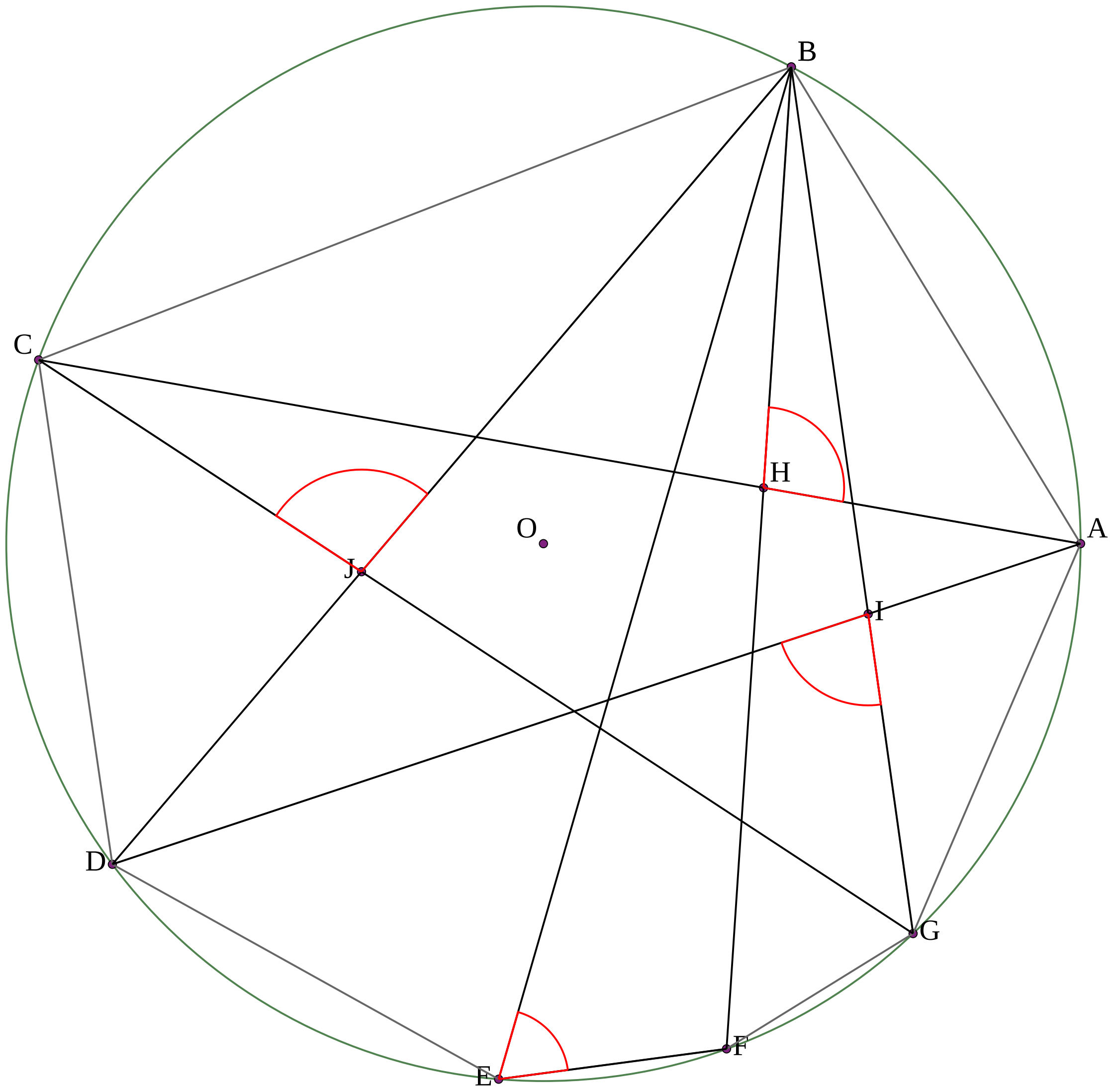
Let ABCDEFG be a cyclic heptagon with center O. Let I be the intersection of AD and BF. Let J be the intersection of DC and GB. Let K be the intersection of AG and FE.
Prove that CAE+DIF = BJC+FKG

Example 161



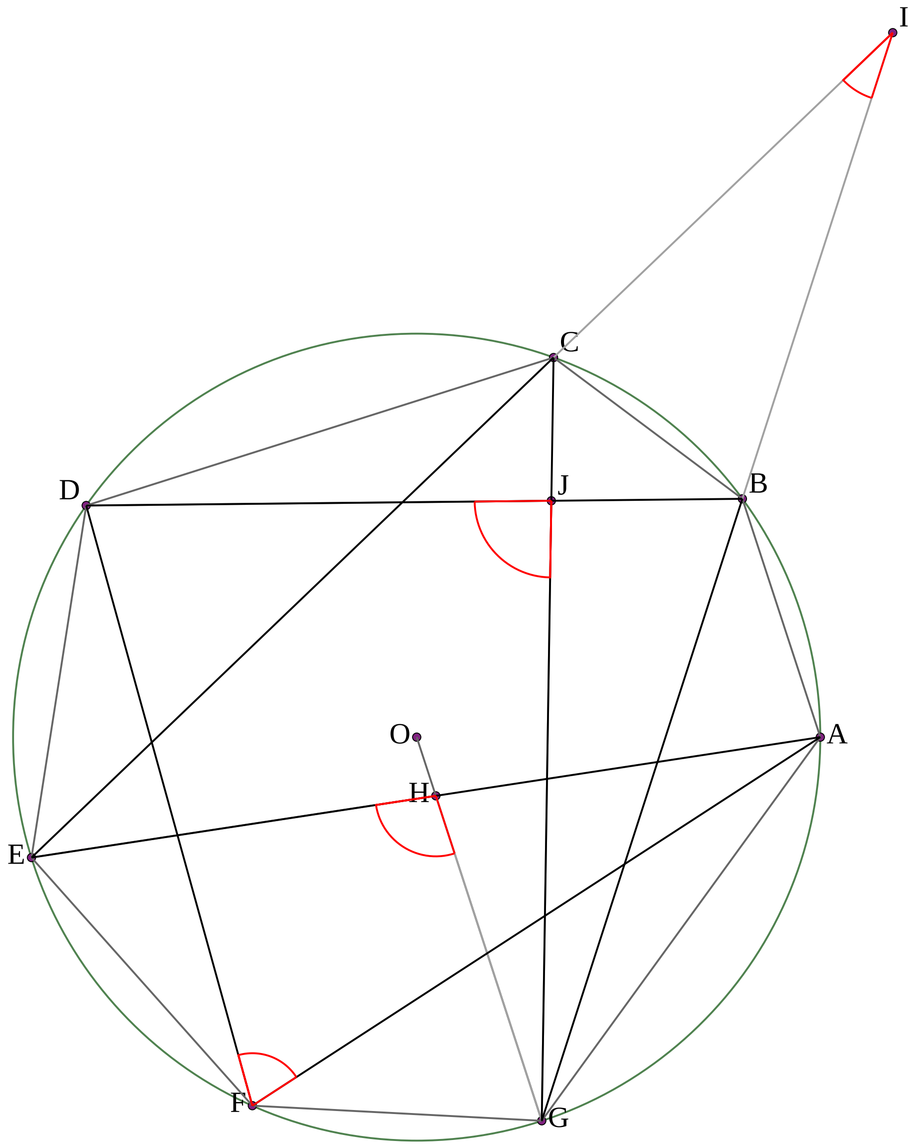
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AB and DC. Let I be the intersection of BF and CE. Let J be the intersection of FD and EG.
Angle GFA = 49°. Angle BIC = 20°. Angle BHC = 113°.
Find angle DJG.

Example 162



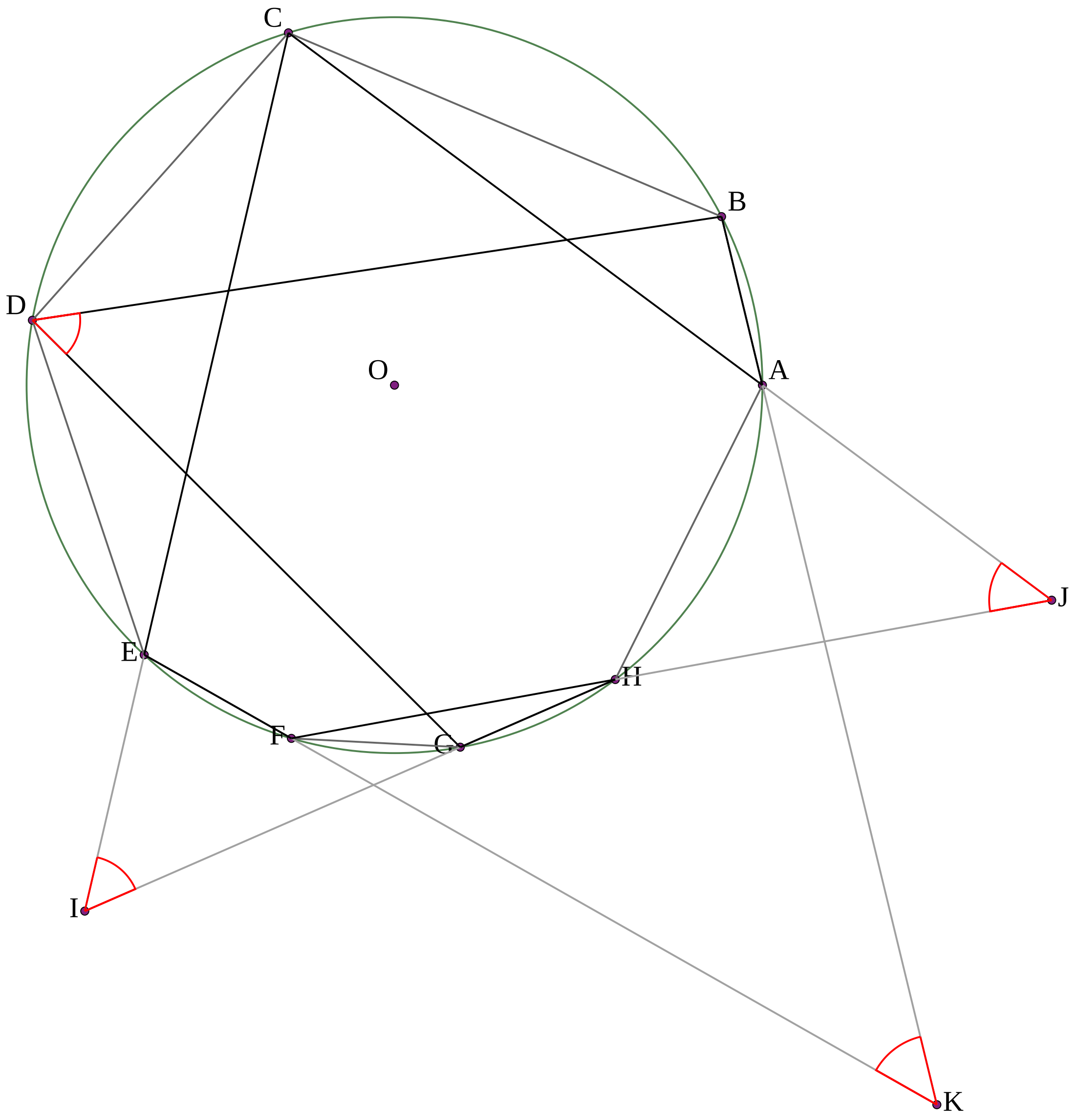
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FB and CA. Let I be the intersection of BG and AD. Let J be the intersection of GC and DB.
Prove that BEF+AHB+BJC = DIG+180

Example 163



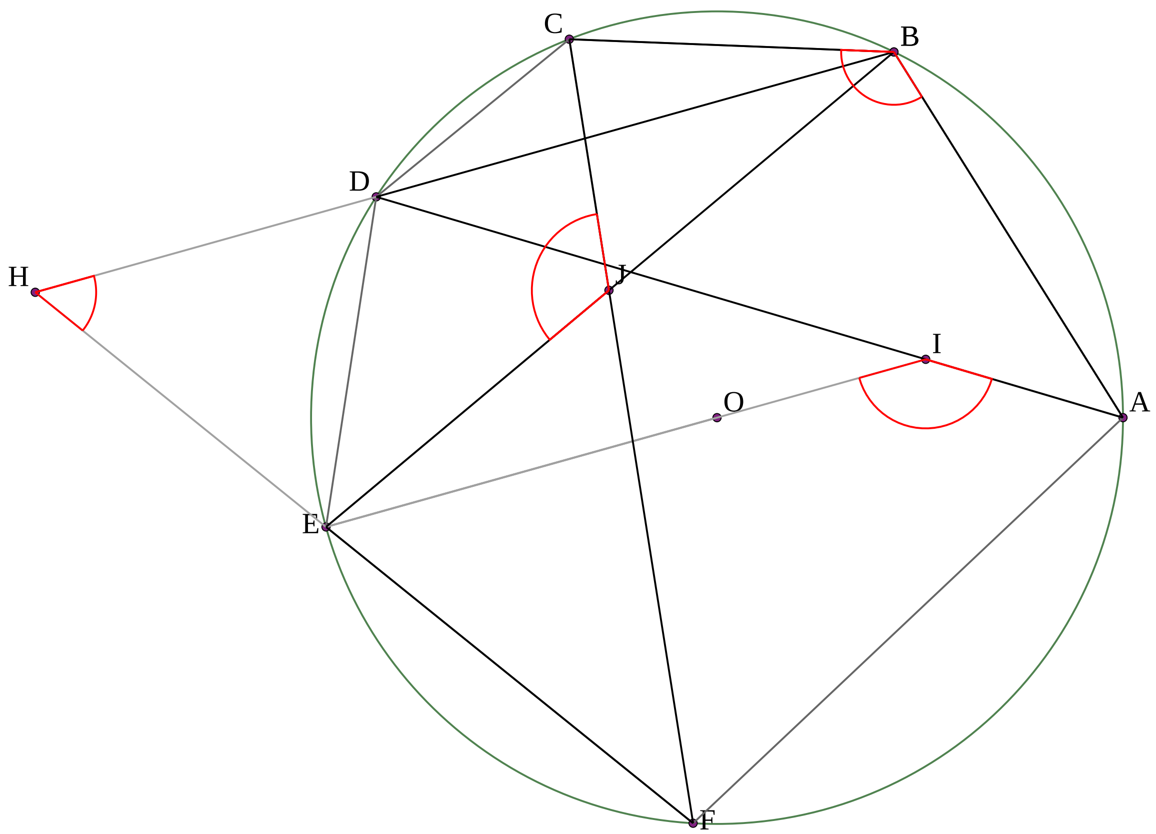
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AE and GO. Let I be the intersection of EC and GB. Let J be the intersection of CG and BD.
Angle EHG = 99°. Angle CIB = 28°. Angle GJD = 89°.
Find angle DFA.

Example 164



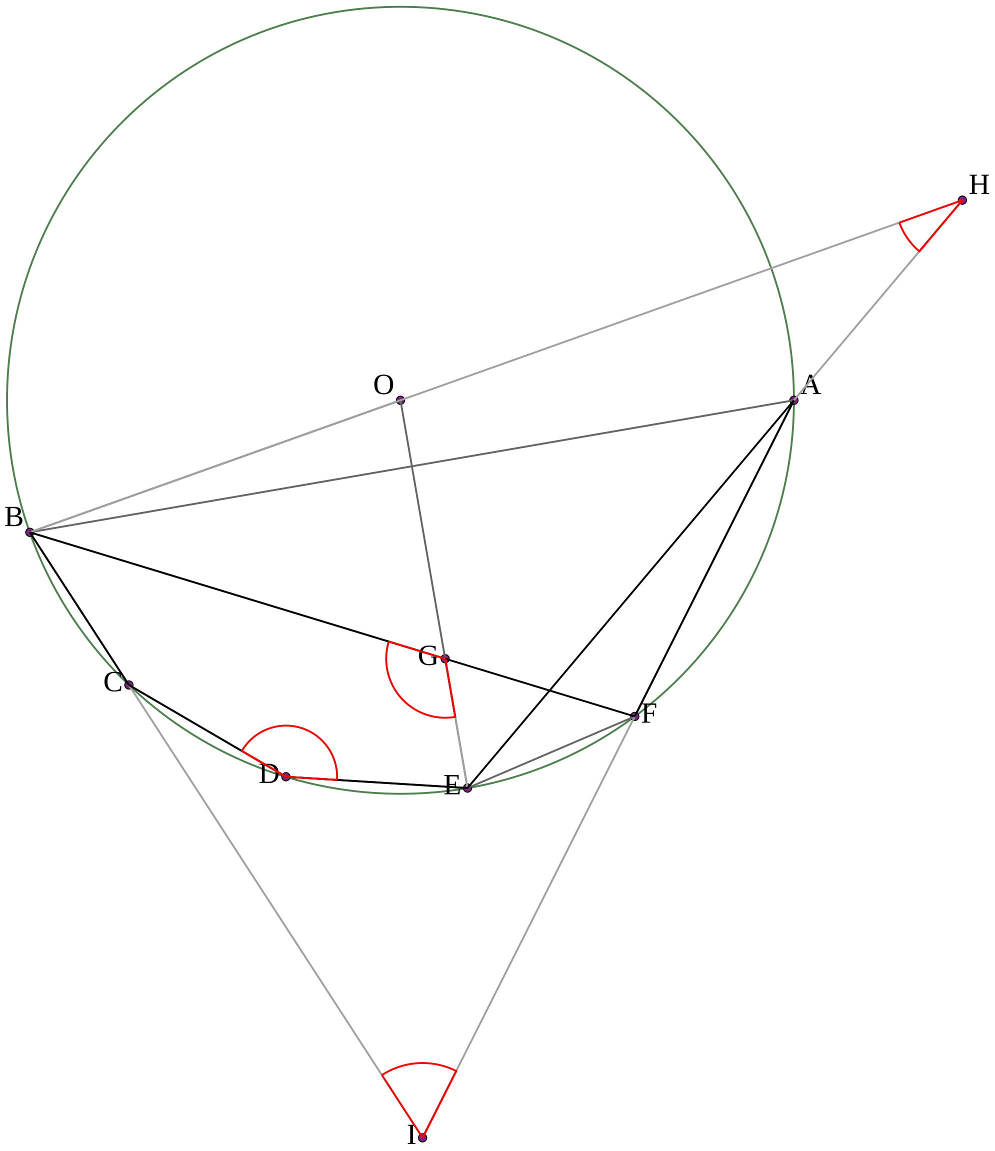
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GH and EC. Let J be the intersection of HF and CA. Let K be the intersection of FE and AB.
Angle GIE = x. Angle HJA = y. Angle BDG = z.
Find angle FKA.

Example 165



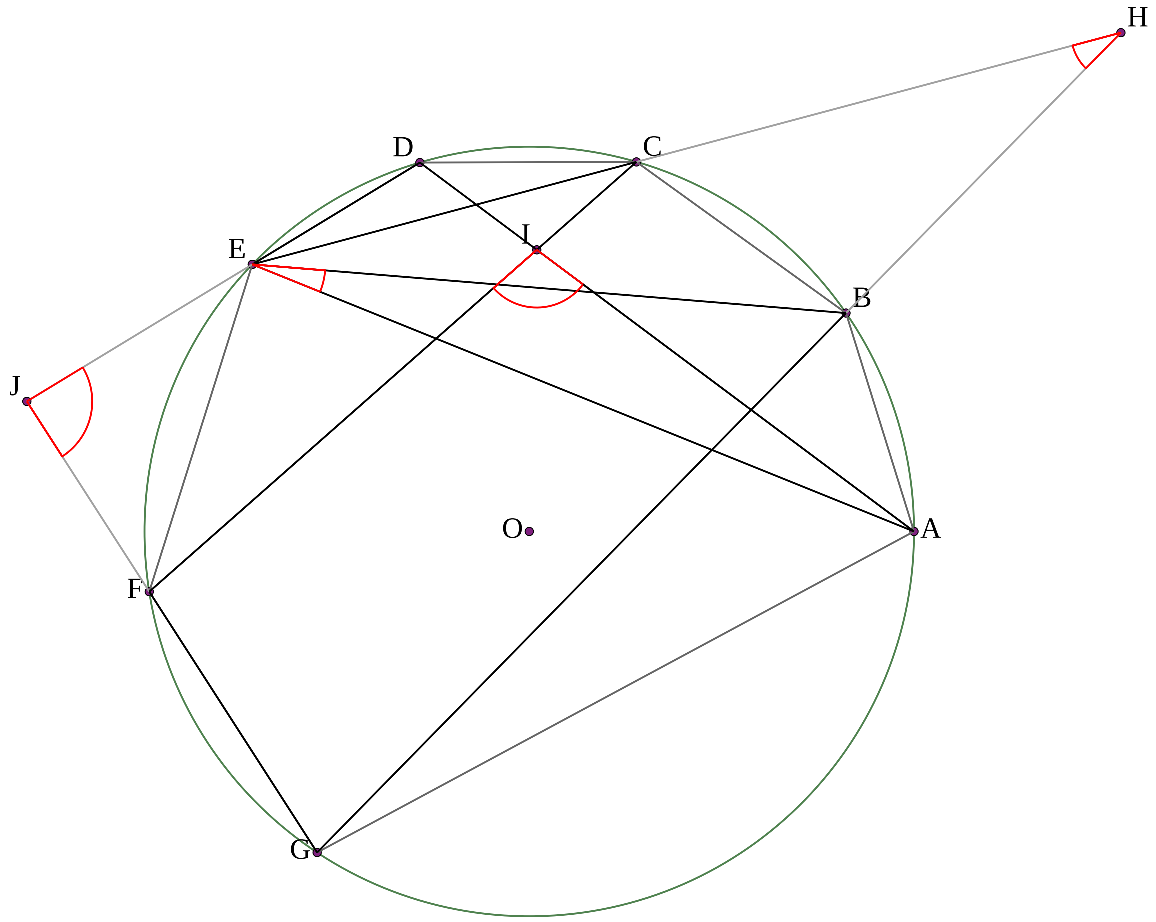
Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of BD and EF. Let I be the intersection of DA and EO. Let J be the intersection of BE and FC.
Angle CBA = x. Angle DHE = y. Angle EJC = z.
Find angle AIE.

Example 166



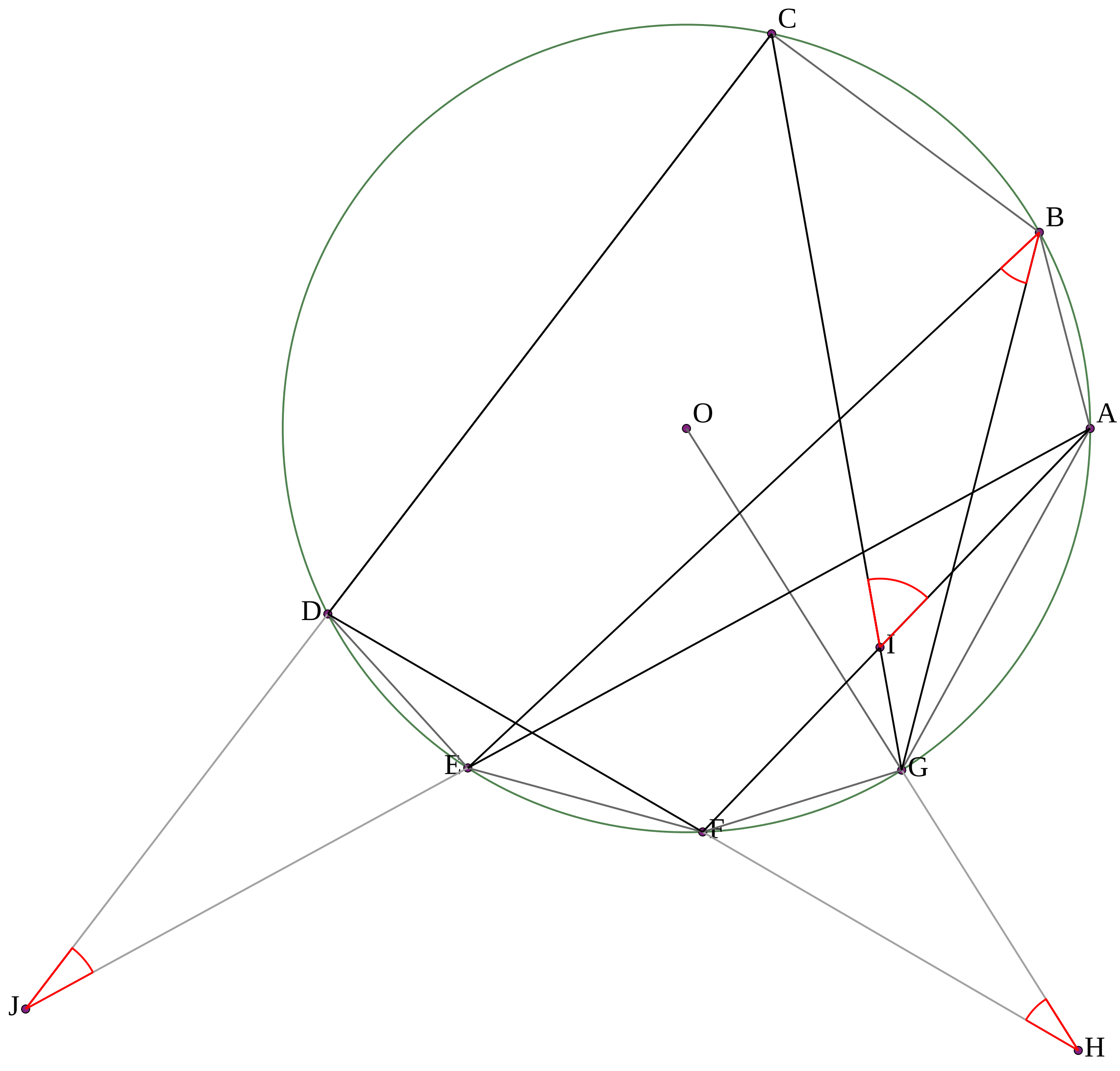
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of OE and FB. Let H be the intersection of EA and BO. Let I be the intersection of AF and BC.
Angle CDE = x. Angle EGB = y. Angle AHB = z.
Find angle FIC.

Example 167



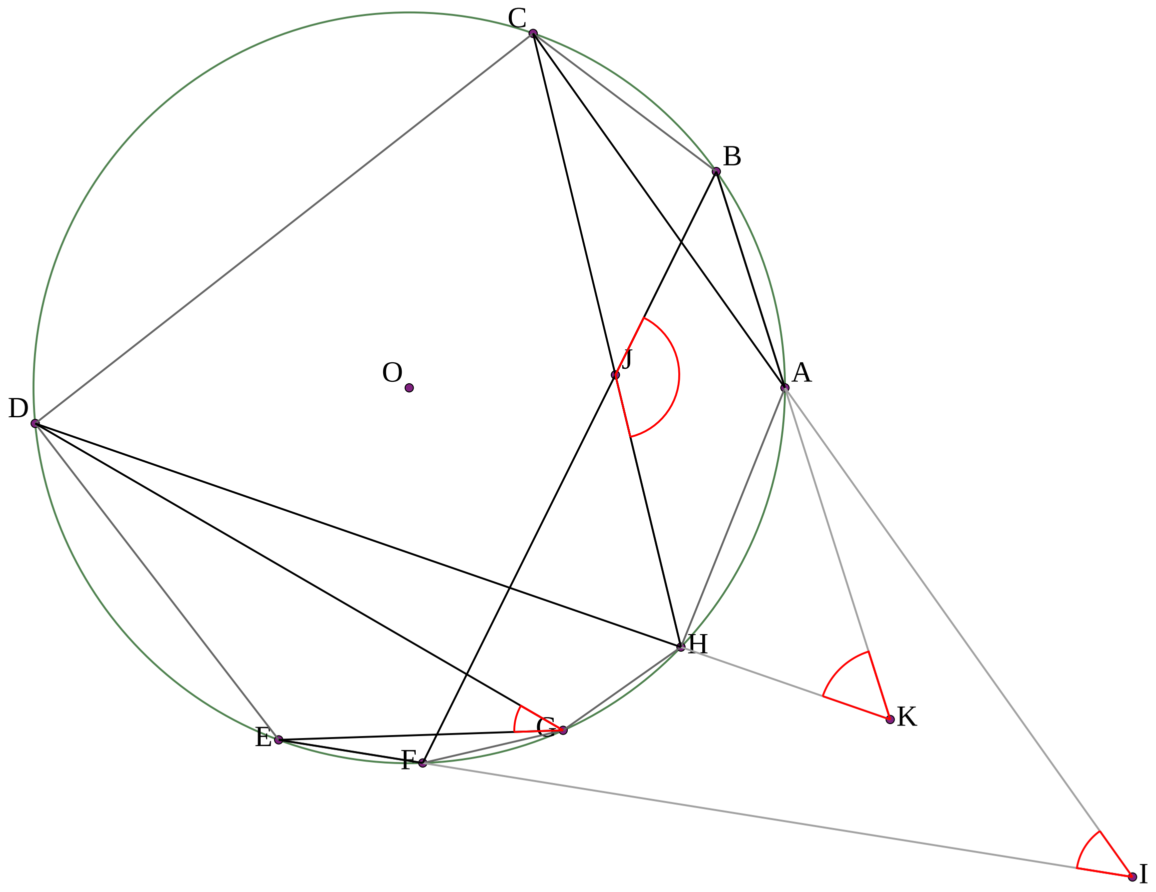
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EC and GB. Let I be the intersection of CF and DA. Let J be the intersection of FG and ED.
Prove that AIF+AEB = BHC+EJF

Example 168



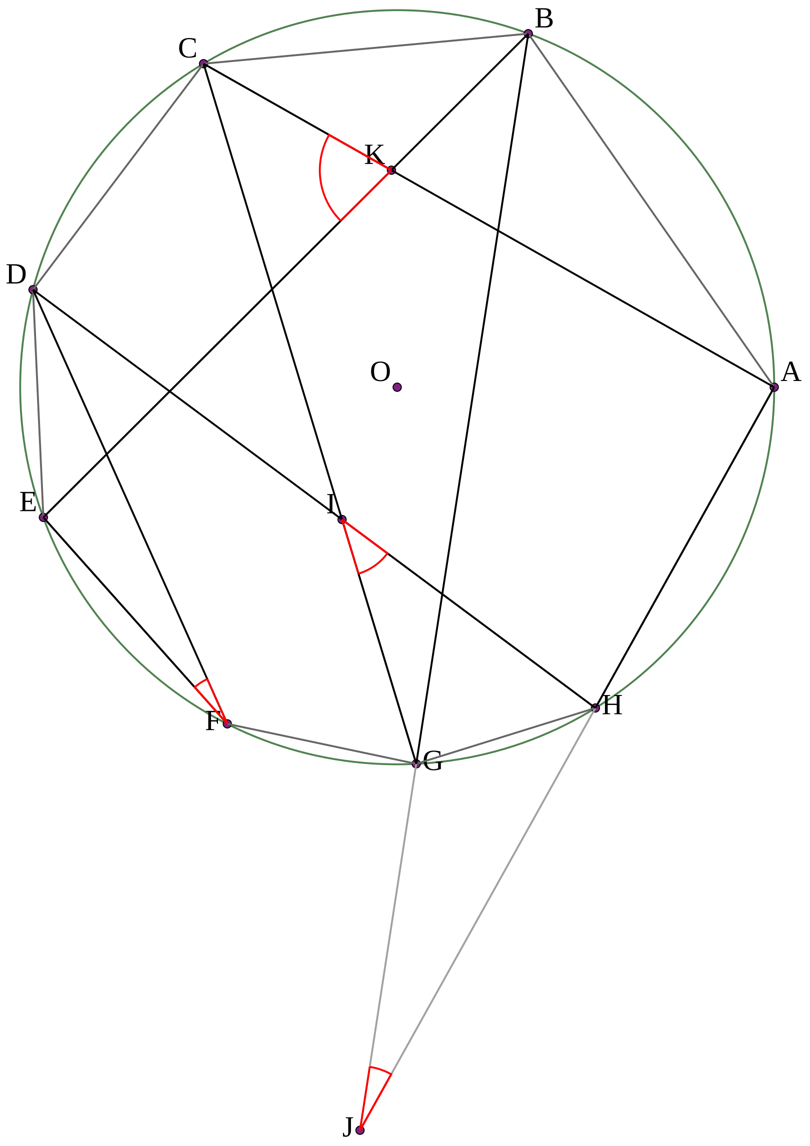
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of OG and DF. Let I be the intersection of GC and FA. Let J be the intersection of CD and AE.
Angle DJE = x. Angle EBG = y. Angle GHF = z.
Find angle CIA.

Example 169



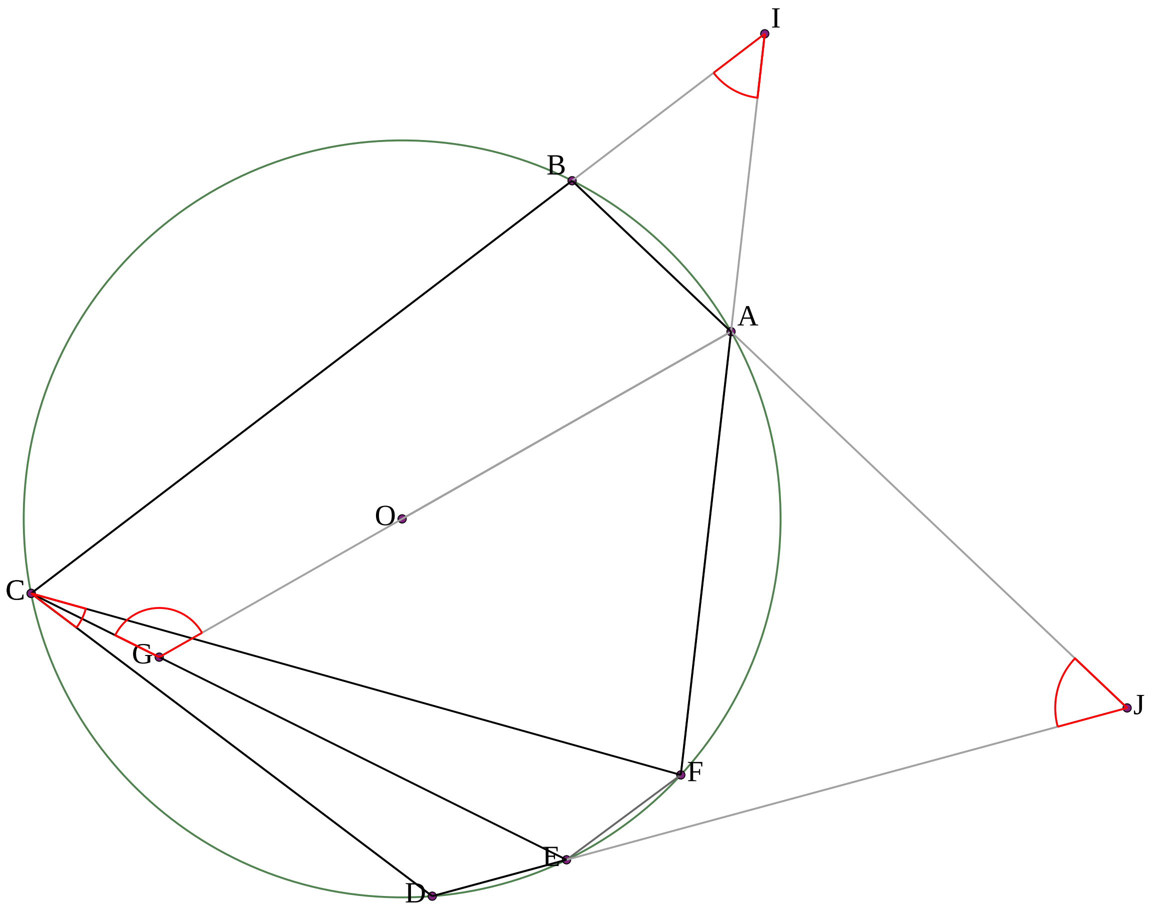
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of EF and AC. Let J be the intersection of FB and CH. Let K be the intersection of BA and HD.
Angle FIA = x. Angle DGE = y. Angle AKH = z.
Find angle BJH.

Example 170



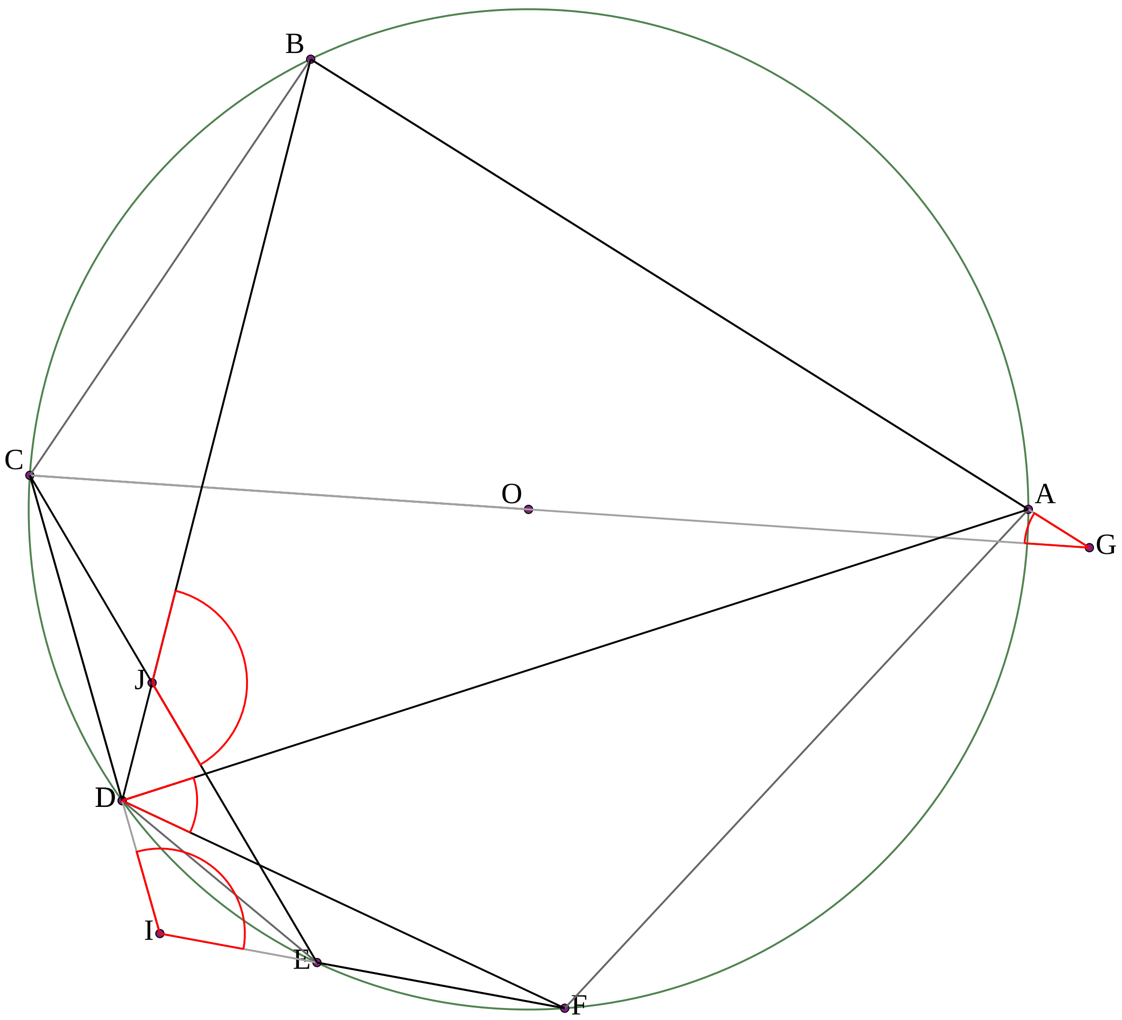
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DH and CG. Let J be the intersection of HA and GB. Let K be the intersection of AC and BE.
Angle EFD = 18°. Angle HIG = 36°. Angle HJG = 20°.
Find angle CKE.

Example 171



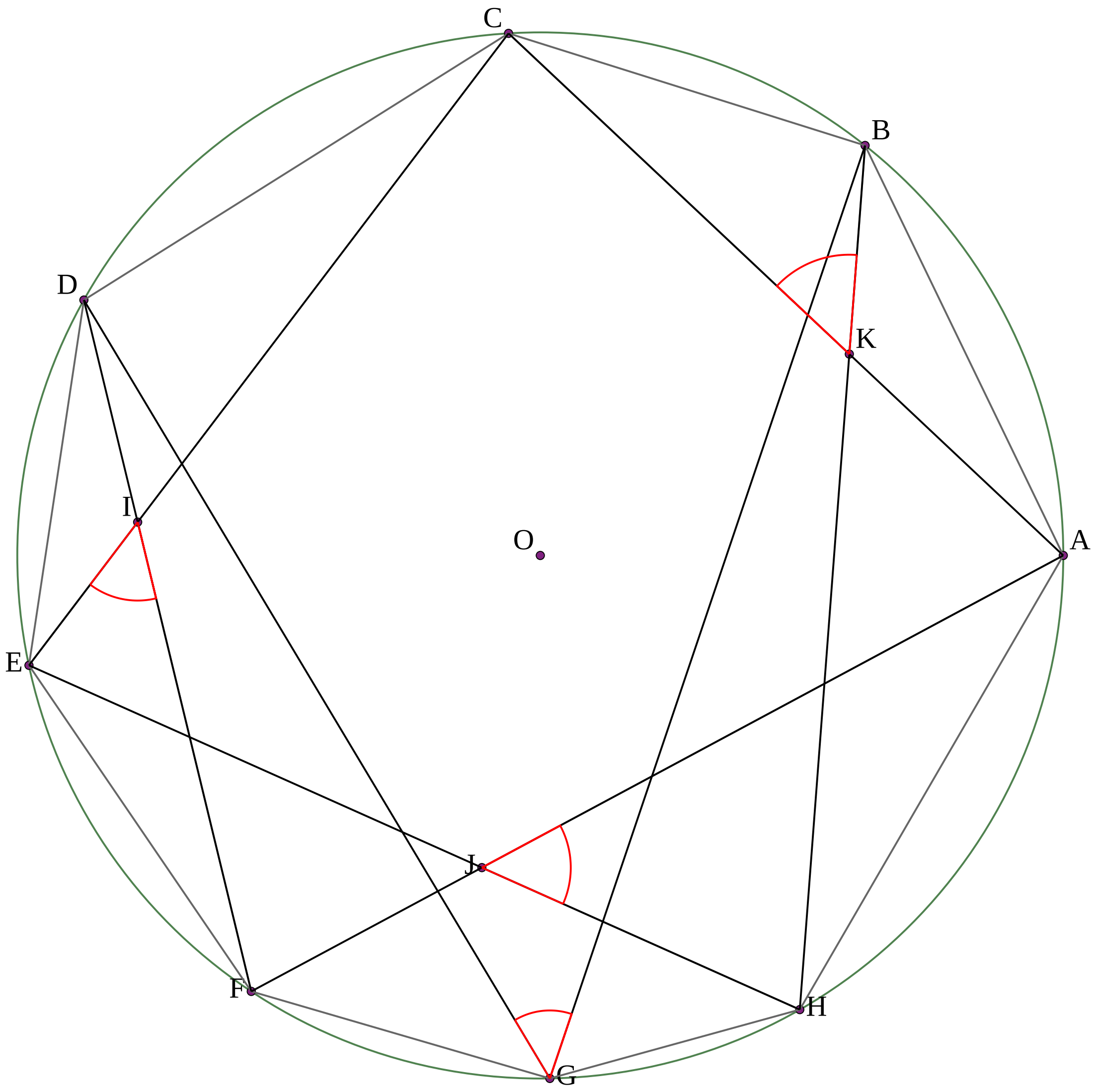
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and AO. Let I be the intersection of FA and BC. Let J be the intersection of AB and DE.
Prove that AGC+AIB = DCF+AJE+90

Example 172



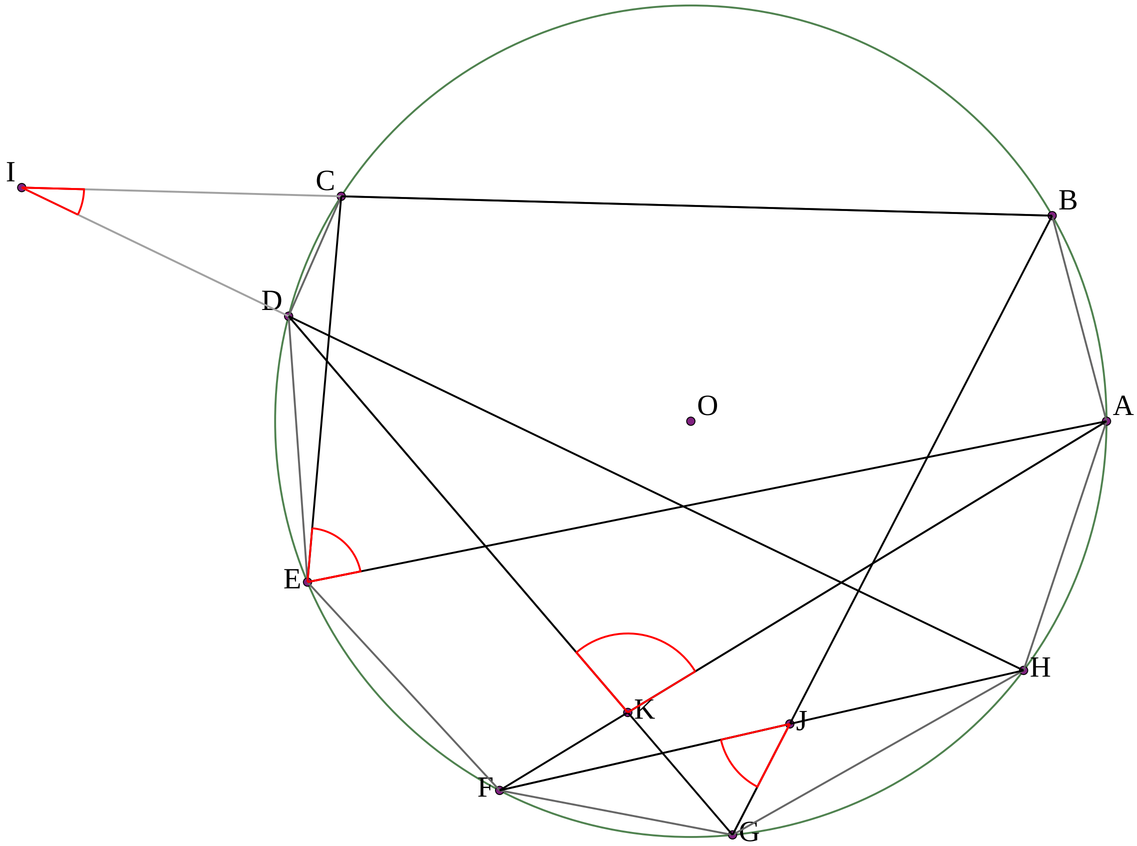
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BA and CO. Let I be the intersection of DC and EF. Let J be the intersection of CE and DB.
Angle DIE = x. Angle AGC = y. Angle ADF = z.
Find angle EJB.

Example 173



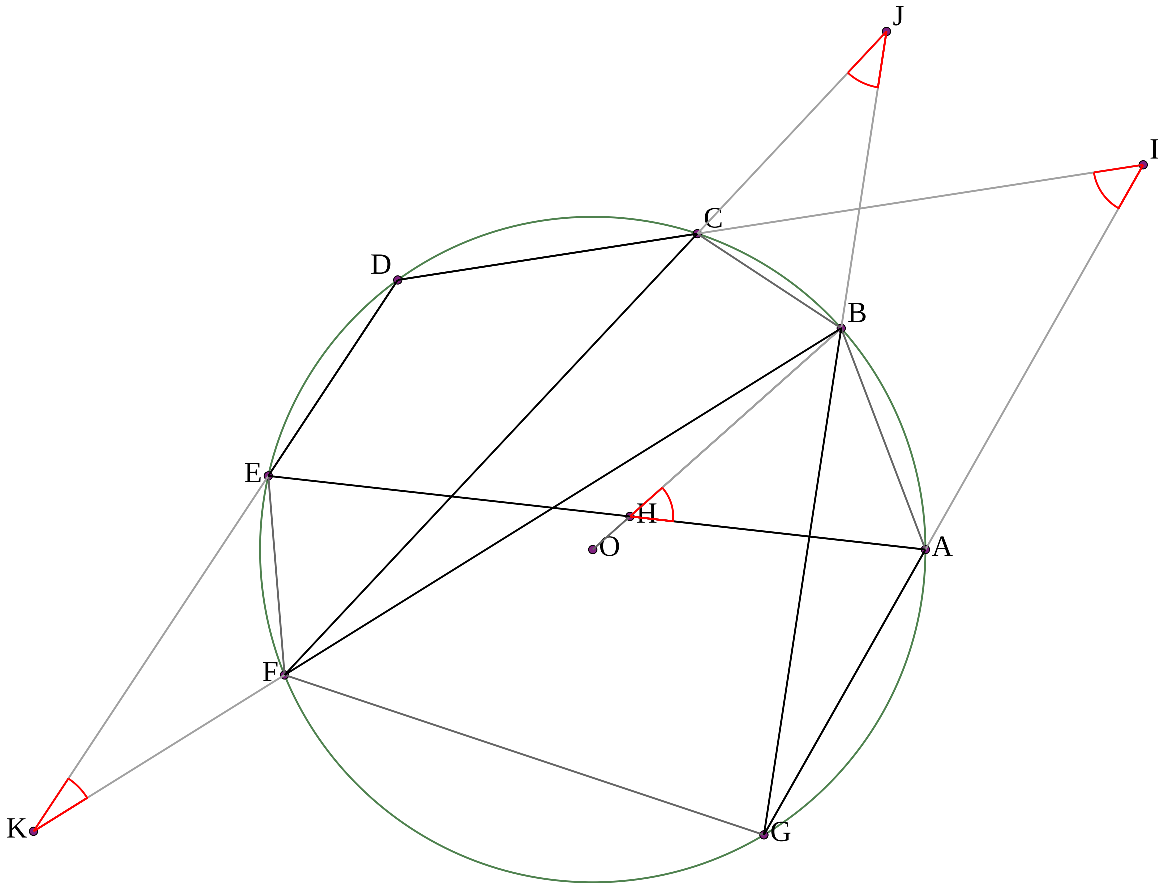
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DF and CE. Let J be the intersection of FA and EH. Let K be the intersection of AC and HB.
Angle BGD = 50°. Angle AJH = 52°. Angle FIE = 51°.
Find angle CKB.

Example 174



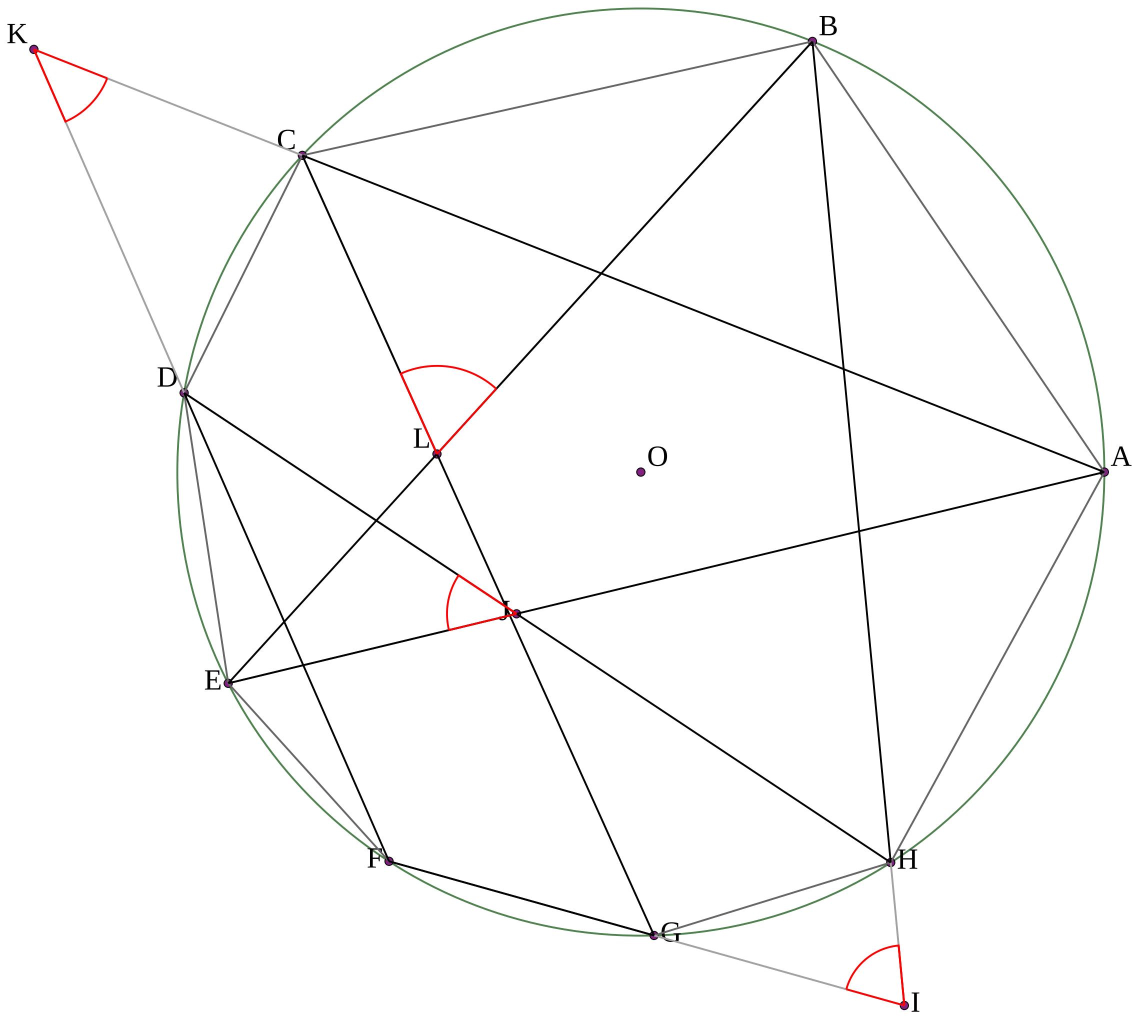
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CB and DH. Let J be the intersection of BG and HF. Let K be the intersection of GD and FA.
Angle CID = 24°. Angle DKA = 99°. Angle AEC = 74°.
Find angle GJF.

Example 175



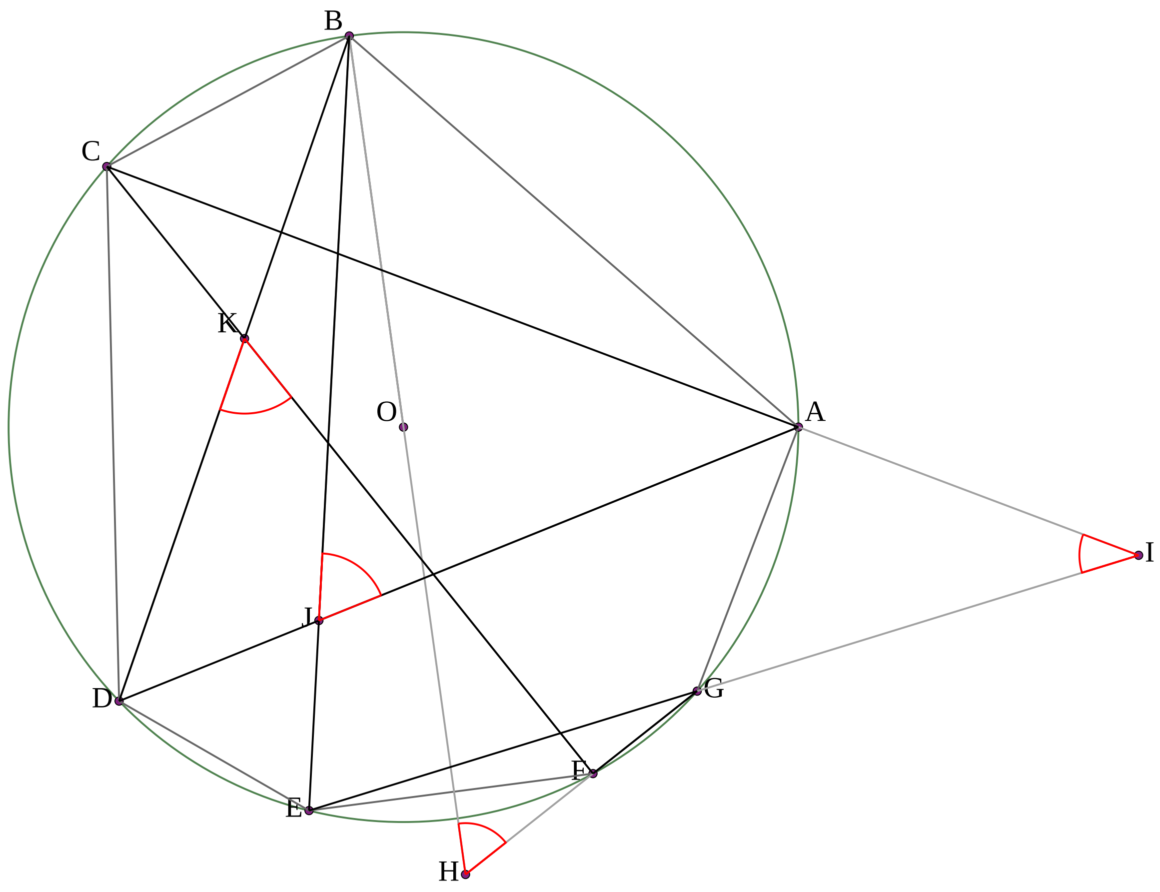
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EA and BO. Let I be the intersection of AG and CD. Let J be the intersection of GB and FC. Let K be the intersection of BF and DE.
Angle AHB = 48°. Angle AIC = 52°. Angle FKE = 25°.
Find angle BJC.

Example 176



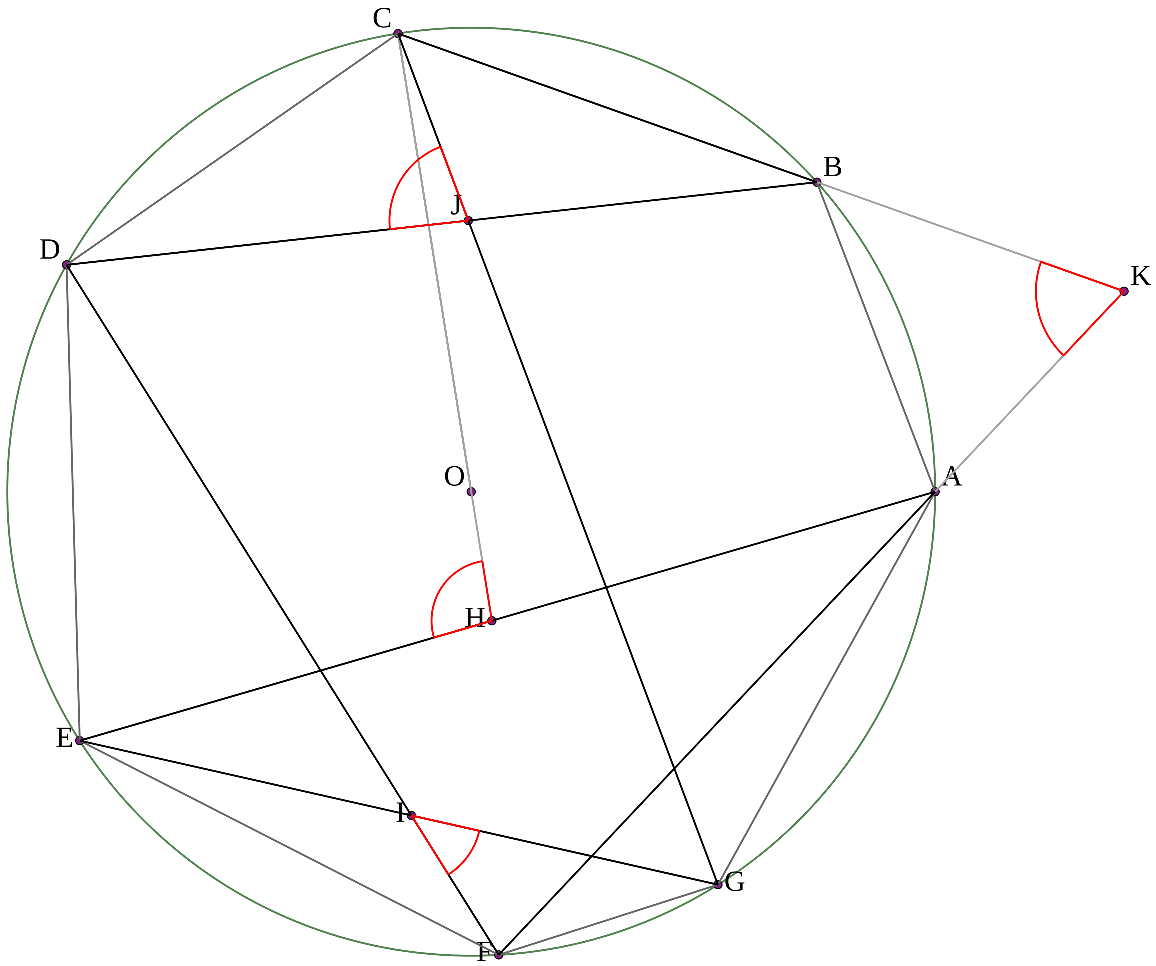
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of BH and FG. Let J be the intersection of HD and AE. Let K be the intersection of DF and CA. Let L be the intersection of GC and EB.
Angle DJE = x. Angle CLB = y. Angle DKC = z.
Find angle HIG.

Example 177



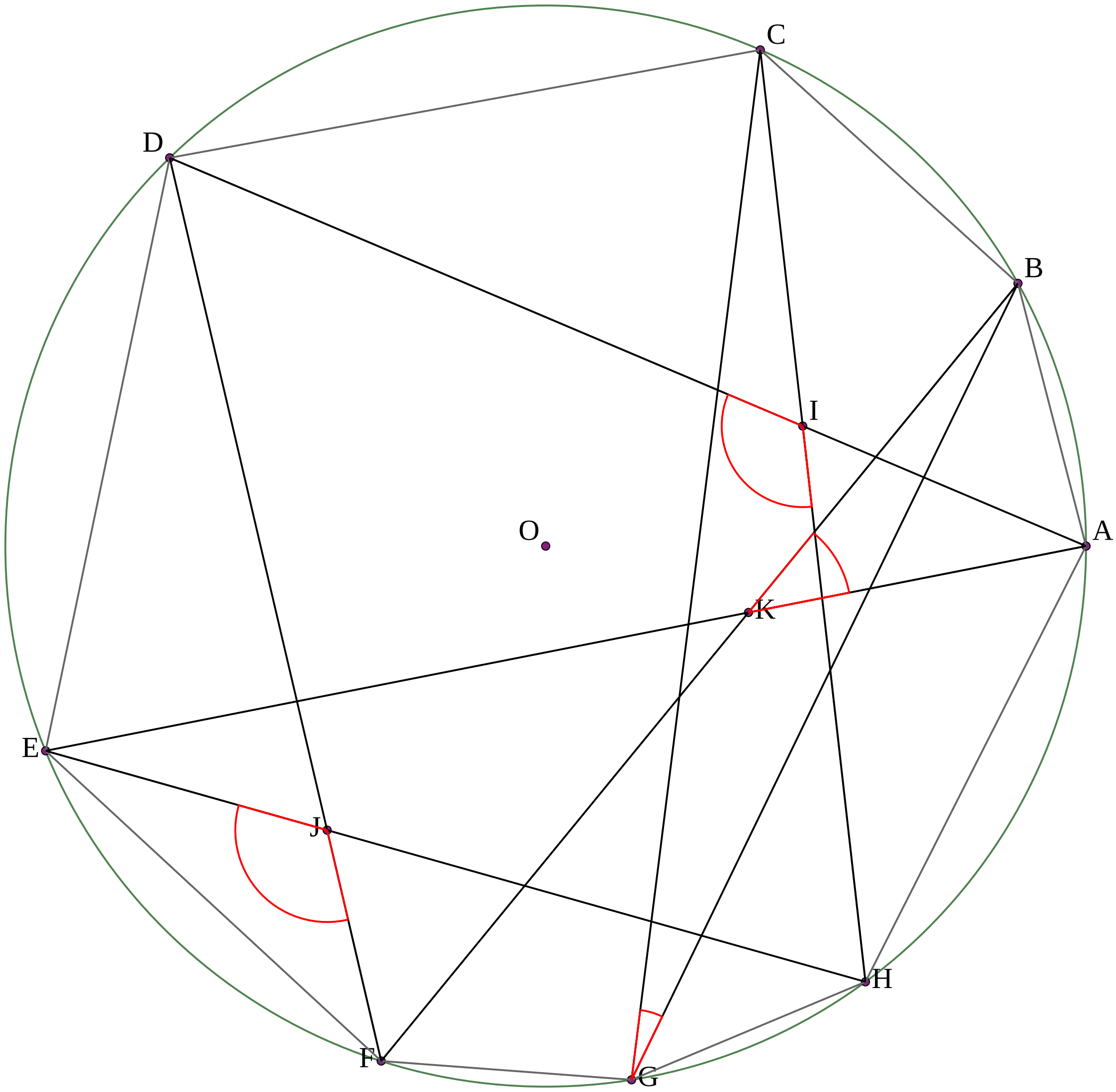
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and BO. Let I be the intersection of GE and AC. Let J be the intersection of EB and DA. Let K be the intersection of BD and CF.
Angle FHB = 60°. Angle GIA = 38°. Angle DKF = 58°.
Find angle BJA.

Example 178



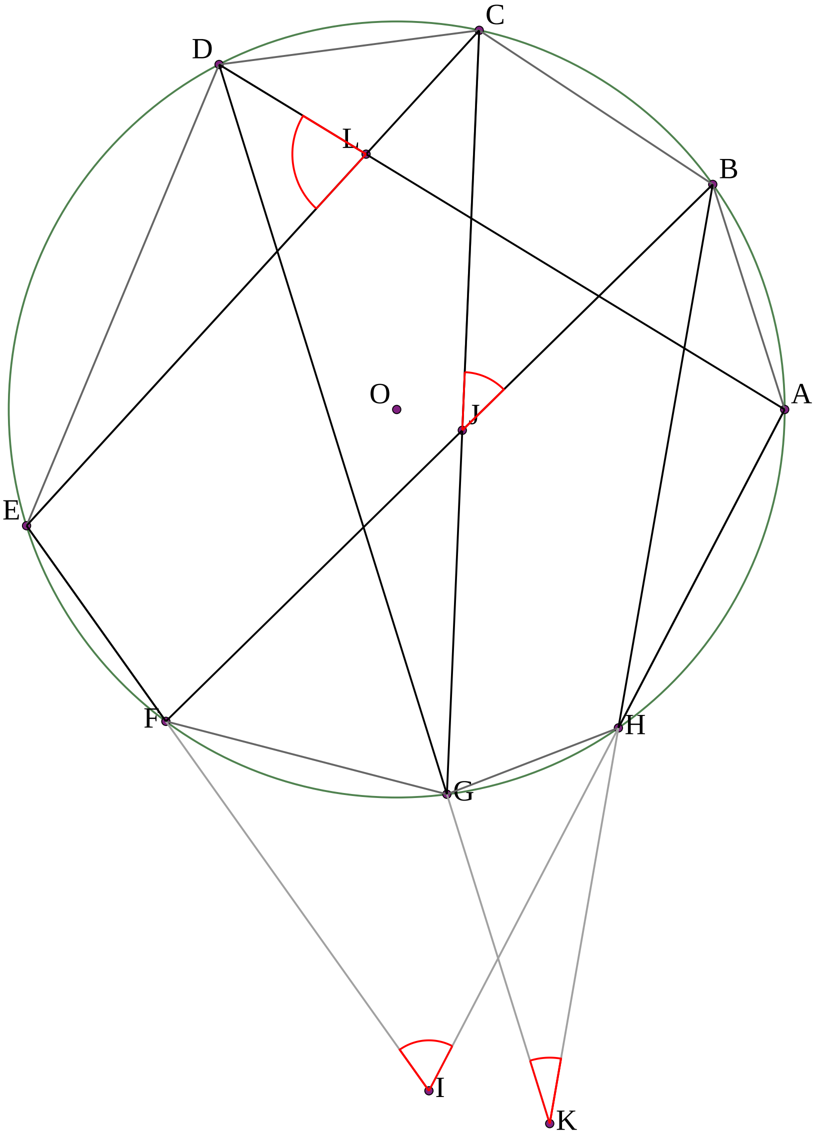
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AE and CO. Let I be the intersection of EG and DF. Let J be the intersection of GC and BD. Let K be the intersection of CB and FA.
Angle EHC = 97°. Angle GIF = 45°. Angle CJD = 76°.
Find angle BKA.

Example 179



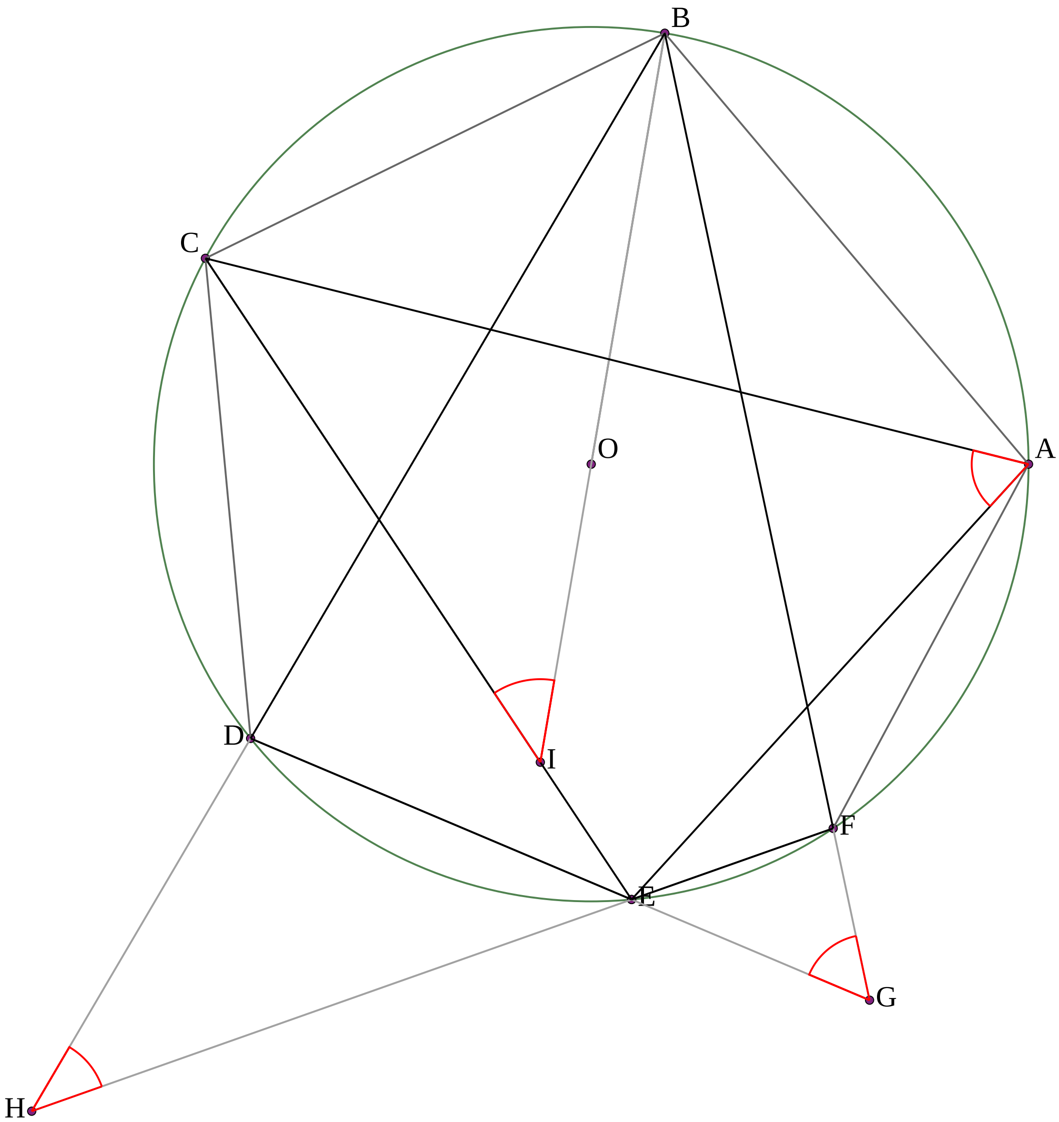
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CH and AD. Let J be the intersection of HE and DF. Let K be the intersection of EA and FB.
Prove that DIH+EJF = BGC+AKB+180

Example 180



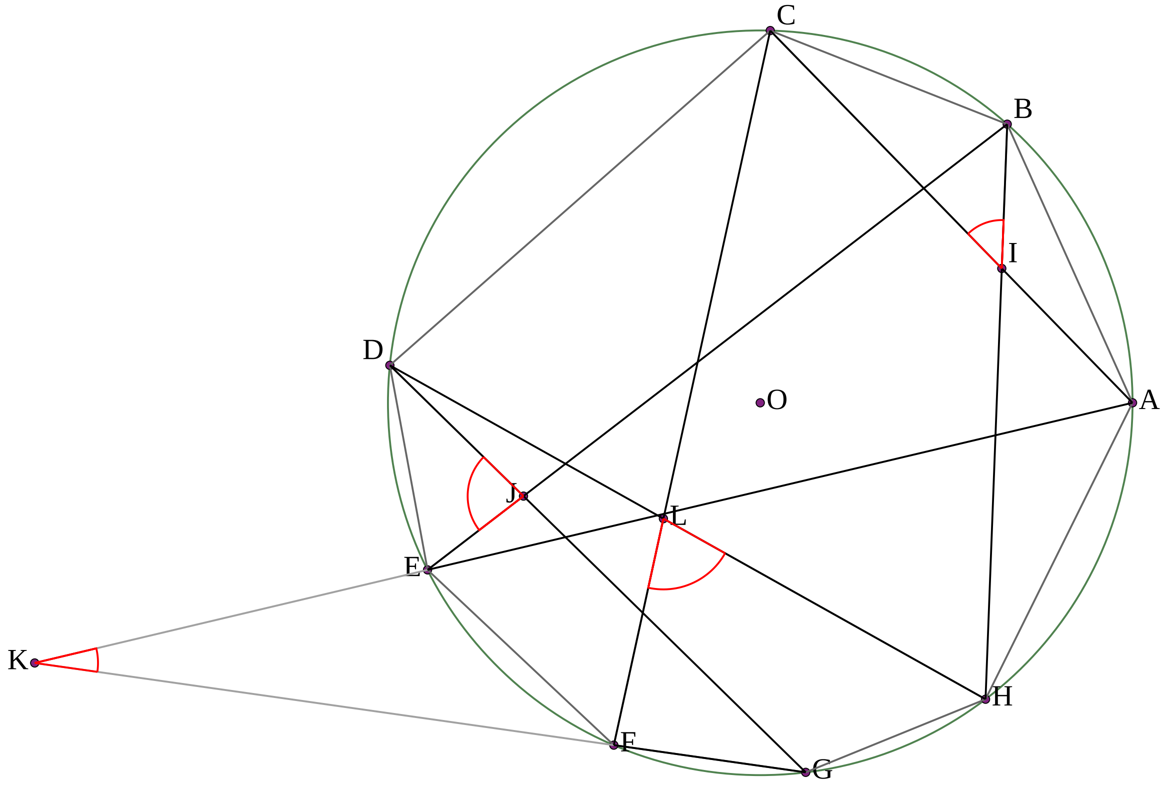
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of EF and HA. Let J be the intersection of FB and GC. Let K be the intersection of BH and DG. Let L be the intersection of AD and CE.
Angle HKG = 27°. Angle DLE = 79°. Angle FIH = 63°.
Find angle BJC.

Example 181



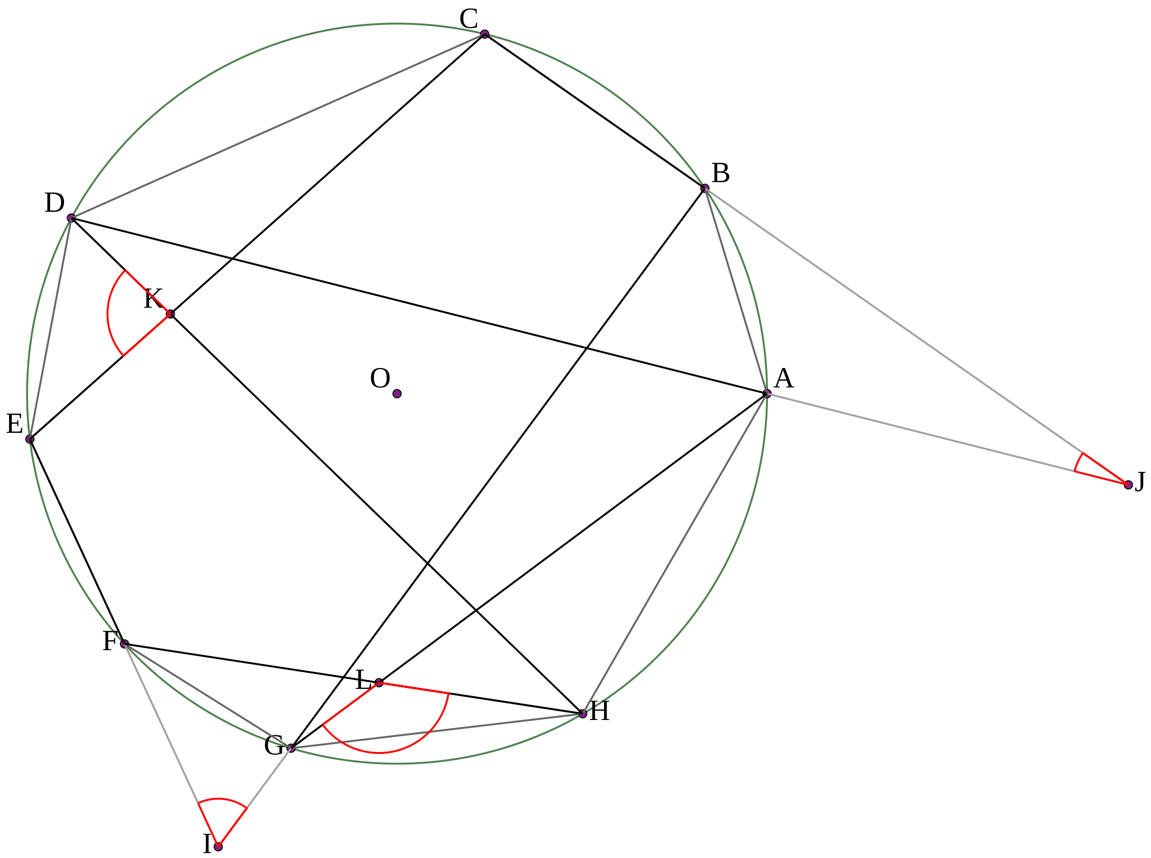
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and BF. Let H be the intersection of DB and FE. Let I be the intersection of OB and EC.
Angle EGF = 55°. Angle CAE = 62°. Angle DHE = 40°.
Find angle BIC.

Example 182



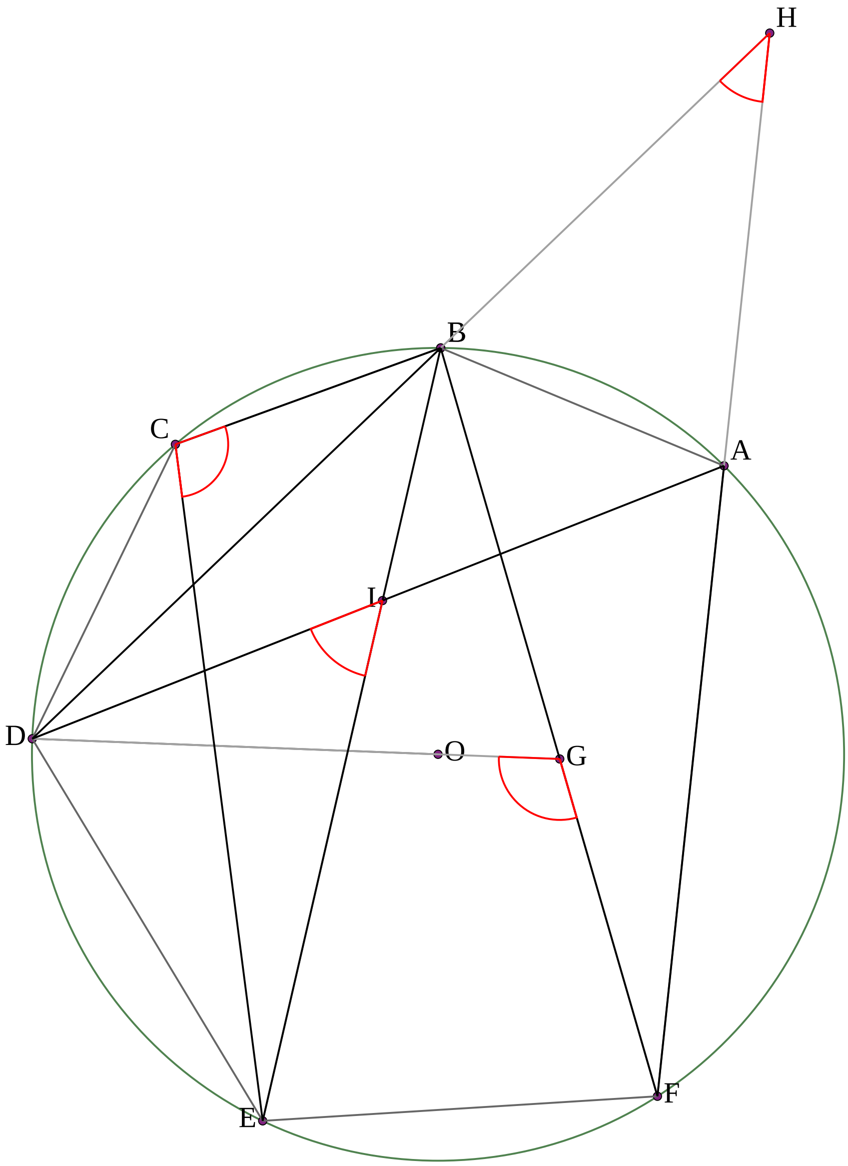
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of HB and AC. Let J be the intersection of BE and GD. Let K be the intersection of EA and FG. Let L be the intersection of CF and DH.
Angle EKF = x. Angle BIC = y. Angle EJD = z.
Find angle FLH.

Example 183



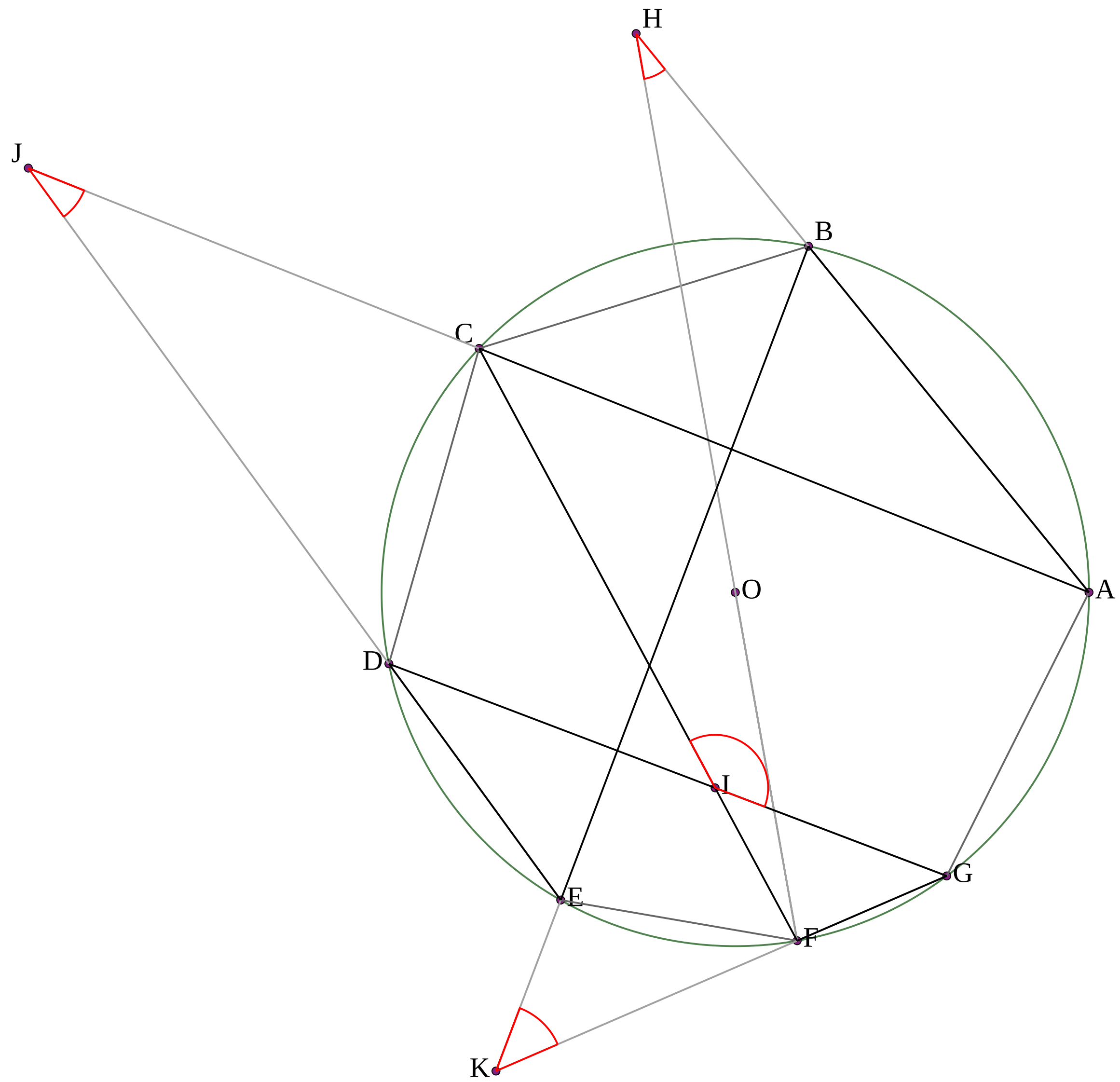
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GB and EF. Let J be the intersection of BC and DA. Let K be the intersection of CE and HD. Let L be the intersection of FH and AG.
Angle GIF = 61°. Angle EKD = 86°. Angle HLG = 135°.
Find angle BJA.

Example 184



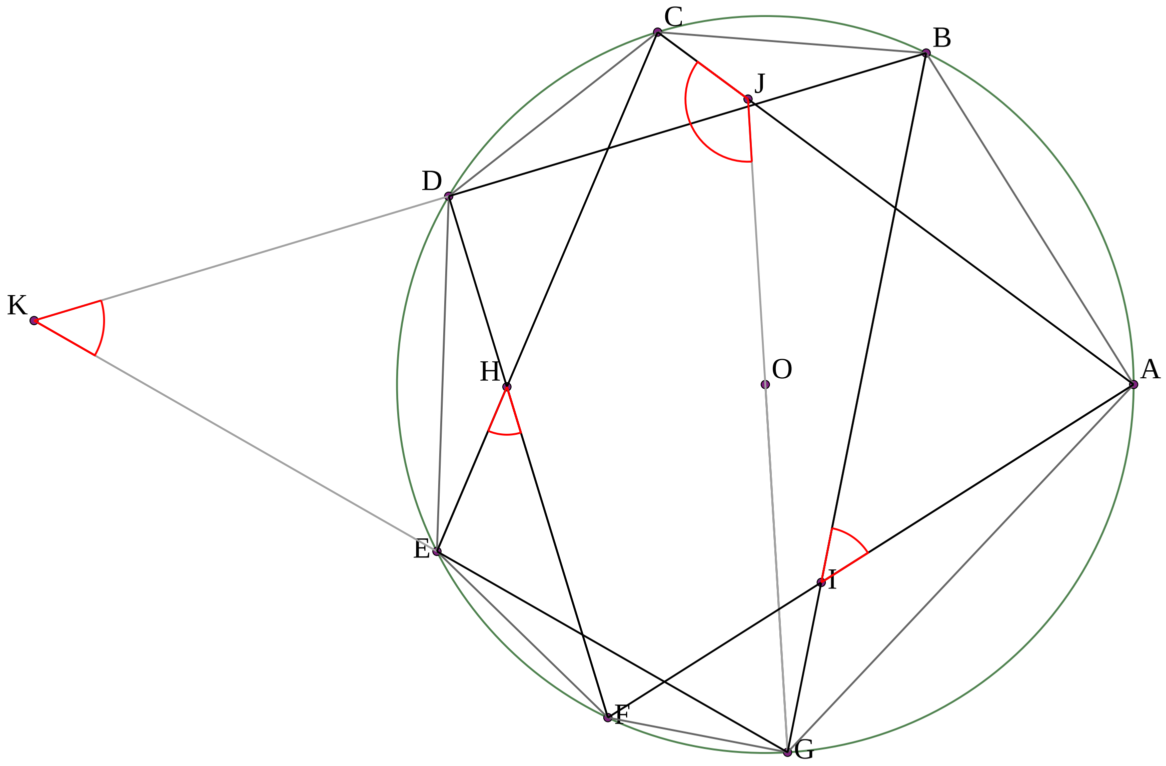
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BF and DO. Let H be the intersection of FA and DB. Let I be the intersection of AD and BE.
Angle FGD = 108°. Angle ECB = 103°. Angle DIE = 56°.
Find angle AHB.

Example 185



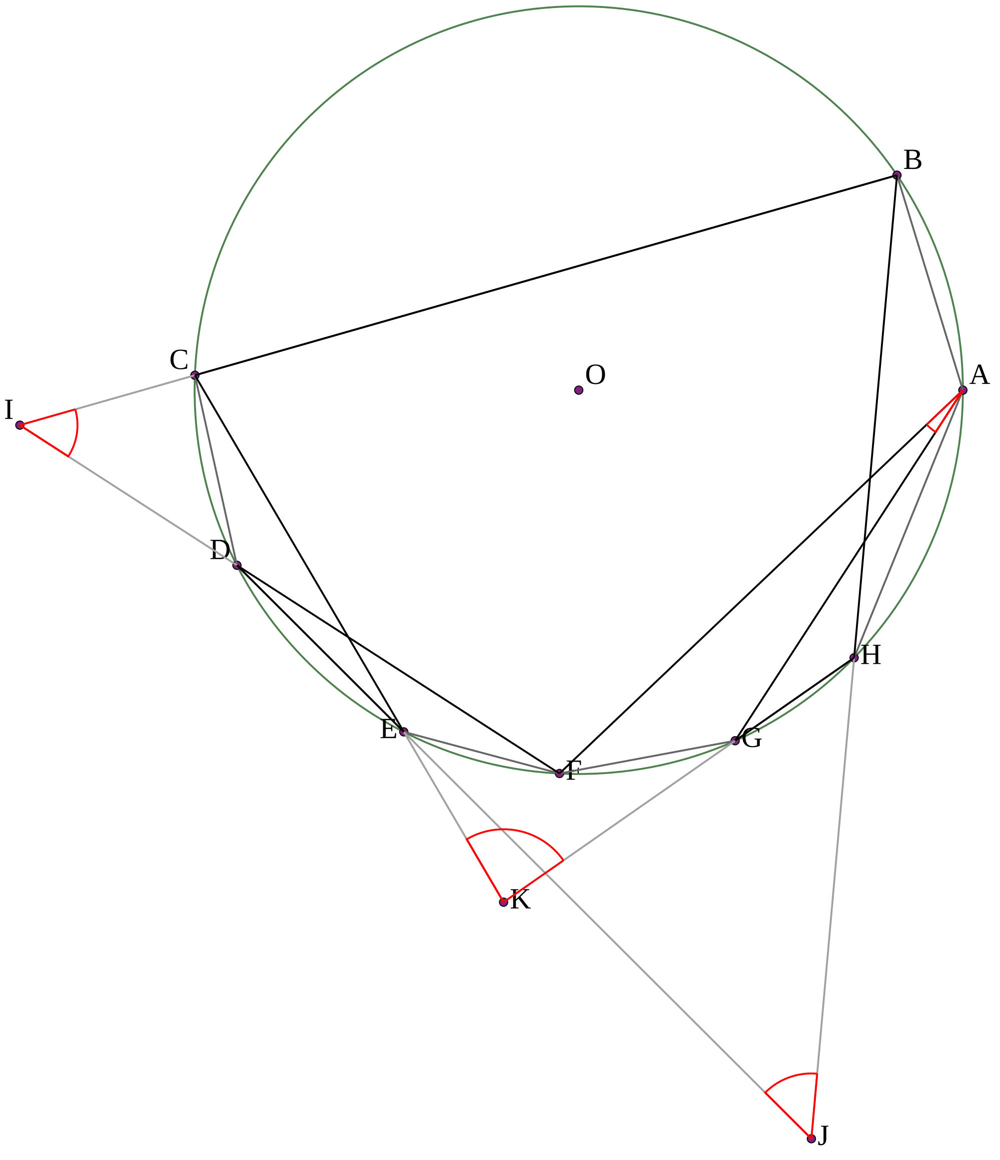
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of OF and AB. Let I be the intersection of FC and DG. Let J be the intersection of CA and ED. Let K be the intersection of BE and GF. Prove that BHF+CIG = CJD+EKF+90

Example 186



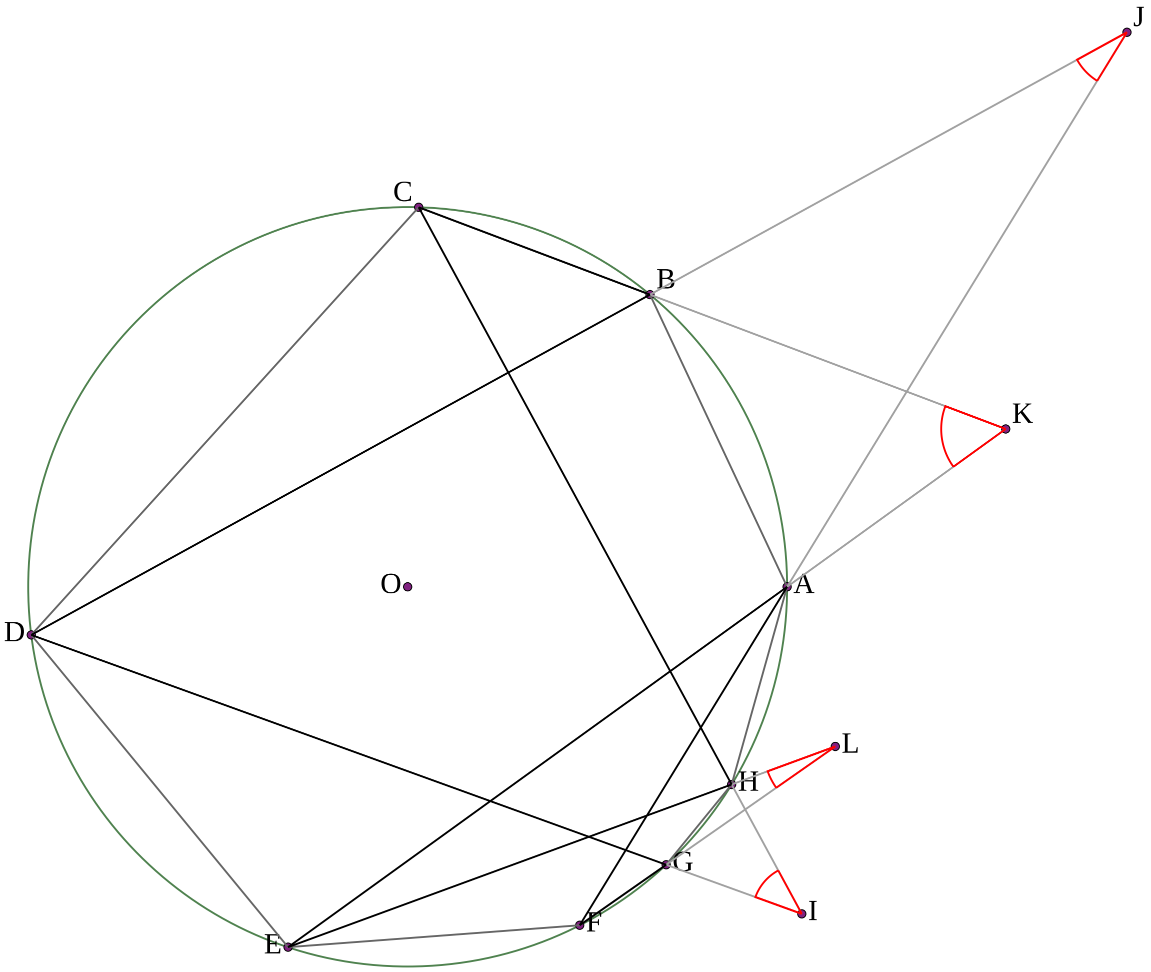
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of DF and CE. Let I be the intersection of FA and GB. Let J be the intersection of AC and GO. Let K be the intersection of EG and BD. Prove that CJG+DKE = EHF+AIB+90

Example 187



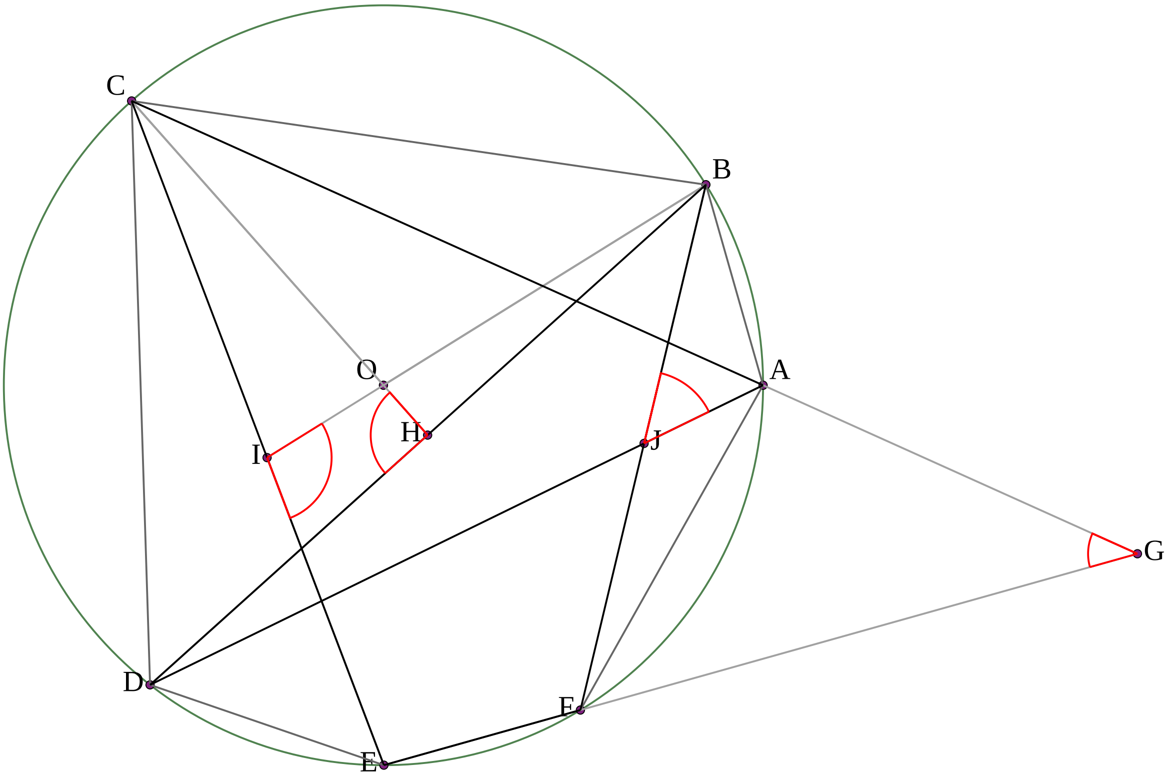
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FD and CB. Let J be the intersection of DE and BH. Let K be the intersection of EC and HG.
Angle GAF = x. Angle EKG = y. Angle DIC = z.
Find angle EJH.

Example 188



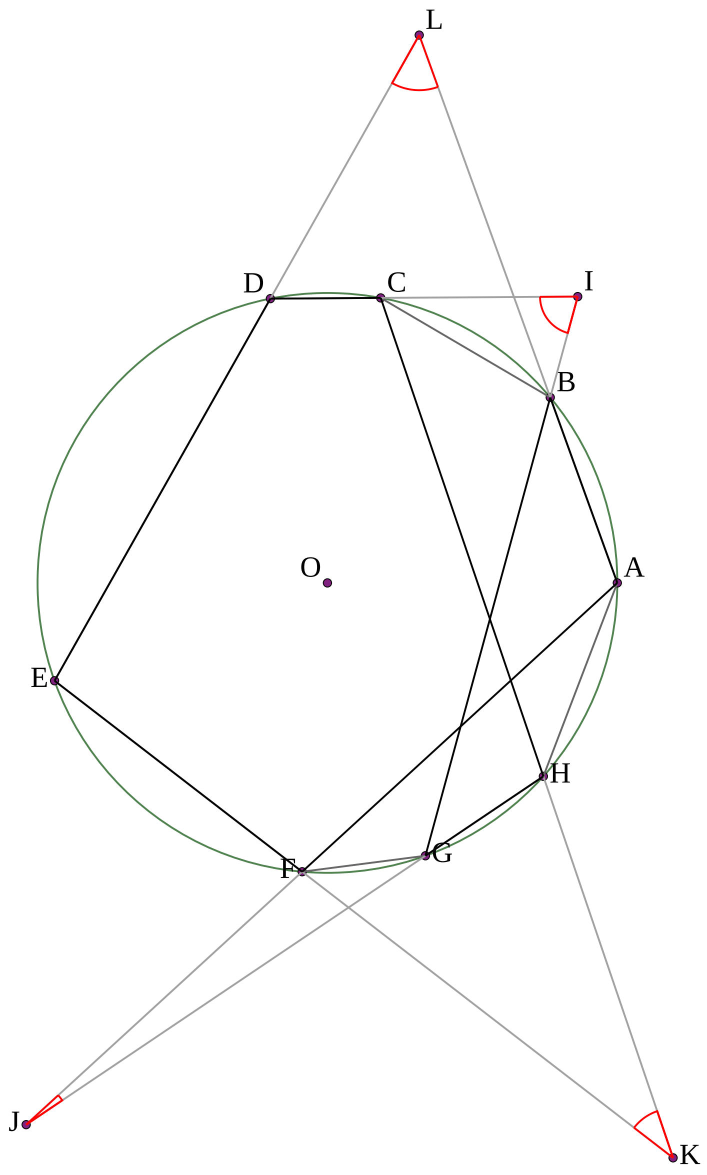
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GD and CH. Let J be the intersection of DB and AF. Let K be the intersection of BC and EA. Let L be the intersection of HE and FG.
Angle GIH = x. Angle BJA = y. Angle BKA = z.
Find angle HLG.

Example 189



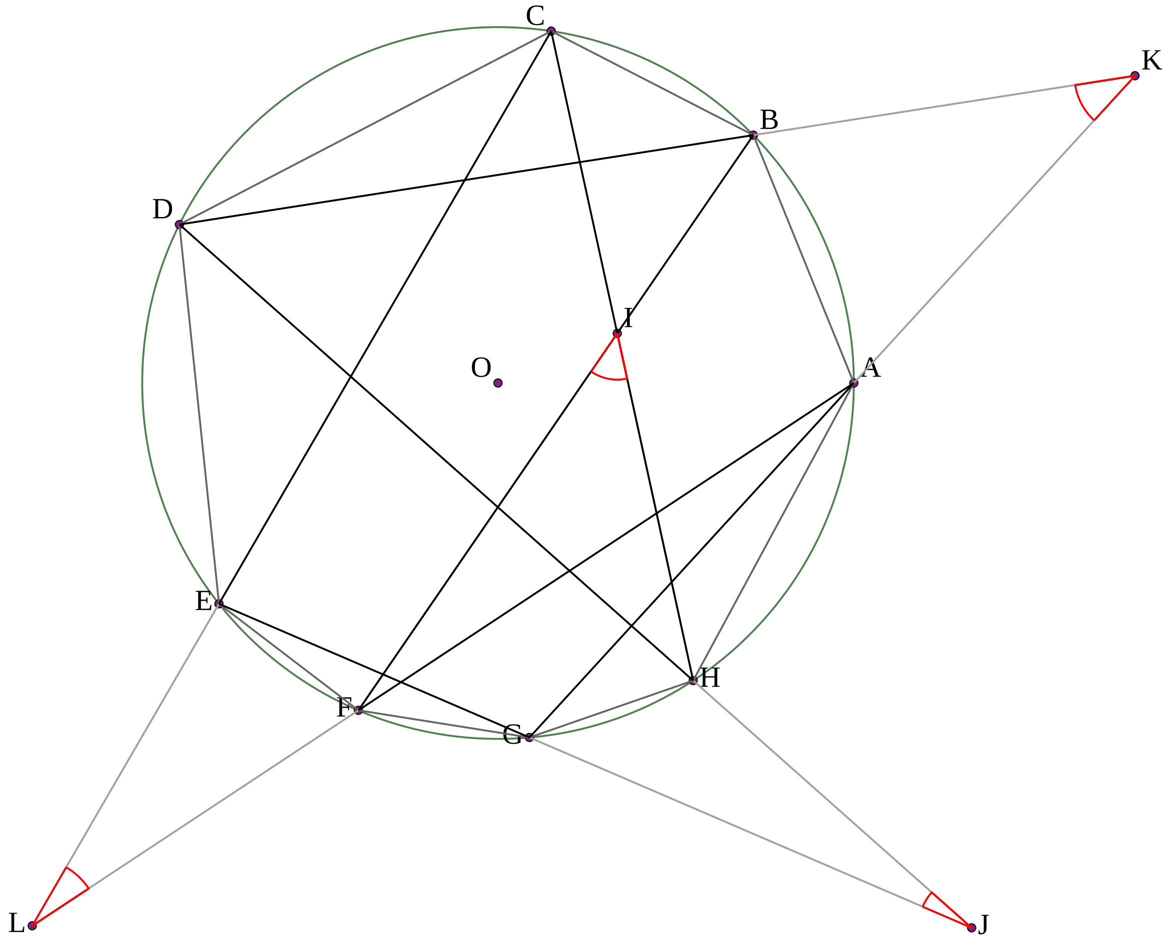
Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AC and EF. Let H be the intersection of OC and BD. Let I be the intersection of CE and BO. Let J be the intersection of FB and DA.
Angle AGF = x. Angle EIB = y. Angle BJA = z.
Find angle CHD.

Example 190



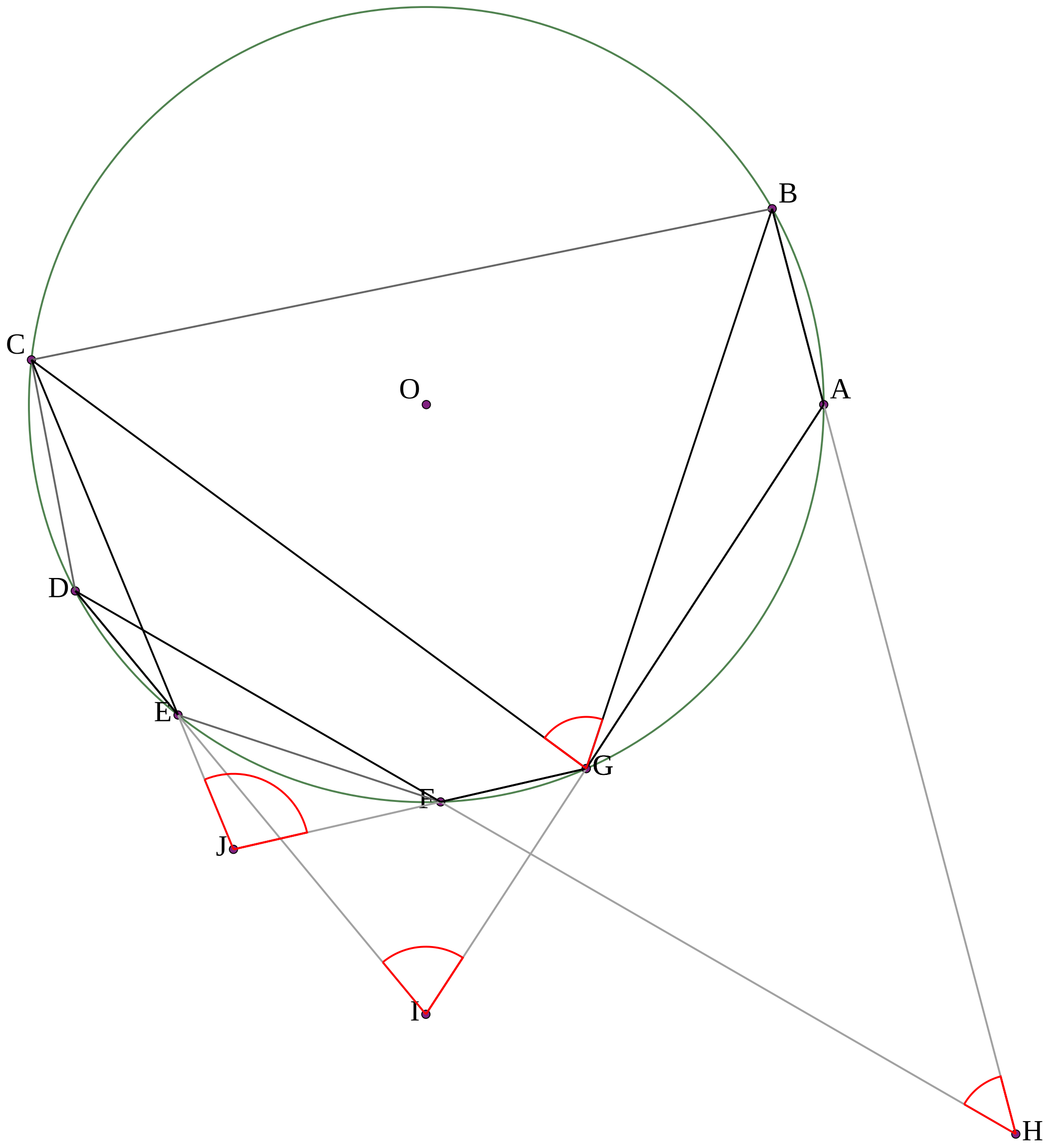
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of BG and CD. Let J be the intersection of GH and FA. Let K be the intersection of HC and EF. Let L be the intersection of DE and AB. Angle HKF = 34°. Angle DLB = 49°. Angle BIC = 74°.
Find angle GJF.

Example 191



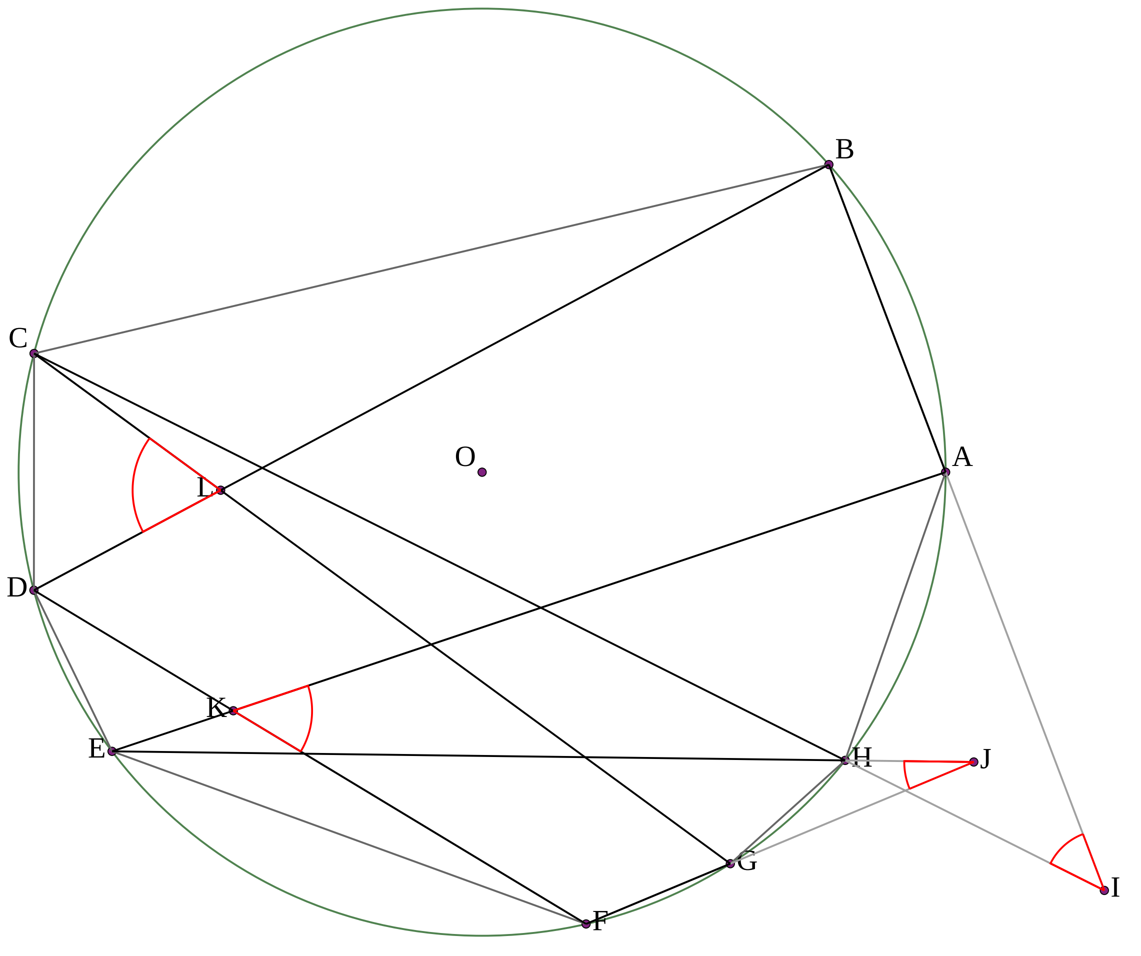
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CH and BF. Let J be the intersection of HD and GE. Let K be the intersection of DB and AG. Let L be the intersection of FA and EC.
Angle HIF = x. Angle HJG = y. Angle BKA = z.
Find angle FLE.

Example 192



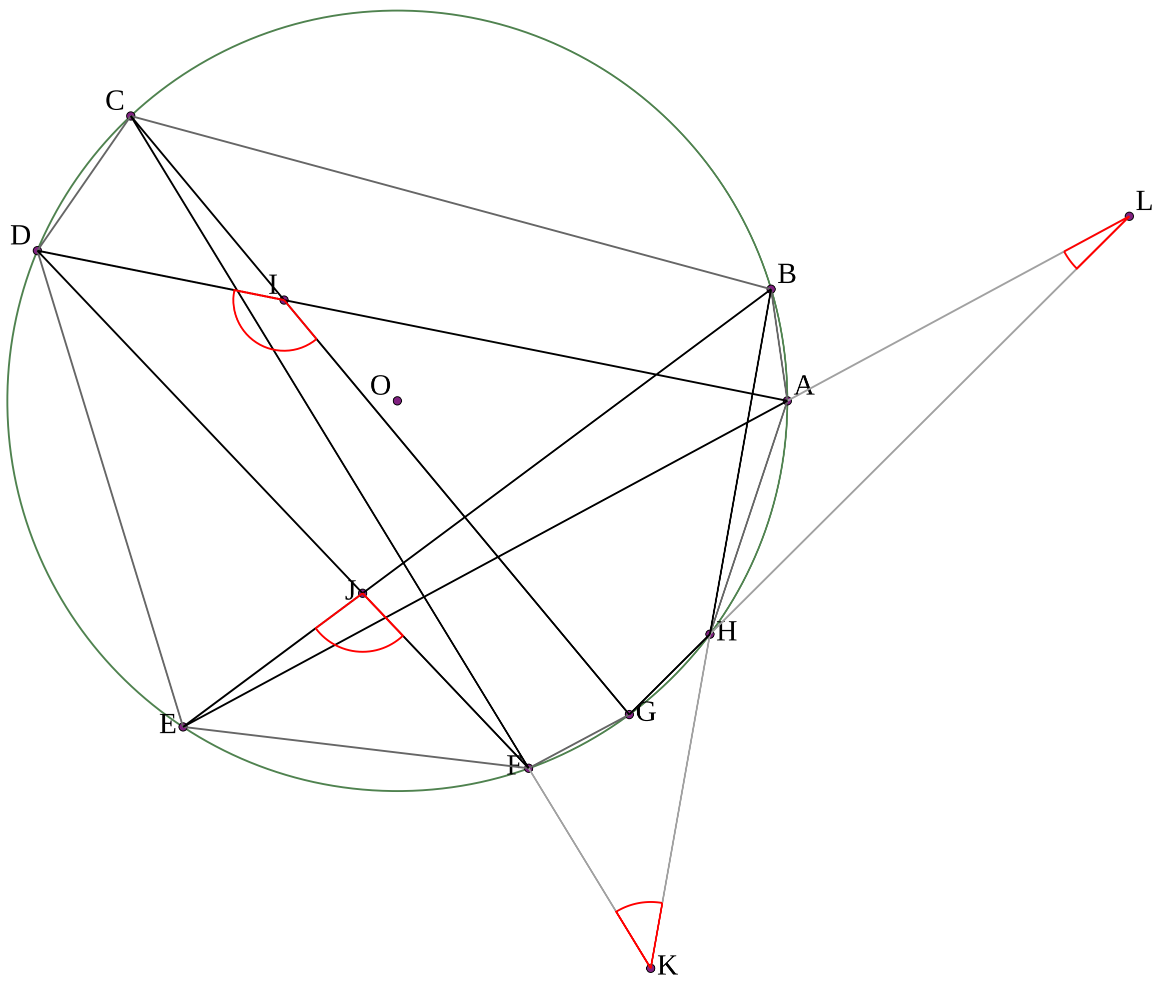
Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of BA and FD. Let I be the intersection of AG and DE. Let J be the intersection of GF and EC.
Prove that BGC+EIG = AHF+EJF

Example 193



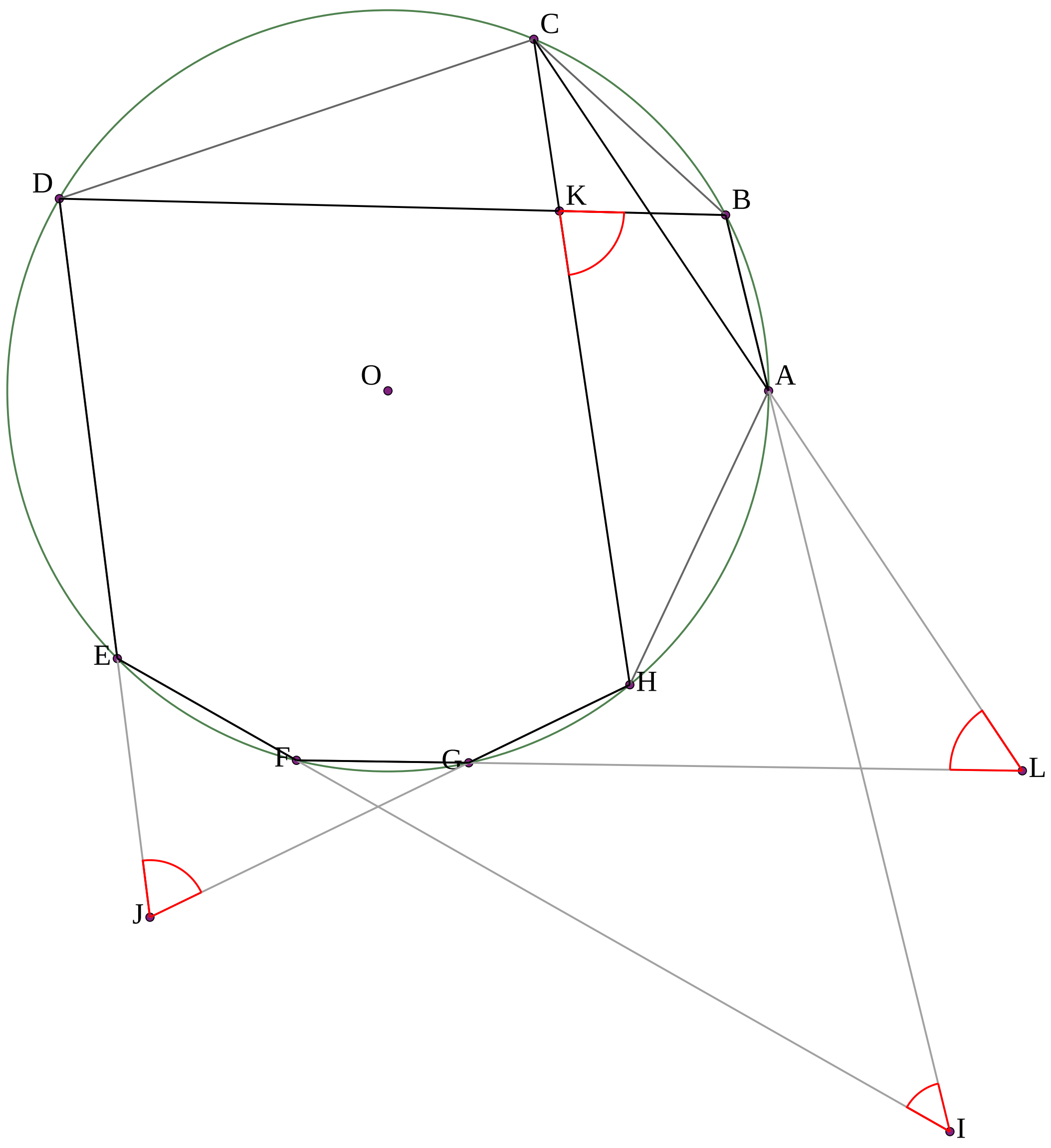
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CH and AB. Let J be the intersection of HE and FG. Let K be the intersection of EA and DF. Let L be the intersection of BD and GC. Prove that AIH+GJH+AKF+CLD = 180

Example 194



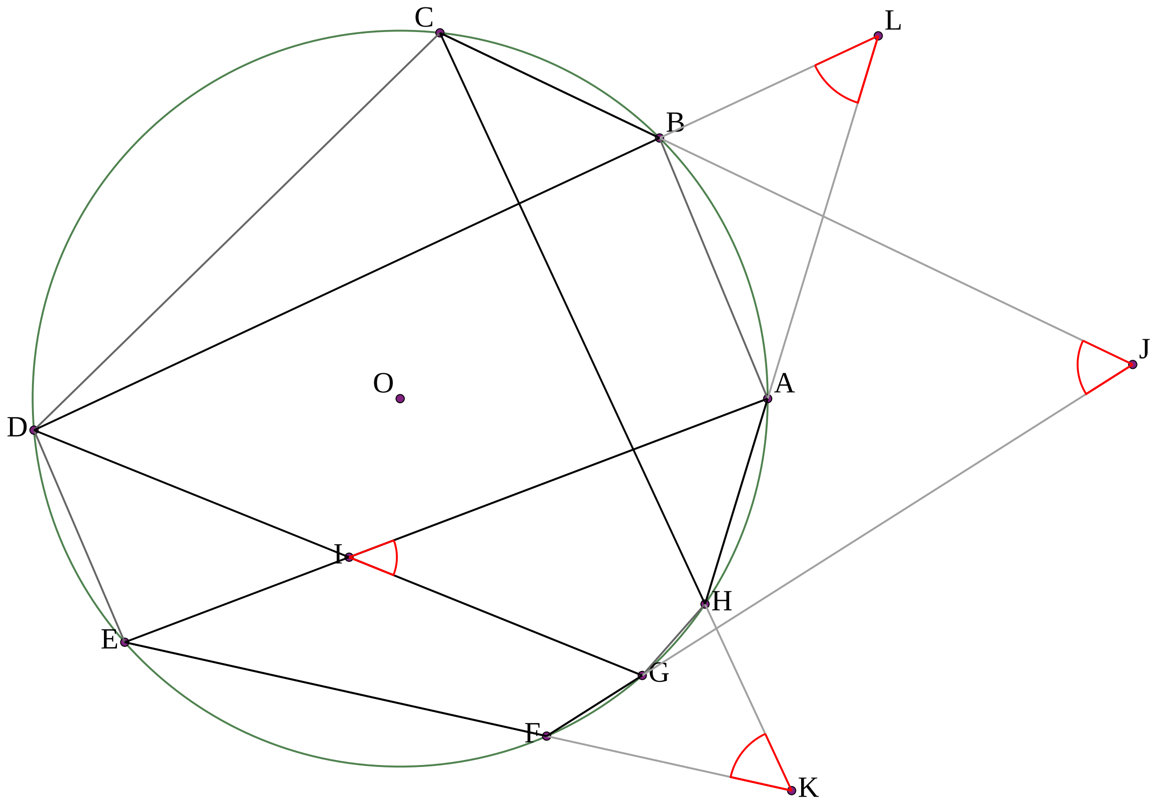
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of AD and CG. Let J be the intersection of DF and BE. Let K be the intersection of FC and HB. Let L be the intersection of GH and EA. Prove that DIG+EJF = FKH+ALH+180

Example 195



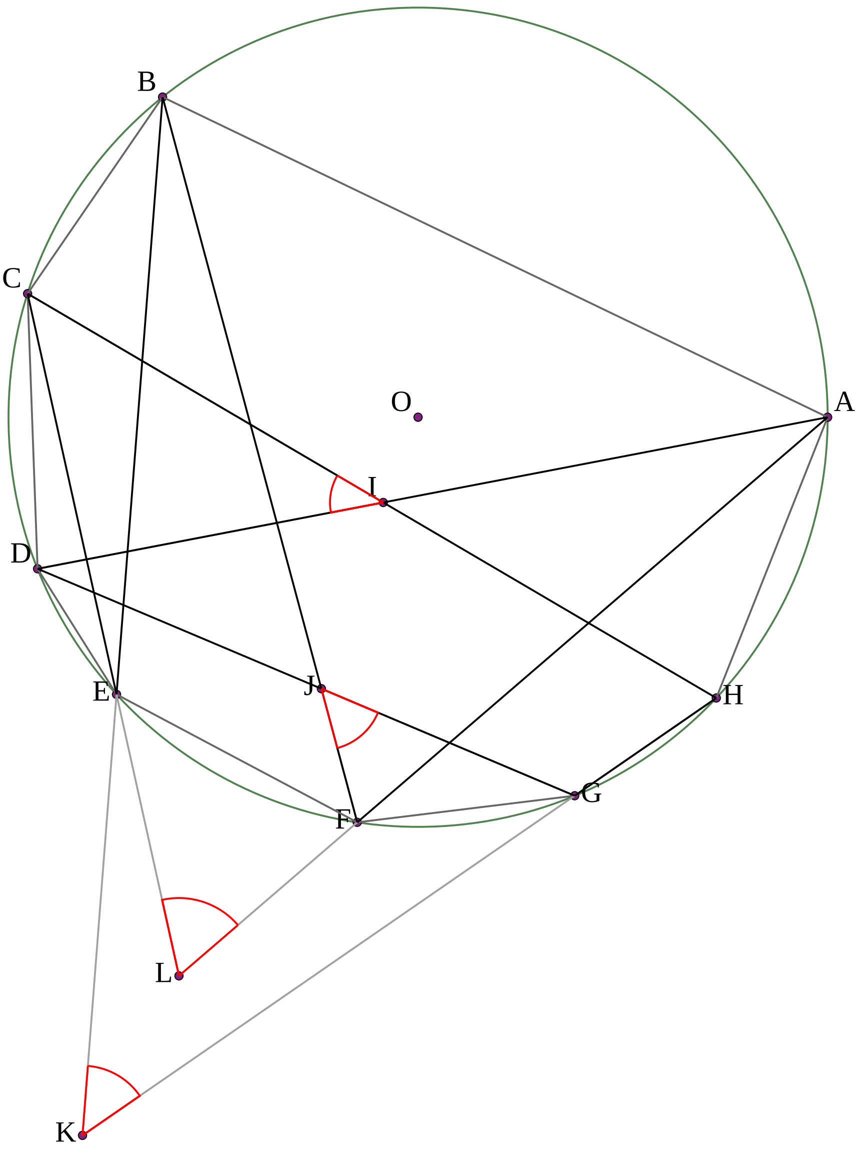
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FE and BA. Let J be the intersection of ED and HG. Let K be the intersection of DB and CH. Let L be the intersection of AC and GF.
Angle EJG = 71°. Angle ALG = 55°. Angle FIA = 47°.
Find angle BKH.

Example 196



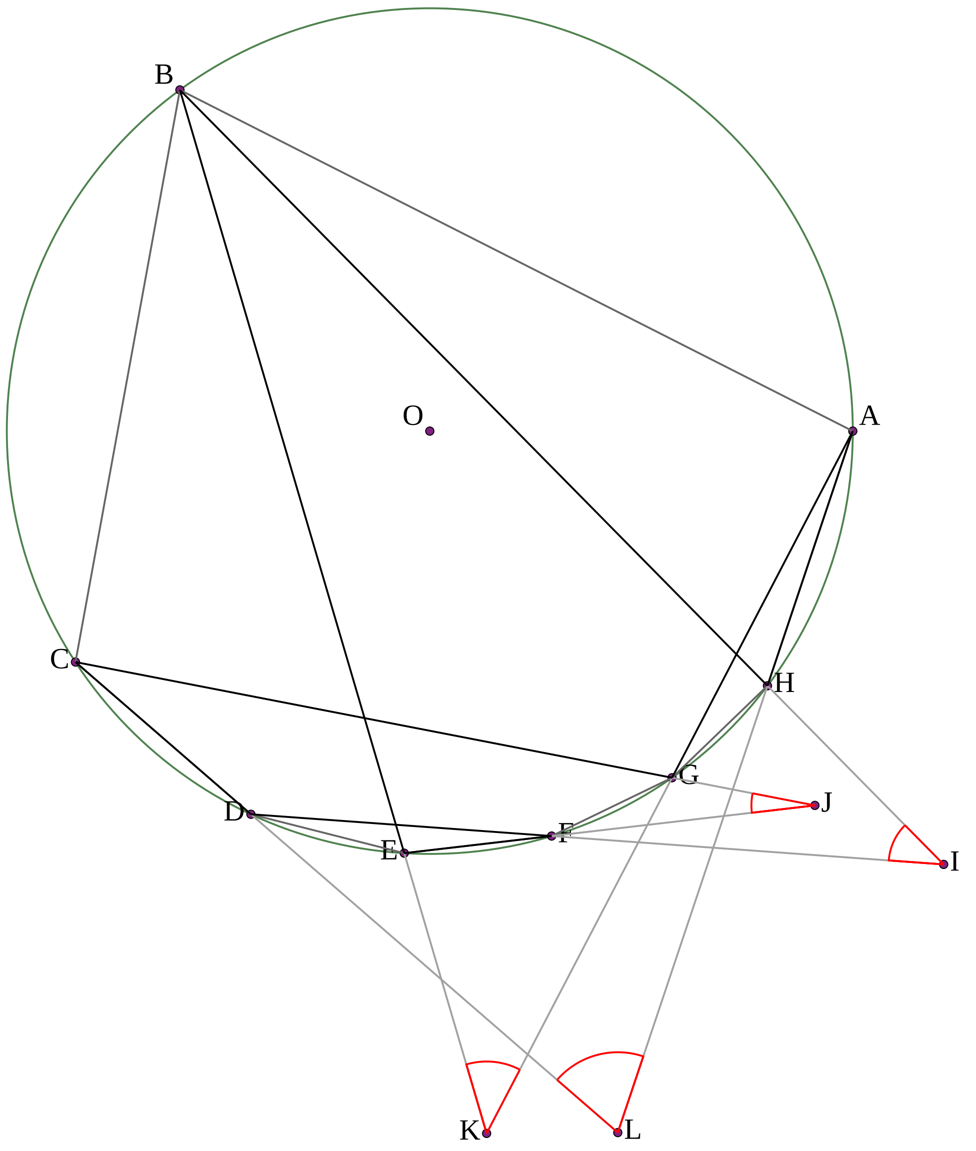
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DG and EA. Let J be the intersection of GF and CB. Let K be the intersection of FE and HC. Let L be the intersection of AH and BD.
Angle GIA = x. Angle GJB = y. Angle ALB = z.
Find angle FKH.

Example 197



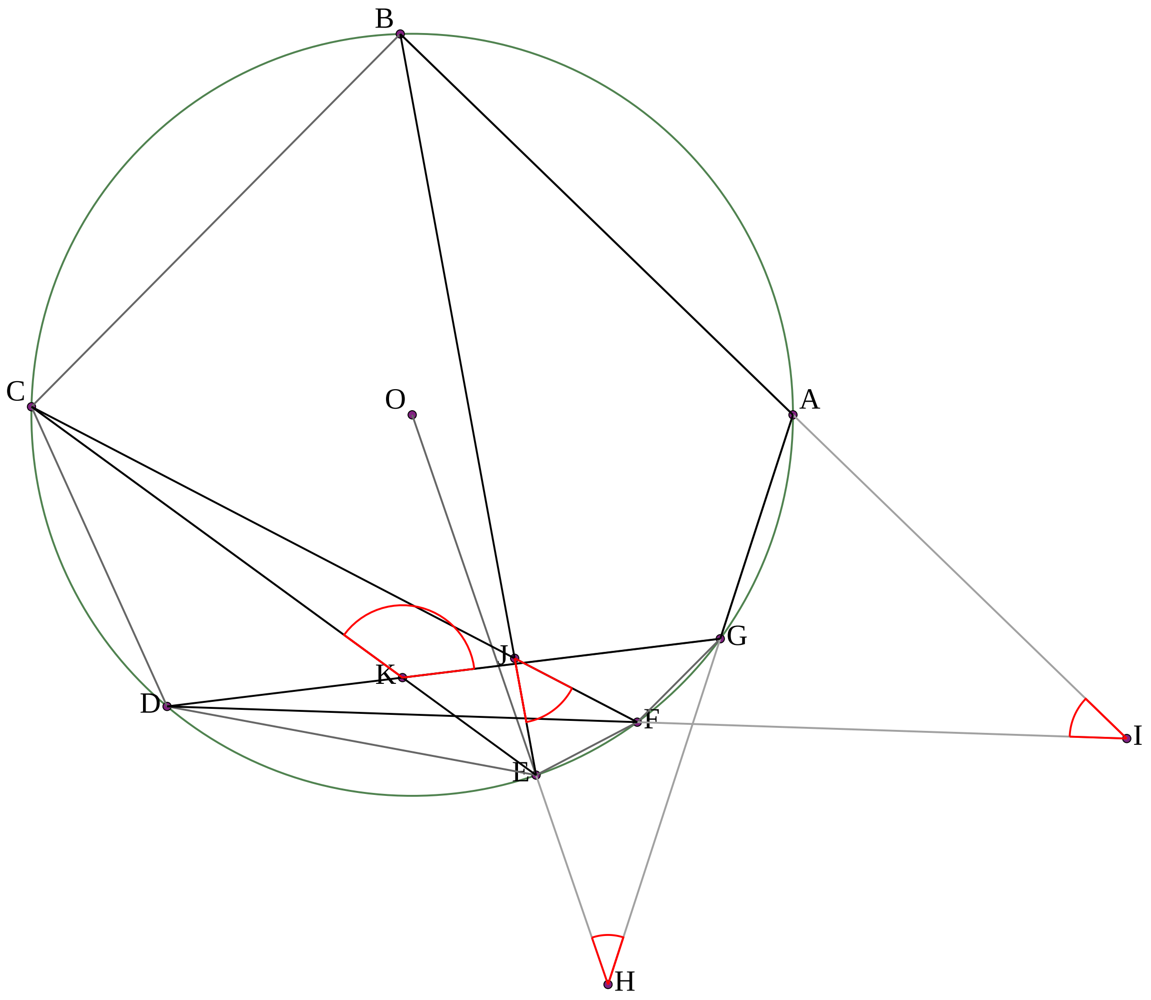
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of AD and HC. Let J be the intersection of DG and BF. Let K be the intersection of GH and EB. Let L be the intersection of CE and FA. Prove that CID+ELF = FJG+EKG

Example 198



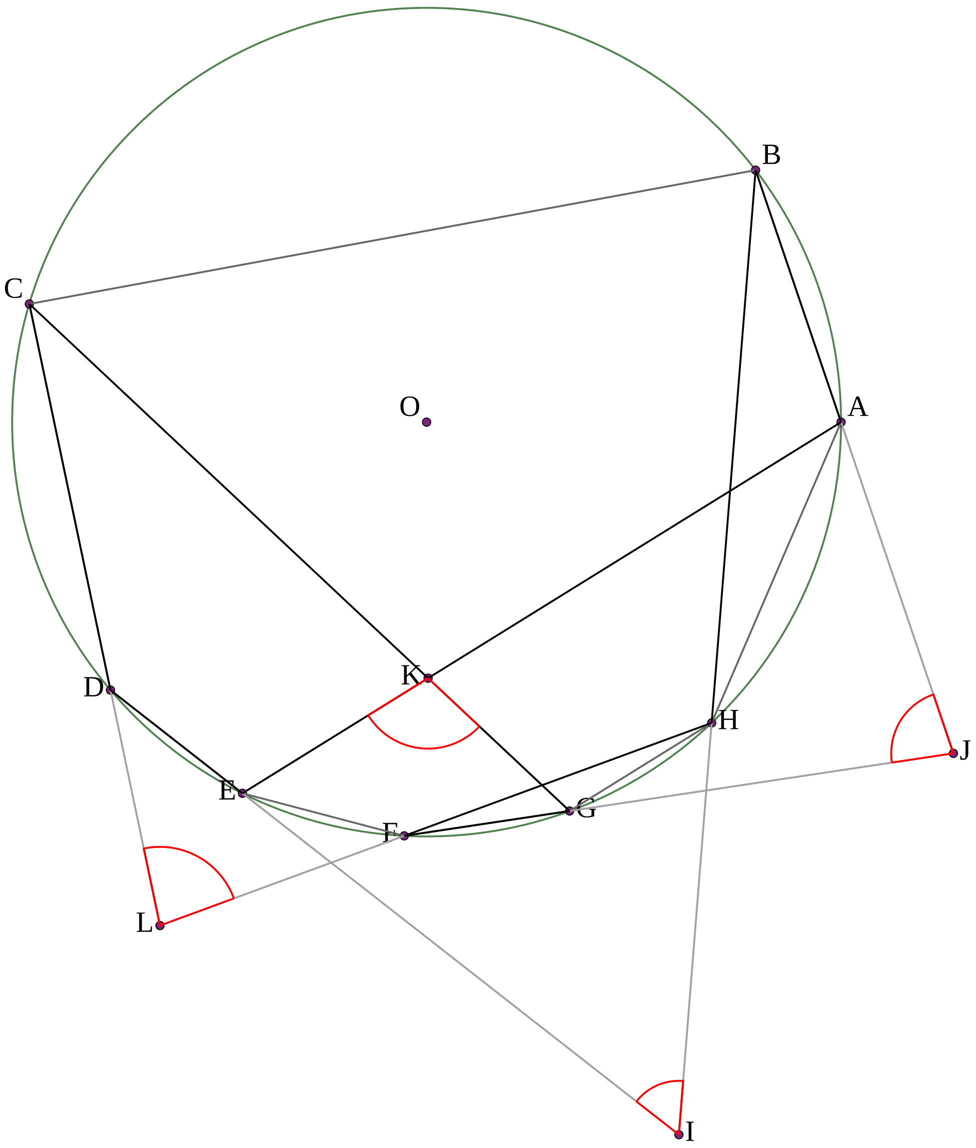
Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DF and BH. Let J be the intersection of FE and GC. Let K be the intersection of EB and AG. Let L be the intersection of HA and CD. Prove that FIH+EKG = FJG+DLH

Example 199



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GA and EO. Let I be the intersection of AB and FD. Let J be the intersection of BE and CF. Let K be the intersection of EC and DG. Prove that AIF+CKG = EHG+EJF+90

Example 200

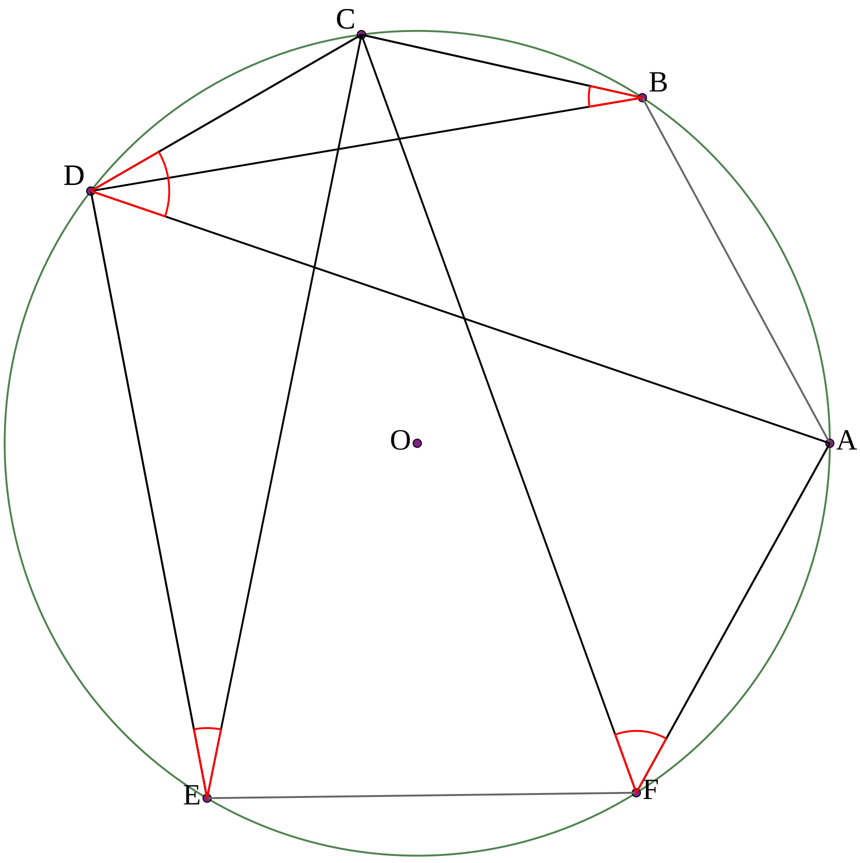


Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of HB and ED. Let J be the intersection of BA and GF. Let K be the intersection of AE and CG. Let L be the intersection of DC and FH. Prove that AJG+DLF = EIH+EKG

Answers

1. DBC = 22°
2. ECB = 180-z
3. EBF = 21°
4. DCA = 180-y
5. CBF = 70°
6. EDF = 19°
7. EFA = 105°
8. CDA = x+y-z
10. CBA = x-y-z+270
15. EAC = x+y-z
16. BCD = 127°
17. GAH = x-y-z
18. HFG = 180-x-y-z
19. HGE = 132°
20. ACD = 99°
22. OFB = 34°
23. DAE = x-y-z+90
24. CFG = x-y-z
25. CFD = x+y-z
27. GCA = x+y+z-360
28. GED = 103°
29. FEH = 35°
30. DEF = 360-x-y-z
31. FAE = 19°
33. DBE = x-y-z+90
34. CFE = 52°
36. EAC = 55°
38. BDO = 48°
39. GHA = 119°
44. OBC = x-y-z+270
46. DAO = x+y-z-90
53. CEF = 108°
55. ECD = 21°
56. ECD = 20°
57. AHB = 76°
58. FCE = 17°
59. DEF = 131°
60. AHG = x+y+z-90
61. CAF = 61°
62. DBO = 33°
63. BDC = 22°
64. CBF = 107°
65. BIF = x+y+z-90
67. FBD = 29°
68. CFA = x-y-z+180
69. DFC = x+y-z
70. FHD = 41°
72. GHF = x+y-z-180
73. BHE = 67°
74. EHG = 27°
76. FCE = x+y-z
77. DCB = x-y-z+180
78. DAC = 180-x-y-z
80. DGE = x+y-z
81. CGE = 270-x-y-z
82. DCE = x-y-z
83. ABF = 60°
86. ODE = x+y-z+90
87. BAF = x+y-z
89. ADG = 44°
90. EJG = x+y-z
91. CDE = 270-x-y-z
92. EDB = x+y+z-90
93. EIF = 76°
96. AFO = x+y-z
97. BHE = x-y-z+180
98. BEG = 64°
99. EBF = x-y-z+90
100. FCA = 51°
101. CID = 47°
102. CED = x-y-z
104. GHC = 180-x-y-z
106. FHG = x+y-z
107. DCH = 95°
108. FIA = 56°
110. EHD = 51°
111. AGE = 141°
112. FBC = 72°
115. DEB = 270-x-y-z
117. AEC = 36°
118. ECD = x+y+z-90
119. CHE = 89°
120. EDF = x-y-z+180
121. ODE = 75°
122. FBA = 43°
123. AJF = 61°
126. DFE = x+y-z
130. FHD = x-y-z+180
132. GJF = x+y-z
133. FHA = x-y-z
135. AIG = x+y-z
136. GBE = x+y-z
137. FHB = x-y-z+90
140. DHF = 46°
144. EIF = 83°
146. FHE = x-y-z+180
147. EID = x+y+z-90
148. EIF = 66°
149. FIG = x+y+z-180
150. DIF = 180-x-y-z
152. AJF = x+y-z
153. BJE = x-y-z+90
154. AIG = 146°
155. AJG = 40°
156. OFC = 30°
157. AGB = 141°
159. GJF = x+y-z-90
161. DJG = 136°
163. DFA = 72°
164. FKA = x+y-z
165. AIE = x+y-z+90
166. FIC = x+y-z-180
168. CIA = x-y-z+90
169. BJH = x-y-z+180
170. CKE = 74°
172. EJB = x-y-z+90
173. CKB = 51°
174. GJF = 49°
175. BJC = 35°
176. HIG = x+y-z
177. BJA = 66°
178. BKA = 66°
180. BJC = 43°
181. BIC = 43°
182. FLH = x-y-z+180
183. BJA = 20°
184. AHB = 39°
187. EJH = x+y-z
188. HLG = x+y-z
189. CHD = x+y-z
190. GJF = 9°
191. FLE = x+y-z
195. BKH = 79°
196. FKH = x+y-z

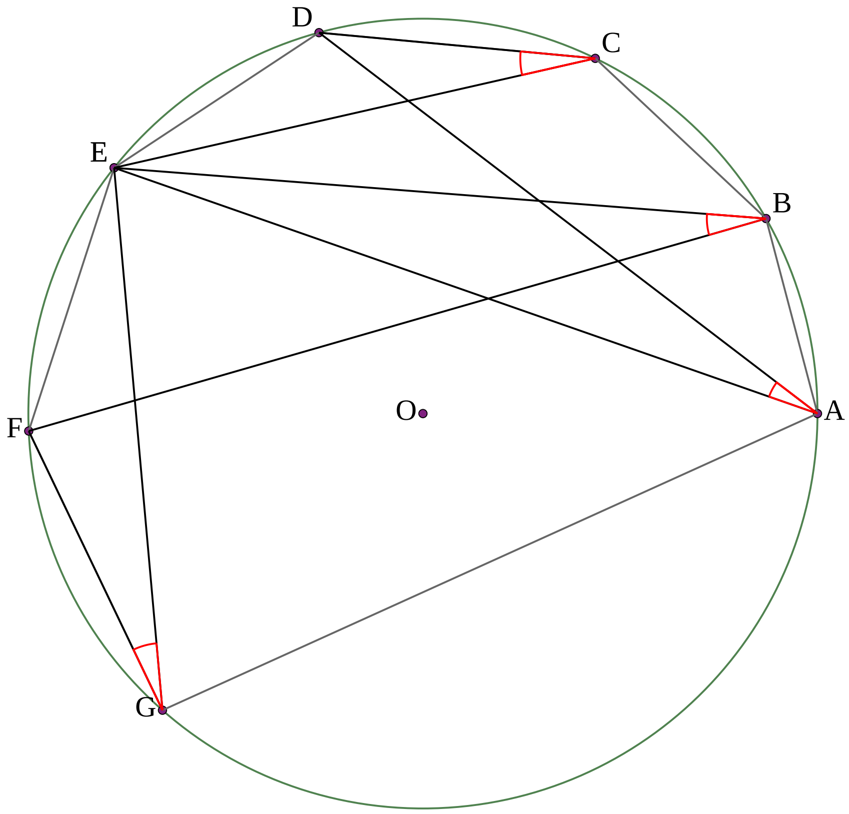
Solution to example 1



Let ABCDEF be a cyclic hexagon with center O.
Angle DEC = 22°. Angle ADC = 49°. Angle CFA = 49°.
Find angle DBC.

As CED and CBD stand on the same chord, CBD=CED, so CBD=22.

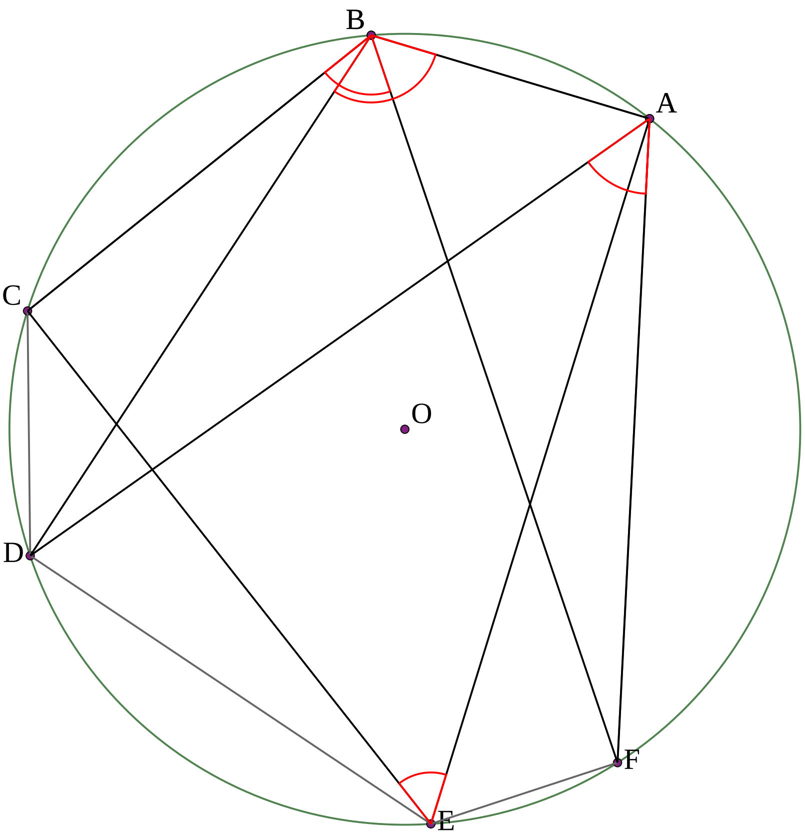
Solution to example 3



Let ABCDEFG be a cyclic heptagon with center O.
Angle FGE = 21°. Angle ECD = 18°. Angle DAE = 18°.
Find angle EBF.

As EGF and EBF stand on the same chord, EBF=EGF, so EBF=21.

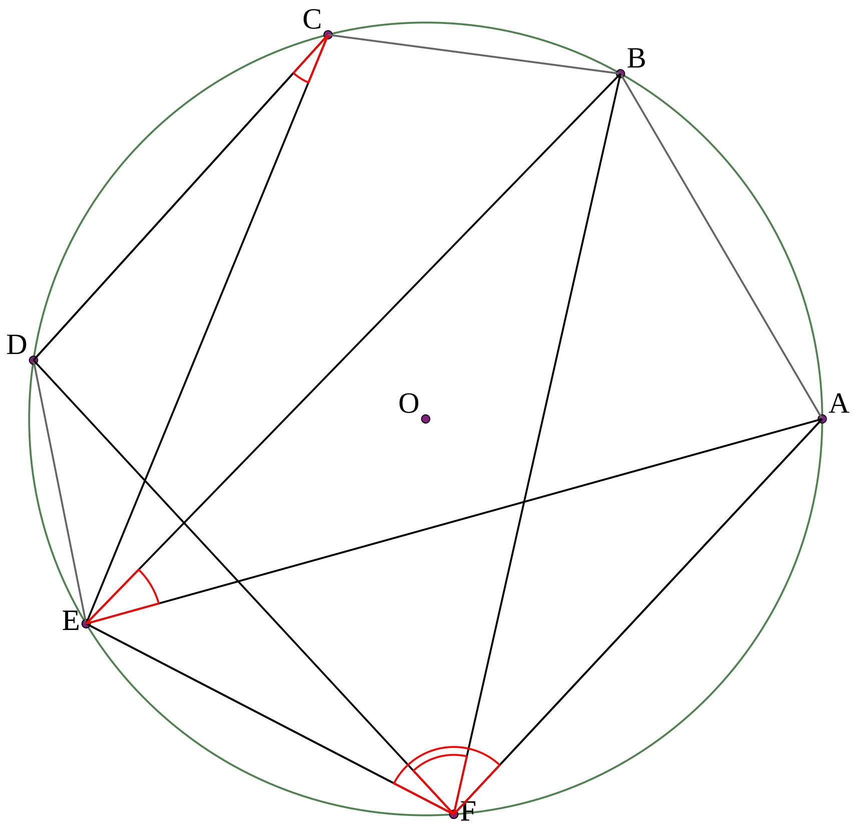
Solution to example 5



Let ABCDEF be a cyclic hexagon with center O.
Angle FAD = 52°. Angle AEC = 55°. Angle ABD = 107°.
Find angle CBF.

As AECB is a cyclic quadrilateral, ABC=180-AEC, so ABC=125.
As DAF and DBF stand on the same chord, DBF=DAF, so DBF=52.
As ABD=107, ABF=55.
As ABC=125, CBF=70.

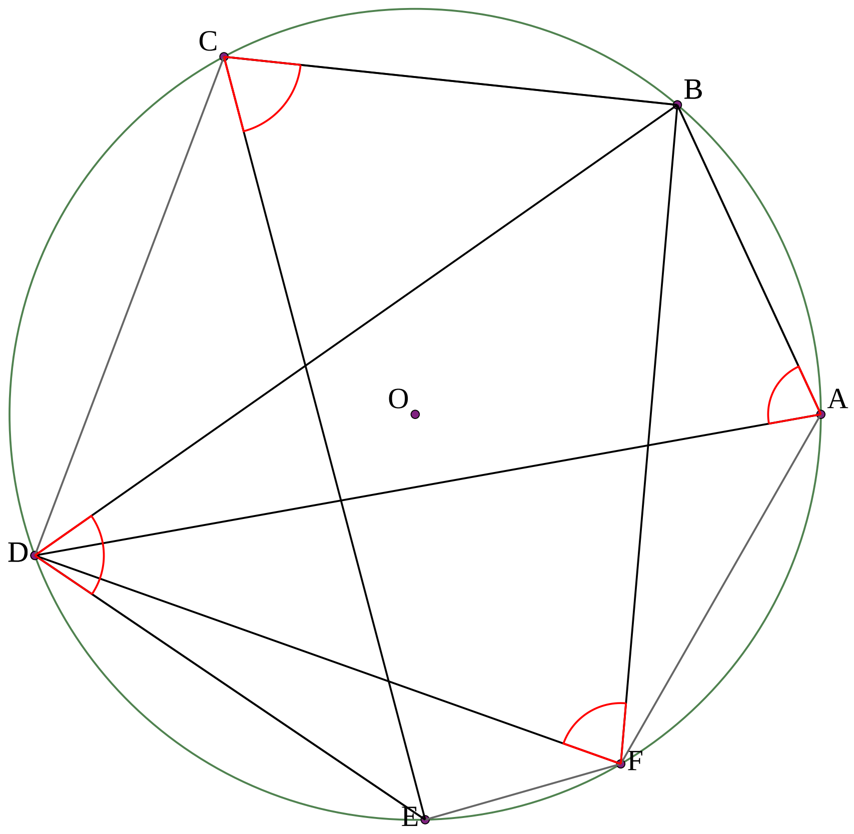
Solution to example 7



Let ABCDEF be a cyclic hexagon with center O.
Angle ECD = 20°. Angle DFB = 55°. Angle BEA = 30°.
Find angle EFA.

As DCE and DFE stand on the same chord, DFE=DCE, so DFE=20.
As AEB and AFB stand on the same chord, AFB=AEB, so AFB=30.
As BFD=55, DFA=85.
As DFE=20, EFA=105.

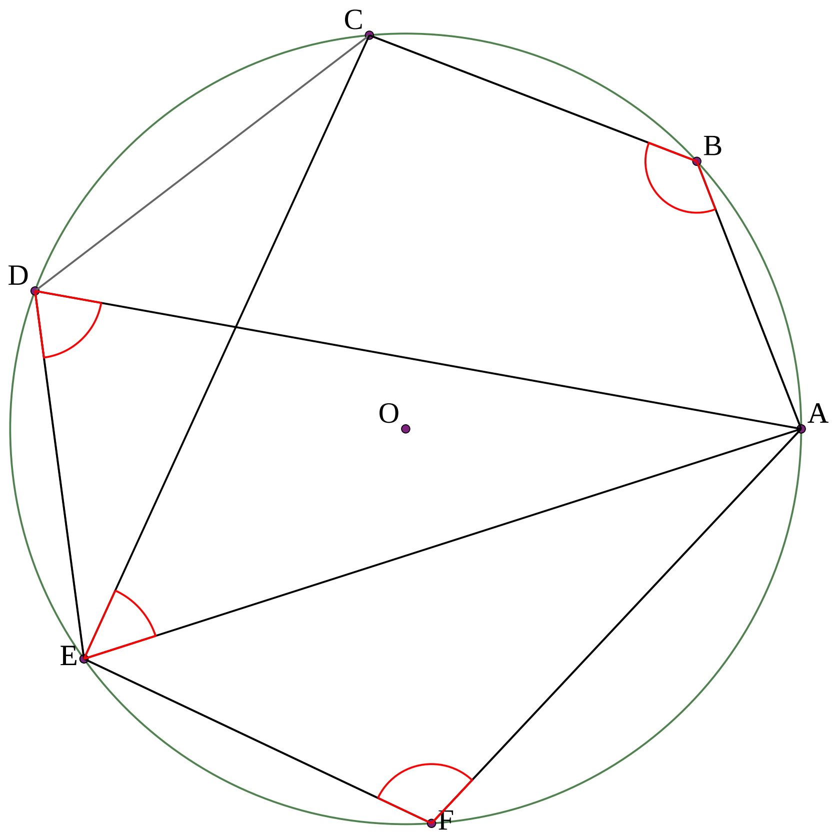
Solution to example 9



Let ABCDEF be a cyclic hexagon with center O.
Prove that BFD+BDE = BAD+BCE

Let BAD=x. Let BFD=y. Let BDE=z. Let BCE=w.
As BDE and BCE stand on the same chord, BCE=BDE, so BCE=z.
But BCE=w, so z=w.
As BAD and BFD stand on the same chord, BFD=BAD, so BFD=x.
But BFD=y, so x=y.
We have these equations: w-z=0 (E1), y-x=0 (E2).
Hence y+z-x-w=0 (E2-E1), or y+z=x+w, or BFD+BDE=BAD+BCE.

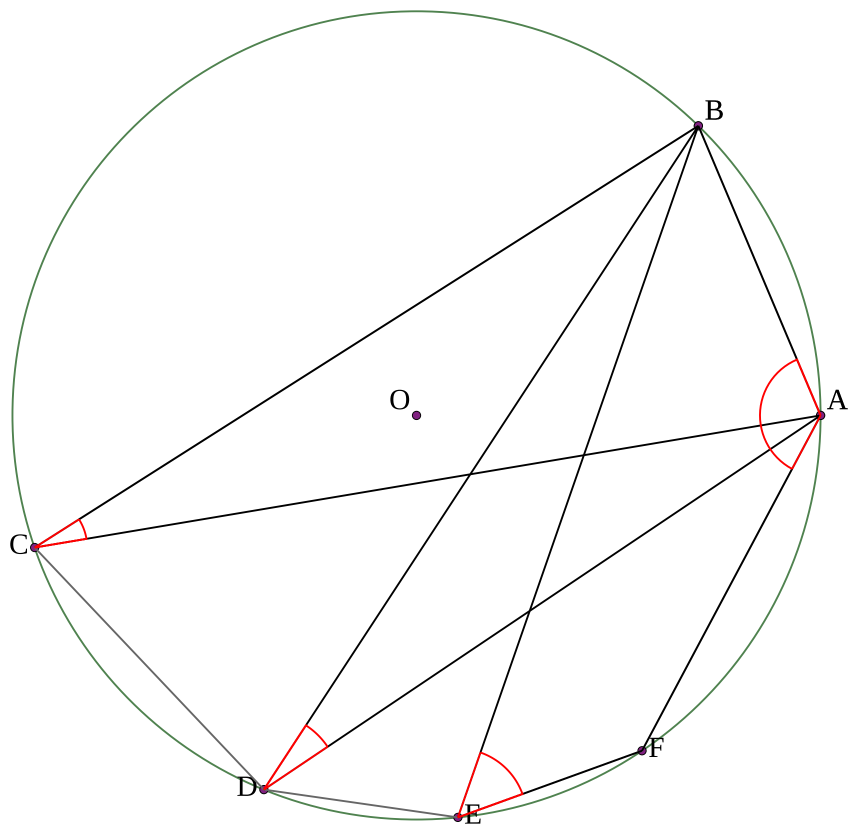
Solution to example 11



Let ABCDEF be a cyclic hexagon with center O.
Prove that ABC+AFE+ADE+AEC = 360

Let ABC=x. Let AFE=y. Let ADE=z. Let AEC=w.
As AFED is a cyclic quadrilateral, ADE=180-AFE, so ADE=180-y.
But ADE=z, so 180-y=z, or 180=y+z.
As ABCE is a cyclic quadrilateral, AEC=180-ABC, so AEC=180-x.
But AEC=w, so 180-x=w, or 180=x+w.
We have these equations: y+z=180 (E1), x+w=180 (E2).
Hence x+y+z+w=360 (E1+E2), or ABC+AFE+ADE+AEC=360.

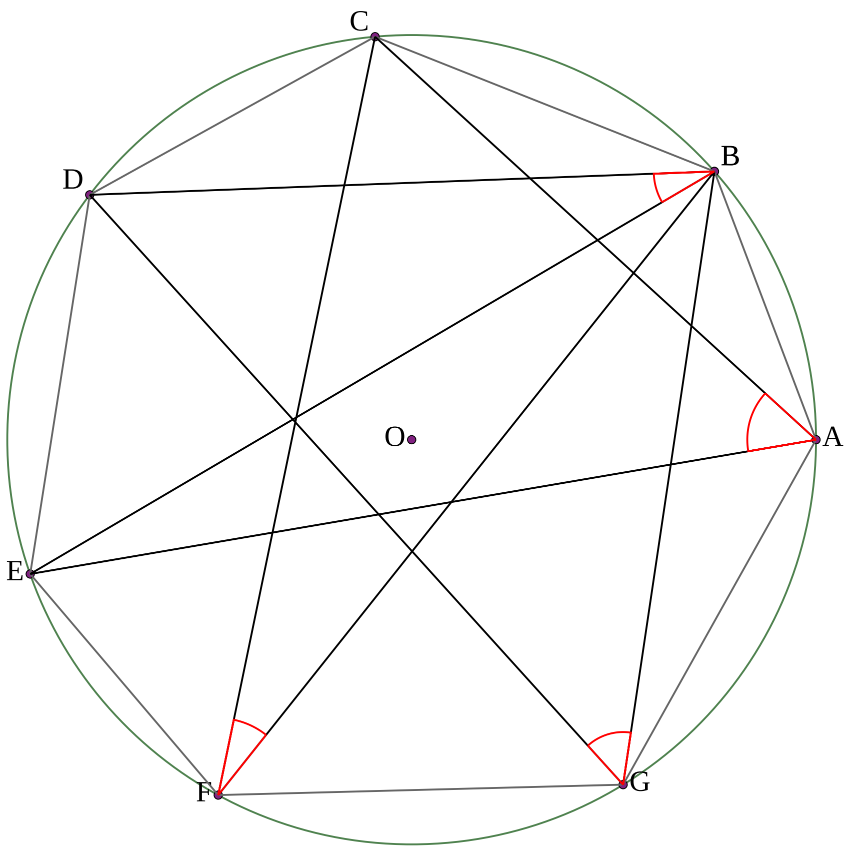
Solution to example 13



Let ABCDEF be a cyclic hexagon with center O.
Prove that ACB+BAF+BEF = ADB+180

Let ACB=x. Let BAF=y. Let BEF=z. Let ADB=w.
As BAFE is a cyclic quadrilateral, BEF=180-BAF, so BEF=180-y.
But BEF=z, so 180-y=z, or 180=y+z.
As ACB and ADB stand on the same chord, ADB=ACB, so ADB=x.
But ADB=w, so x=w.
We have these equations: y+z=180 (E1), w-x=0 (E2).
Hence x+y+z-w=180 (E2-E1), or x+y+z=w+180, or ACB+BAF+BEF=ADB+180.

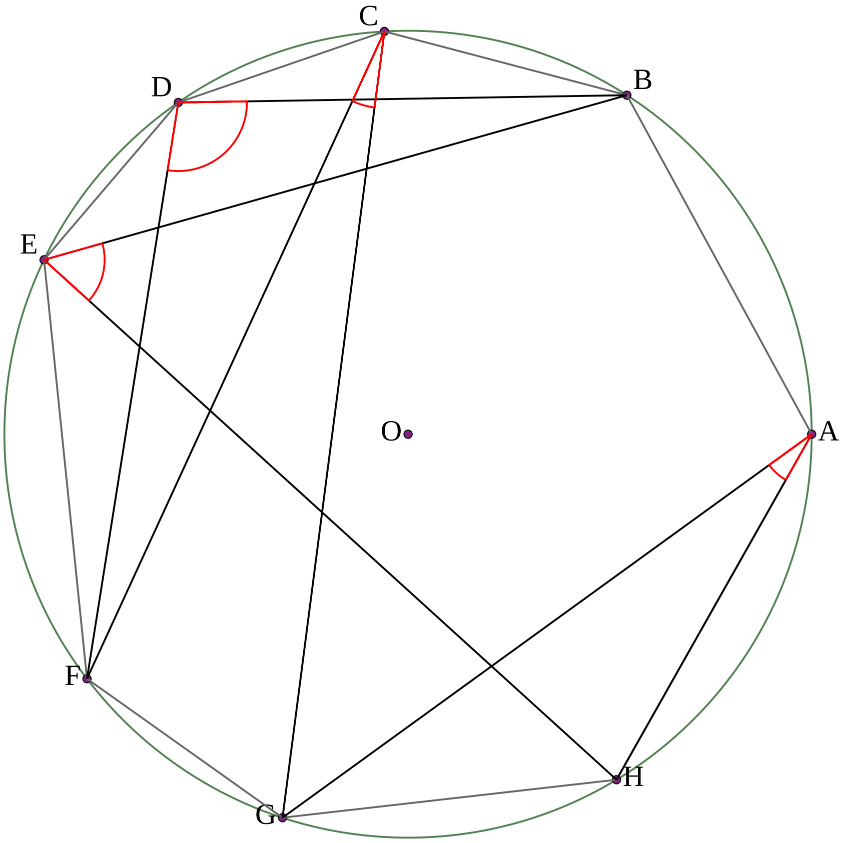
Solution to example 15



Let ABCDEFG be a cyclic heptagon with center O.
Angle BGD = x. Angle DBE = y. Angle CFB = z.
Find angle EAC.

As BFC and BAC stand on the same chord, BAC=BFC, so BAC=z.
As BGD and BED stand on the same chord, BED=BGD, so BED=x.
As DBE=y, BDE=180-x-y.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=x+y.
As BAC=z, CAE=x+y-z.

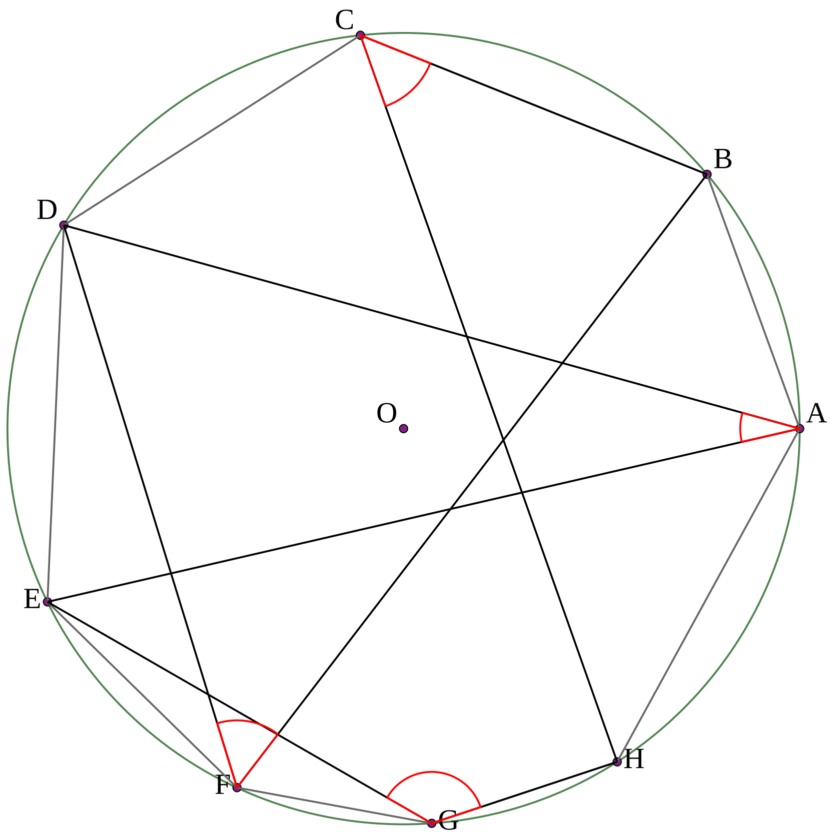
Solution to example 17



Let ABCDEFGH be a cyclic octagon with center O.
Angle BDF = x. Angle HEB = y. Angle FCG = z.
Find angle GAH.

As BEHA is a cyclic quadrilateral, BAH=180-BEH, so BAH=180-y.
As BDF and BCF stand on the same chord, BCF=BDF, so BCF=x.
As FCG=z, GCB=x-z.
As BCGA is a cyclic quadrilateral, BAG=180-BCG, so BAG=z-x+180.
As BAH=180-y, HAG=x-y-z.

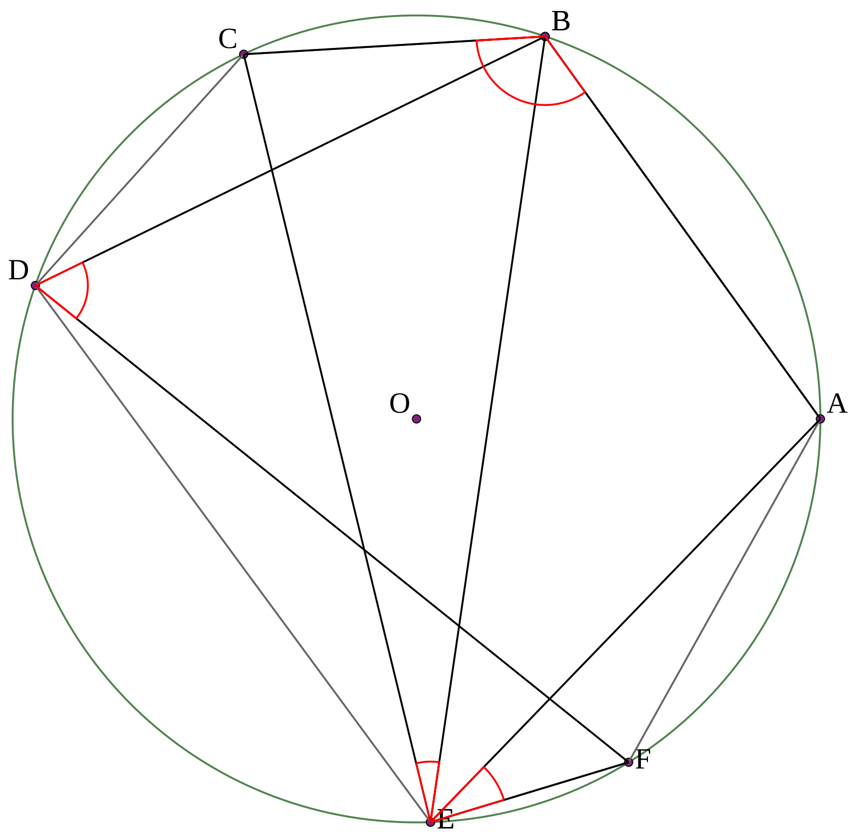
Solution to example 19



Let ABCDEFGH be a cyclic octagon with center O.
Angle EAD = 28°. Angle DFB = 55°. Angle BCH = 49°.
Find angle HGE.

As BCHA is a cyclic quadrilateral, BAH=180-BCH, so BAH=131.
As BFD and BAD stand on the same chord, BAD=BFD, so BAD=55.
As DAE=28, EAB=83.
As BAH=131, HAE=48.
As EAHG is a cyclic quadrilateral, EGH=180-EAH, so EGH=132.

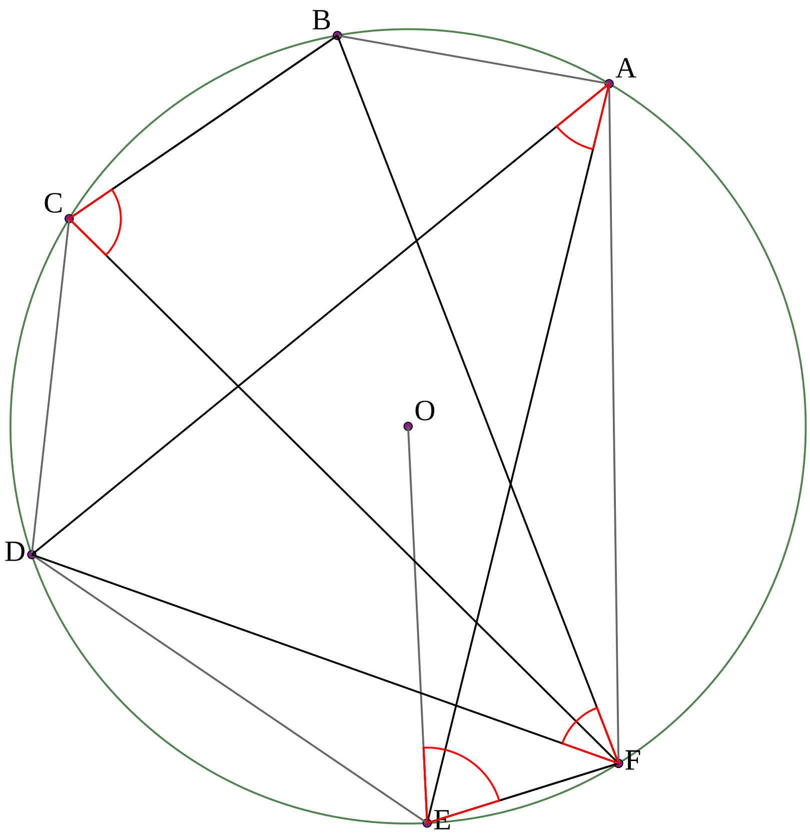
Solution to example 21



Let ABCDEF be a cyclic hexagon with center O.
Prove that BDF+BEC+ABC = AEF+180

Let BDF=x. Let BEC=y. Let ABC=z. Let AEF=w.
As ABCE is a cyclic quadrilateral, AEC=180-ABC, so AEC=180-z.
As BEC=y, BEA=180-y-z.
As BDF and BEF stand on the same chord, BEF=BDF, so BEF=x.
As AEF=w, AEB=x-w.
But AEB=180-y-z, so x-w=180-y-z, or x+y+z=w+180, or BDF+BEC+ABC=AEF+180.

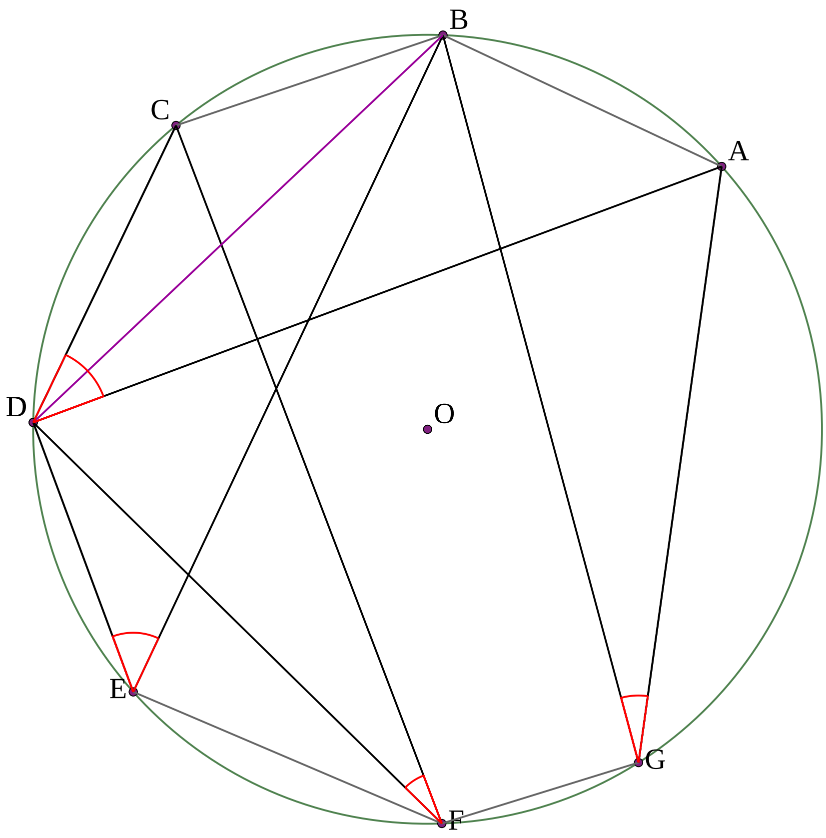
Solution to example 23



Let ABCDEF be a cyclic hexagon with center O.
Angle OEF = x. Angle FCB = y. Angle BFD = z.
Find angle DAE.

As triangle FEO is isosceles, EOF=180-2x.
As EOF is at the center of a circle on the same chord as EAF, EOF=2EAF, so EAF=90-x.
As BFDC is a cyclic quadrilateral, BCD=180-BFD, so BCD=180-z.
As BCF=y, FCD=180-y-z.
As DCF and DAF stand on the same chord, DAF=DCF, so DAF=180-y-z.
As EAF=90-x, EAD=x-y-z+90.

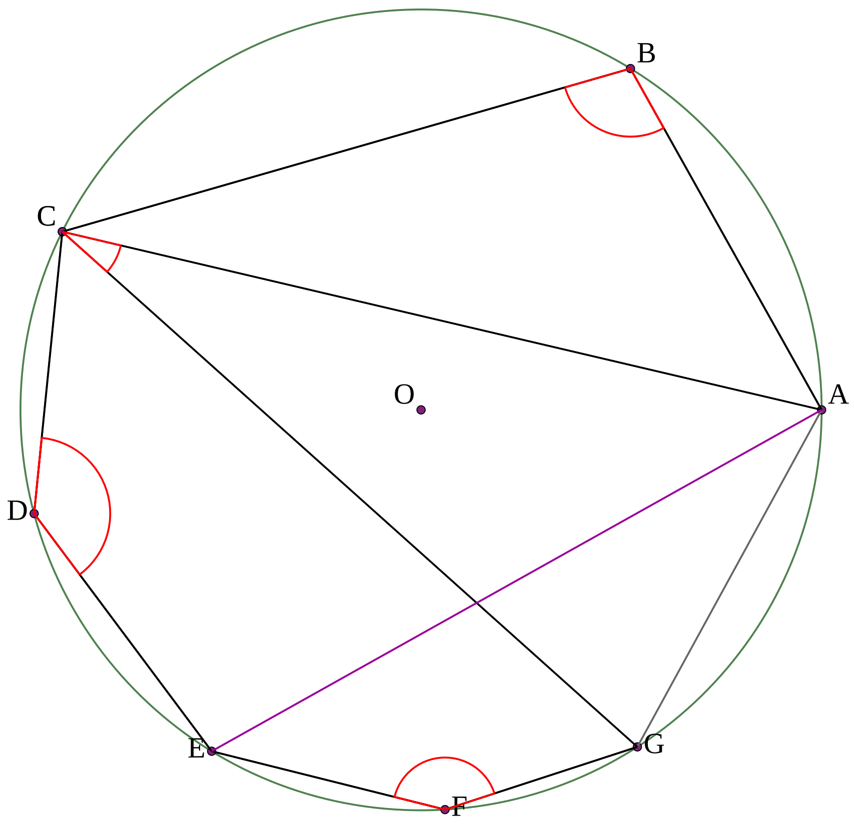
Solution to example 25



Let ABCDEFG be a cyclic heptagon with center O.
Angle DEB = x. Angle BGA = y. Angle ADC = z.
Find angle CFD.

Draw line BD.
As BEDC is a cyclic quadrilateral, BCD=180-BED, so BCD=180-x.
As AGB and ADB stand on the same chord, ADB=AGB, so ADB=y.
As ADC=z, CDB=z-y.
As BCD=180-x, CBD=x+y-z.
As CBD and CFD stand on the same chord, CFD=CBD, so CFD=x+y-z.

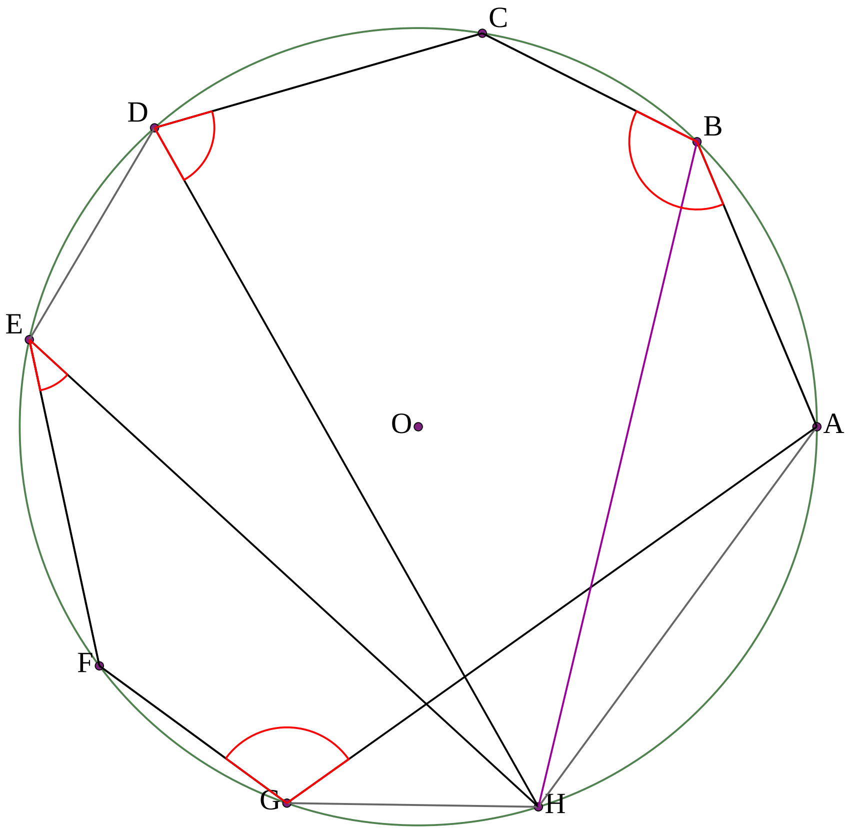
Solution to example 27



Let ABCDEFG be a cyclic heptagon with center O.
Angle EFG = x. Angle ABC = y. Angle CDE = z.
Find angle GCA.

Draw line AE.
As EFGA is a cyclic quadrilateral, EAG=180-EFG, so EAG=180-x.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=180-z.
As EAG=180-x, GAC=360-x-z.
As ABCG is a cyclic quadrilateral, AGC=180-ABC, so AGC=180-y.
As CAG=360-x-z, ACG=x+y+z-360.

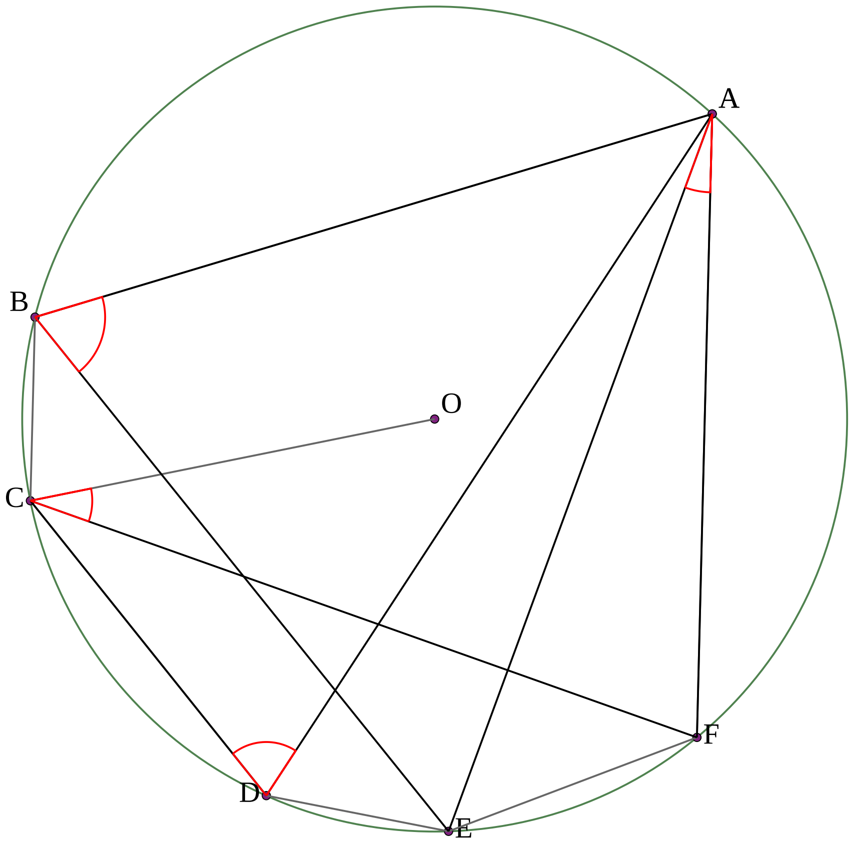
Solution to example 29



Let ABCDEFGH be a cyclic octagon with center O.
Angle HDC = 77°. Angle CBA = 140°. Angle AGF = 108°.
Find angle FEH.

Draw line BH.
As CDHB is a cyclic quadrilateral, CBH=180-CDH, so CBH=103.
As ABC=140, ABH=37.
As ABH and AGH stand on the same chord, AGH=ABH, so AGH=37.
As AGH=37, HGF=145.
As FGHE is a cyclic quadrilateral, FEH=180-FGH, so FEH=35.

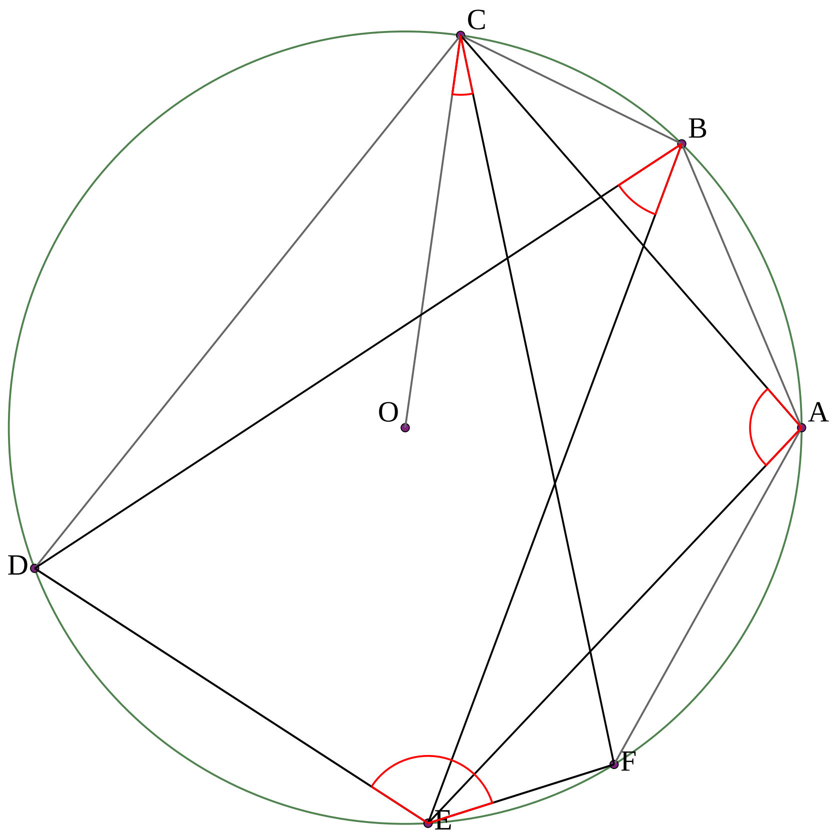
Solution to example 31



Let ABCDEF be a cyclic hexagon with center O.
Angle ADC = 72°. Angle ABE = 68°. Angle OCF = 31°.
Find angle FAE.

As triangle FCO is isosceles, CFO=31.
As ABE and ADE stand on the same chord, ADE=ABE, so ADE=68.
As ADC=72, CDE=140.
As CDEF is a cyclic quadrilateral, CFE=180-CDE, so CFE=40.
As CFO=31, OFE=71.
As triangle EFO is isosceles, EOF=38.
As EOF is at the center of a circle on the same chord as EAF, EOF=2EAF, so EAF=19.

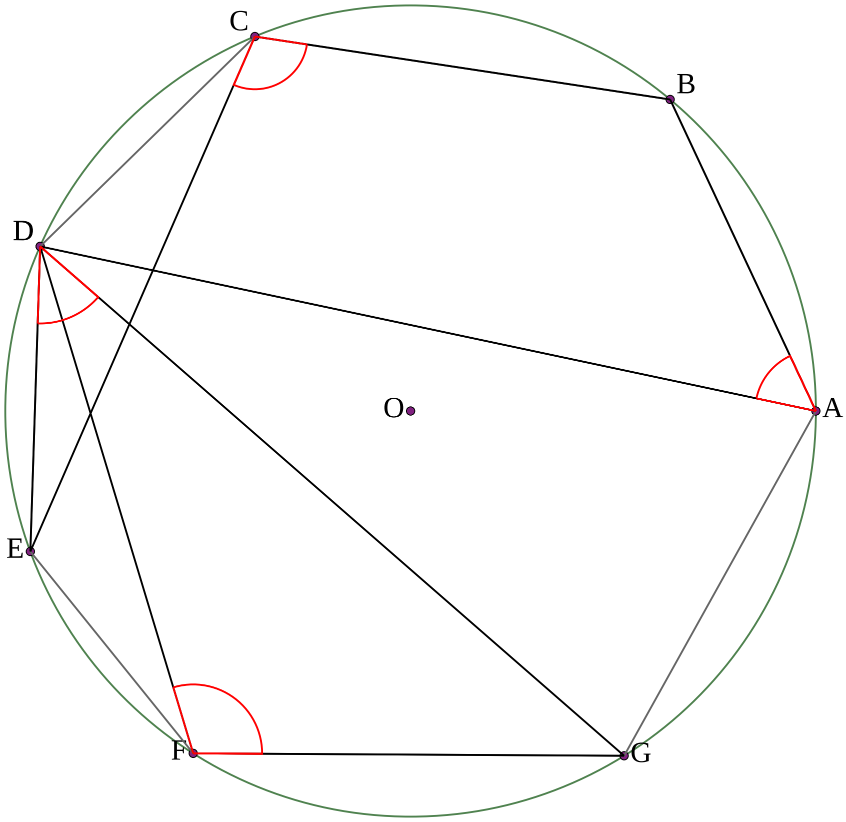
Solution to example 33



Let ABCDEF be a cyclic hexagon with center O.
Angle EAC = x. Angle OCF = y. Angle FED = z.
Find angle DBE.

As CAED is a cyclic quadrilateral, CDE=180-CAE, so CDE=180-x.
As DEFC is a cyclic quadrilateral, DCF=180-DEF, so DCF=180-z.
As FCO=y, OCD=180-y-z.
As triangle DCO is isosceles, CDO=180-y-z.
As CDE=180-x, EDO=y+z-x.
As triangle EDO is isosceles, DOE=2x-2y-2z+180.
As DOE is at the center of a circle on the same chord as DBE, DOE=2DBE, so DBE=x-y-z+90.

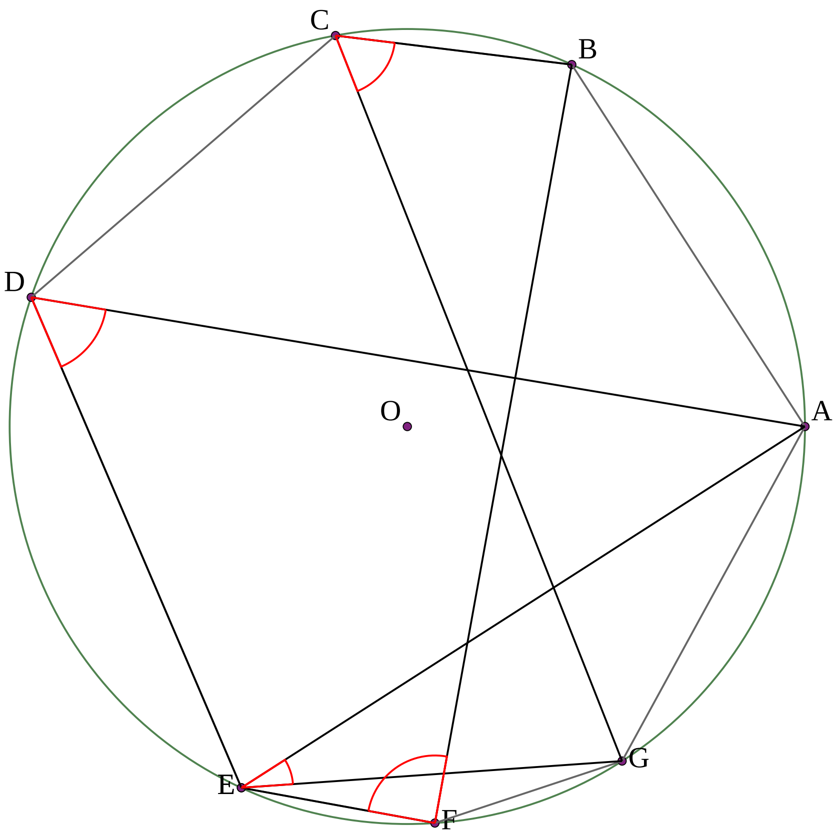
Solution to example 35



Let ABCDEFG be a cyclic heptagon with center O.
Prove that DFG+EDG = BCE+BAD

Let BCE=x. Let BAD=y. Let DFG=z. Let EDG=w.
As EDGF is a cyclic quadrilateral, EFG=180-EDG, so EFG=180-w.
As DFG=z, DFE=180-z-w.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=180-y.
As BCE=x, ECD=180-x-y.
As DCE and DFE stand on the same chord, DFE=DCE, so DFE=180-x-y.
But DFE=180-z-w, so 180-x-y=180-z-w, or z+w=x+y, or DFG+EDG=BCE+BAD.

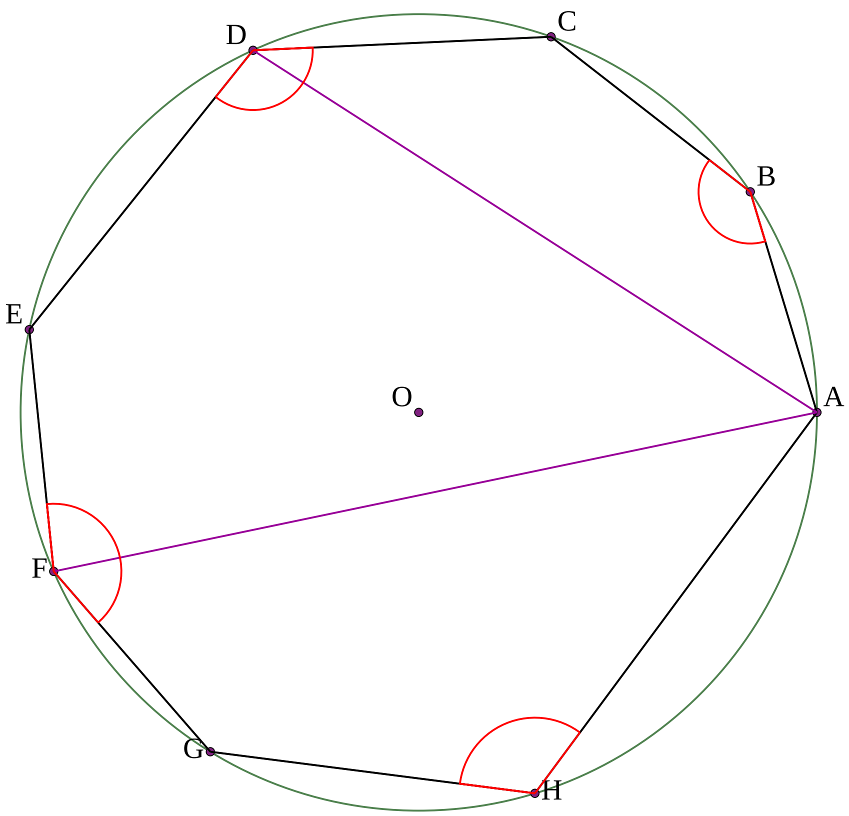
Solution to example 37



Let ABCDEFG be a cyclic heptagon with center O.
Prove that BCG+BFE+ADE = AEG+180

Let AEG=x. Let BCG=y. Let BFE=z. Let ADE=w.
As BCG and BFG stand on the same chord, BFG=BCG, so BFG=y.
As BFE=z, EFG=y+z.
As ADEG is a cyclic quadrilateral, AGE=180-ADE, so AGE=180-w.
As AEG=x, EAG=w-x.
As EAGF is a cyclic quadrilateral, EFG=180-EAG, so EFG=x-w+180.
But EFG=y+z, so x-w+180=y+z, or y+z+w=x+180, or BCG+BFE+ADE=AEG+180.

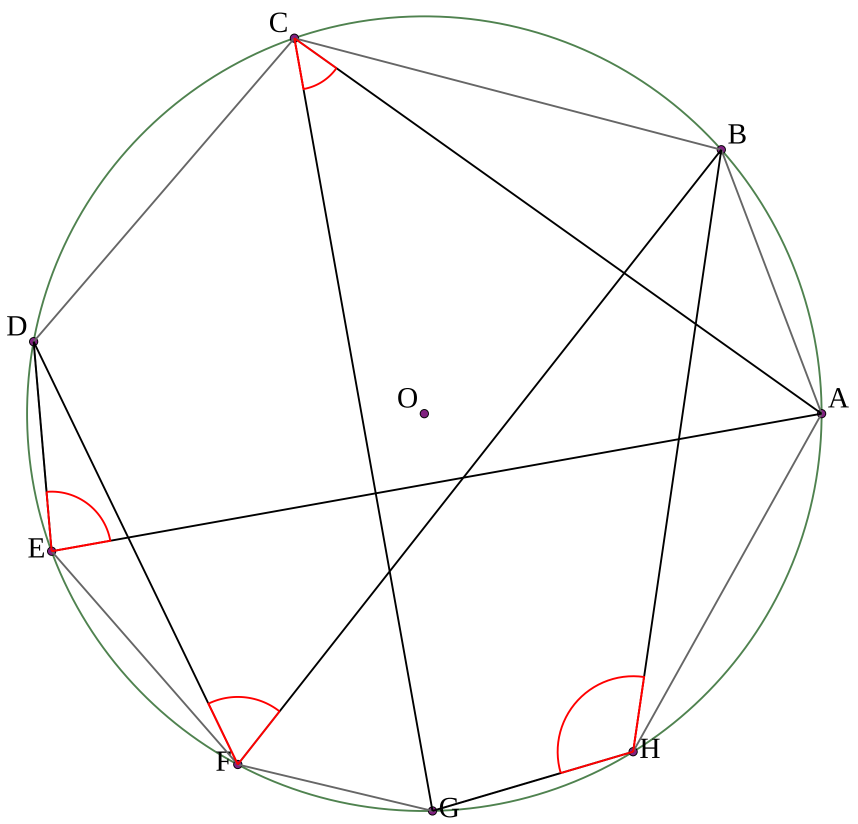
Solution to example 39



Let ABCDEFGH be a cyclic octagon with center O.
Angle ABC = 145°. Angle CDE = 131°. Angle EFG = 145°.
Find angle GHA.

Draw lines AD and AF.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=35.
As CDE=131, EDA=96.
As ADEF is a cyclic quadrilateral, AFE=180-ADE, so AFE=84.
As AFE=84, AFG=61.
As AFGH is a cyclic quadrilateral, AHG=180-AFG, so AHG=119.

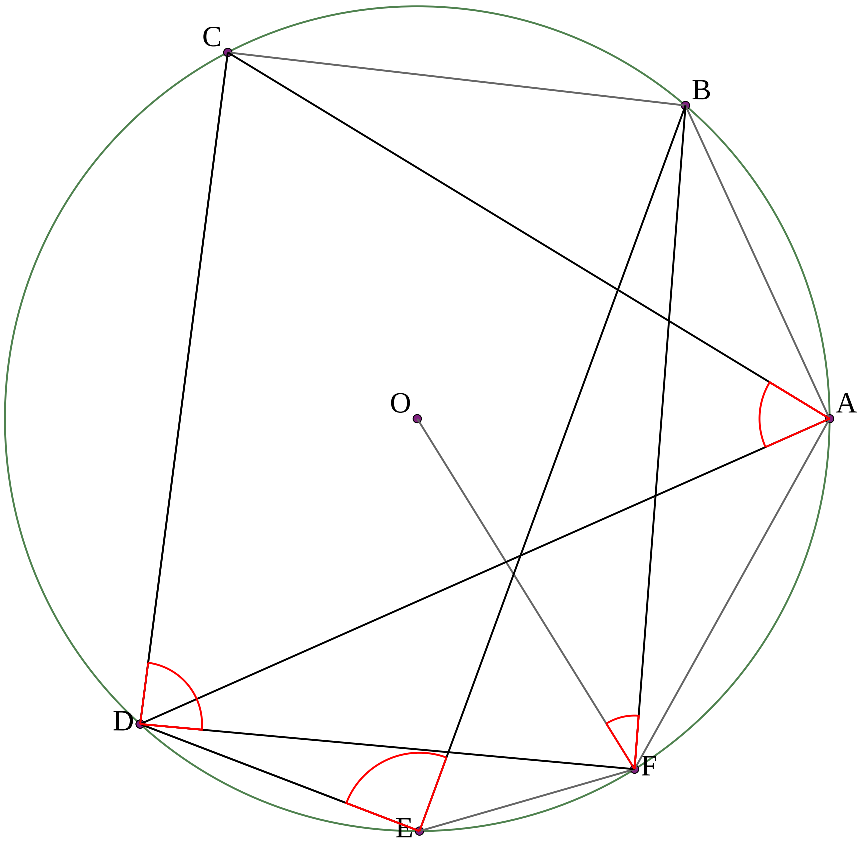
Solution to example 41



Let ABCDEFGH be a cyclic octagon with center O.
Prove that ACG+AED+BHG = BFD+180

Let ACG=x. Let AED=y. Let BFD=z. Let BHG=w.
As BHGF is a cyclic quadrilateral, BFG=180-BHG, so BFG=180-w.
As BFD=z, DFG=z-w+180.
As AEDC is a cyclic quadrilateral, ACD=180-AED, so ACD=180-y.
As ACG=x, GCD=180-x-y.
As DCGF is a cyclic quadrilateral, DFG=180-DCG, so DFG=x+y.
But DFG=z-w+180, so x+y=z-w+180, or x+y+w=z+180, or ACG+AED+BHG=BFD+180.

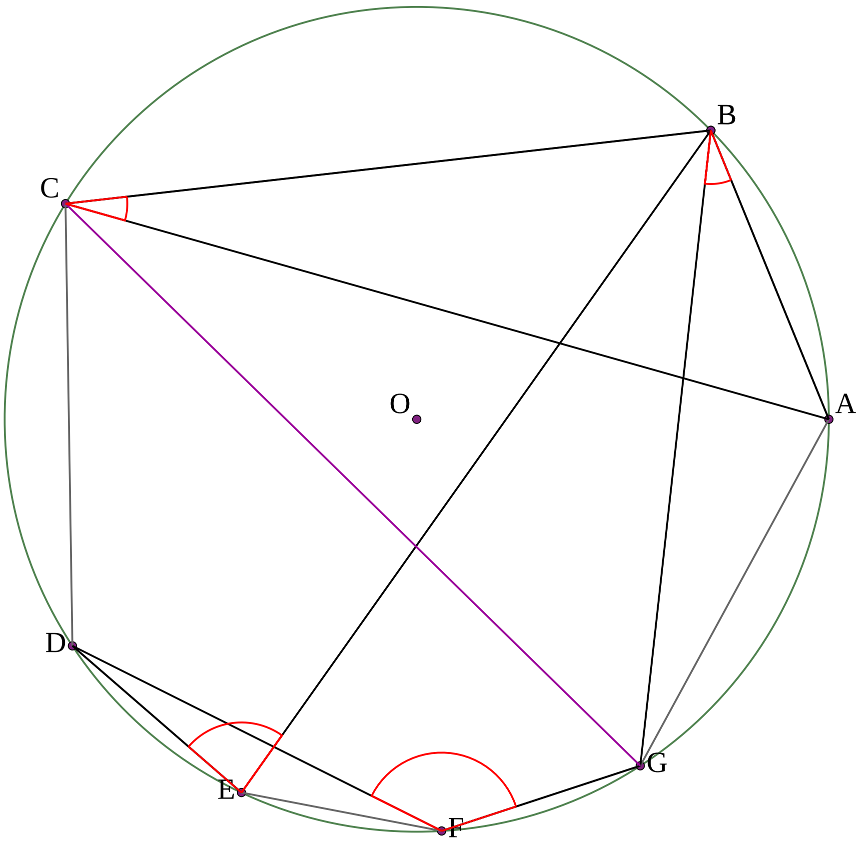
Solution to example 43



Let ABCDEF be a cyclic hexagon with center O.
Prove that BFO+CDF+CAD = BED+90

Let BFO=x. Let CDF=y. Let CAD=z. Let BED=w.
As triangle BFO is isosceles, BOF=180-2x.
As BOF is at the center of a circle on the same chord, but in the opposite direction to BAF, BOF=360-2BAF, so BAF=x+90.
As CDFA is a cyclic quadrilateral, CAF=180-CDF, so CAF=180-y.
As BED and BAD stand on the same chord, BAD=BED, so BAD=w.
As CAD=z, CAB=w-z.
As CAF=180-y, FAB=w-y-z+180.
But BAF=x+90, so w-y-z+180=x+90, or x+y+z=w+90, or BFO+CDF+CAD=BED+90.

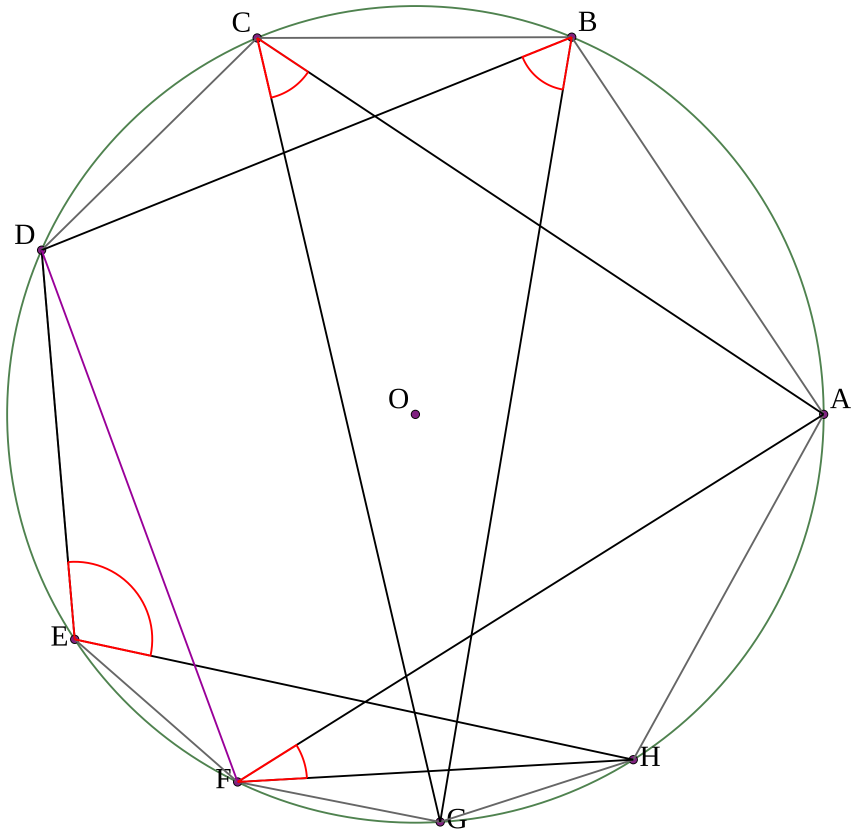
Solution to example 45



Let ABCDEFG be a cyclic heptagon with center O.
Prove that DFG = ABG+ACB+BED

Draw line CG.
Let ABG=x. Let ACB=y. Let BED=z. Let DFG=w.
As BEDC is a cyclic quadrilateral, BCD=180-BED, so BCD=180-z.
As ACB=y, ACD=180-y-z.
As ABG and ACG stand on the same chord, ACG=ABG, so ACG=x.
As DFGC is a cyclic quadrilateral, DCG=180-DFG, so DCG=180-w.
As ACG=x, ACD=x-w+180.
But ACD=180-y-z, so x-w+180=180-y-z, or x+y+z=w, or ABG+ACB+BED=DFG.

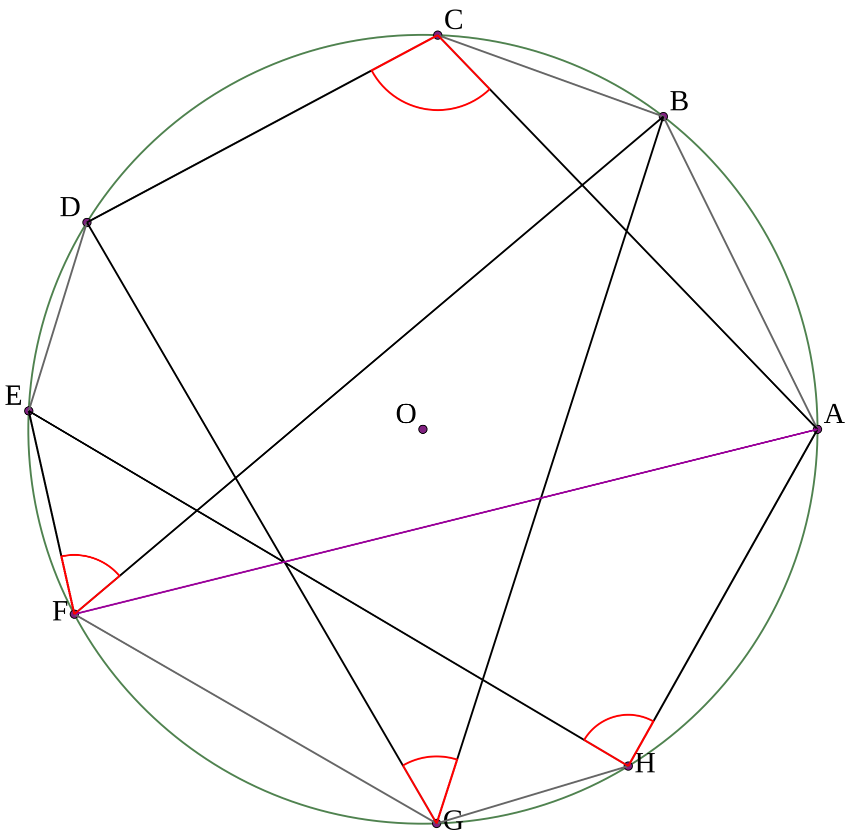
Solution to example 47



Let ABCDEFGH be a cyclic octagon with center O.
Prove that DBG+ACG+DEH = AFH+180

Draw line DF.
Let DBG=x. Let ACG=y. Let AFH=z. Let DEH=w.
As DEH and DFH stand on the same chord, DFH=DEH, so DFH=w.
As AFH=z, AFD=w-z.
As ACG and ABG stand on the same chord, ABG=ACG, so ABG=y.
As DBG=x, DBA=x+y.
As ABDF is a cyclic quadrilateral, AFD=180-ABD, so AFD=180-x-y.
But AFD=w-z, so 180-x-y=w-z, or x+y+w=z+180, or DBG+ACG+DEH=AFH+180.

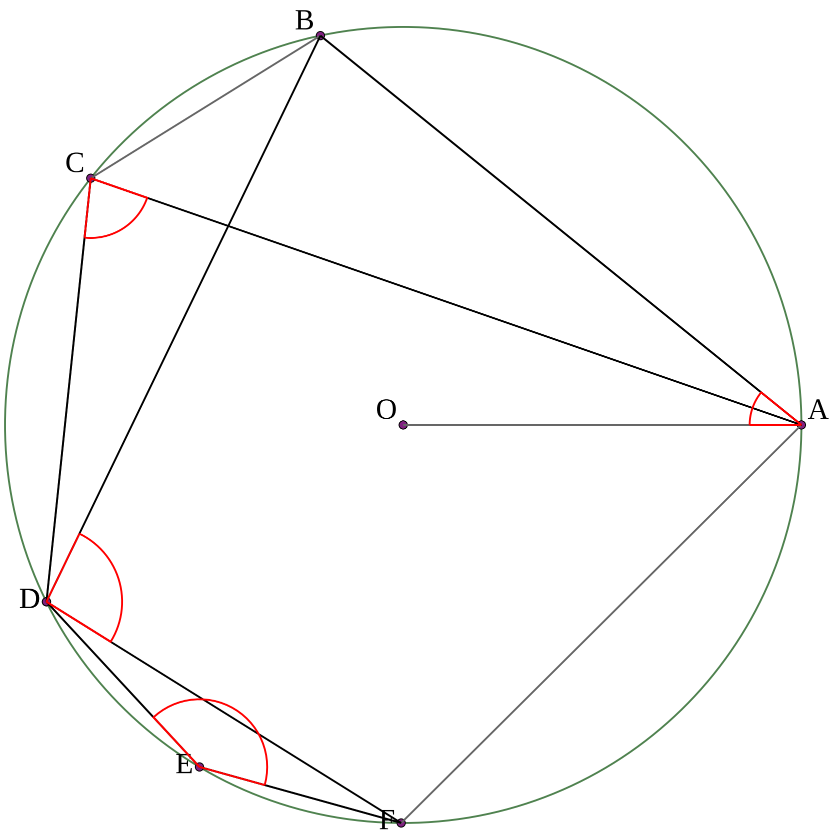
Solution to example 49



Let ABCDEFGH be a cyclic octagon with center O.
Prove that AHE+BGD+ACD = BFE+180

Draw line AF.
Let AHE=x. Let BFE=y. Let BGD=z. Let ACD=w.
As BGDC is a cyclic quadrilateral, BCD=180-BGD, so BCD=180-z.
As ACD=w, ACB=180-z-w.
As AHE and AFE stand on the same chord, AFE=AHE, so AFE=x.
As BFE=y, BFA=x-y.
As AFB and ACB stand on the same chord, ACB=AFB, so ACB=x-y.
But ACB=180-z-w, so x-y=180-z-w, or x+z+w=y+180, or AHE+BGD+ACD=BFE+180.

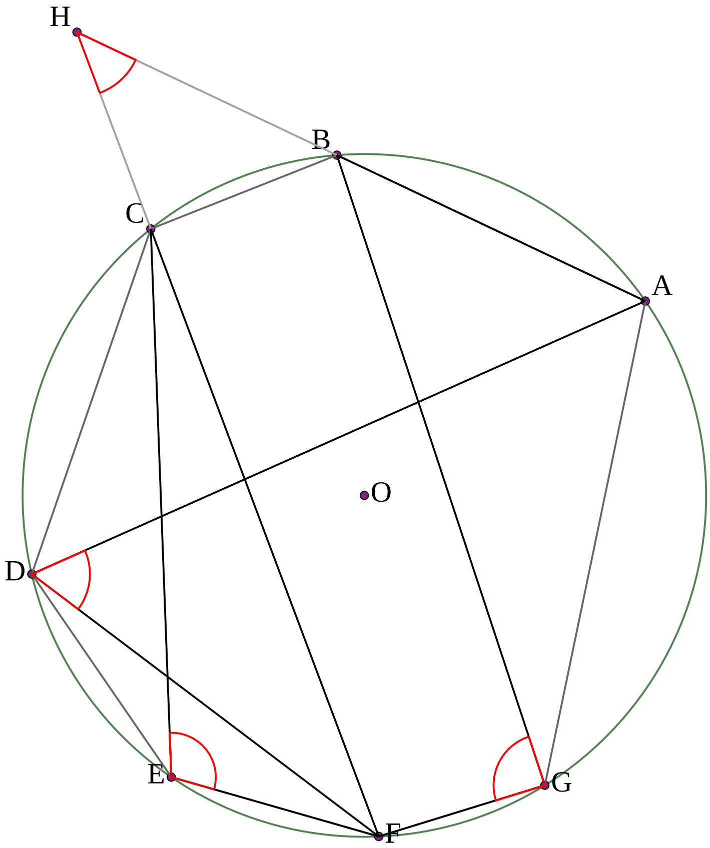
Solution to example 51



Let ABCDEF be a cyclic hexagon with center O.
Prove that ACD+DEF = BAO+BDF+90

Let BAO=x. Let ACD=y. Let DEF=z. Let BDF=w.
As ACDF is a cyclic quadrilateral, AFD=180-ACD, so AFD=180-y.
As BDFA is a cyclic quadrilateral, BAF=180-BDF, so BAF=180-w.
As BAO=x, OAF=180-x-w.
As triangle FAO is isosceles, AFO=180-x-w.
As AFD=180-y, DFO=x+w-y.
As triangle DFO is isosceles, DOF=2y-2x-2w+180.
As DOF is at the center of a circle on the same chord, but in the opposite direction to DEF, DOF=360-2DEF, so DEF=x+w-y+90.
But DEF=z, so x+w-y+90=z, or x+w+90=y+z, or BAO+BDF+90=ACD+DEF.

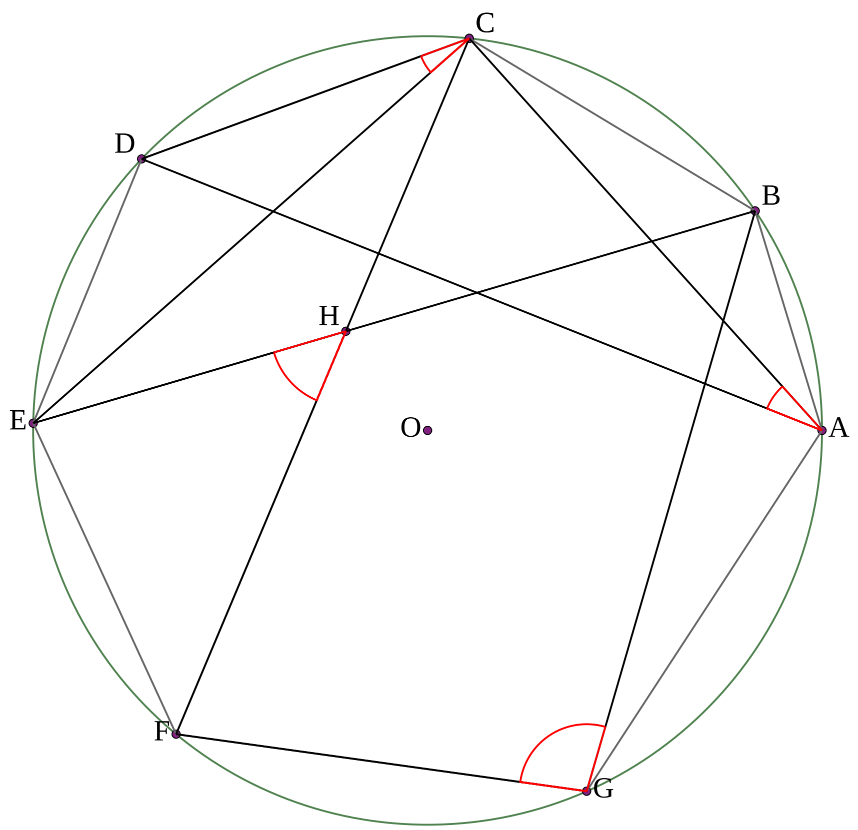
Solution to example 53



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of BA and FC.
Angle ADF = 61°. Angle FGB = 89°. Angle BHC = 44°.
Find angle CEF.

As BGFC is a cyclic quadrilateral, BCF=180-BGF, so BCF=91.
As BCF=91, BCH=89.
As BCH=89, CBH=47.
As CBH=47, CBA=133.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=47.
As ADC=47, CDF=108.
As CDF and CEF stand on the same chord, CEF=CDF, so CEF=108.

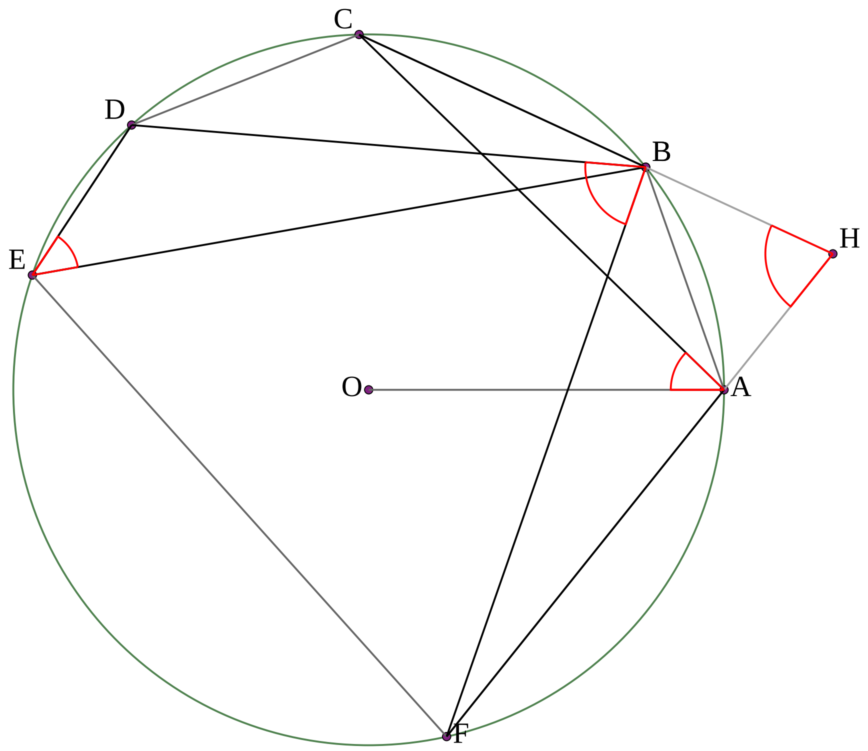
Solution to example 55



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CF and BE.
Angle FGB = 98°. Angle DAC = 26°. Angle FHE = 51°.
Find angle ECD.

As BGFE is a cyclic quadrilateral, BEF=180-BGF, so BEF=82.
As FEH=82, EFH=47.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=133.
As CAD and CED stand on the same chord, CED=CAD, so CED=26.
As CDE=133, DCE=21.

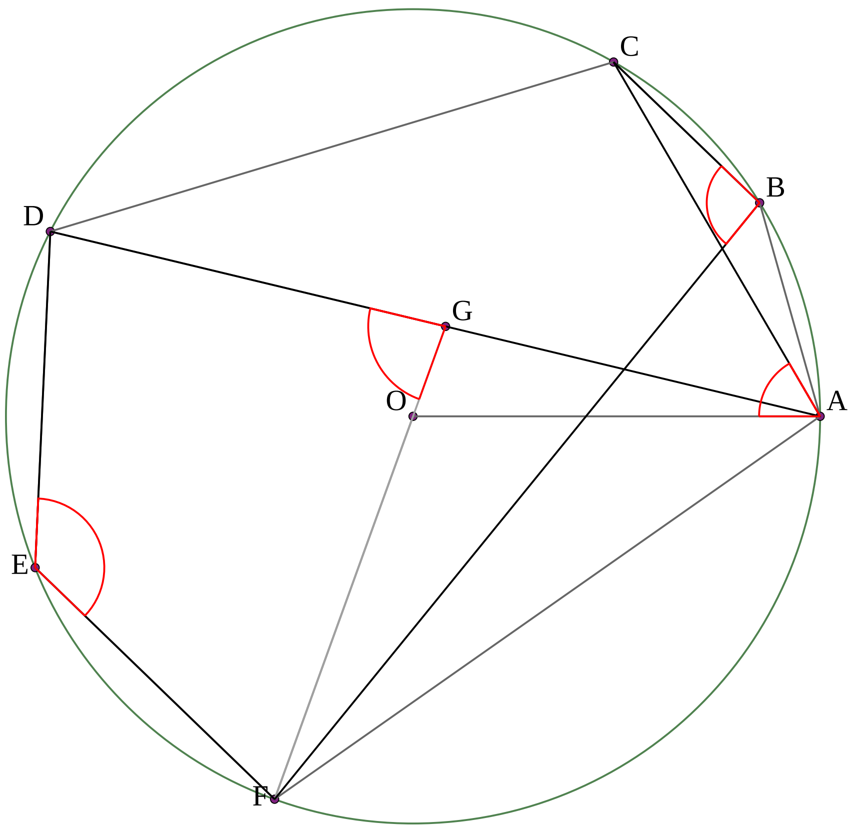
Solution to example 57



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of FA and CB.
Angle DEB = 47°. Angle OAC = 44°. Angle FBD = 75°.
Find angle AHB.

As triangle CAO is isosceles, AOC=92.
As AOC is at the center of a circle on the same chord, but in the opposite direction to ABC, AOC=360-2ABC, so ABC=134.
As ABC=134, ABH=46.
As DBFE is a cyclic quadrilateral, DEF=180-DBF, so DEF=105.
As BED=47, BEF=58.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=122.
As BAF=122, BAH=58.
As ABH=46, AHB=76.

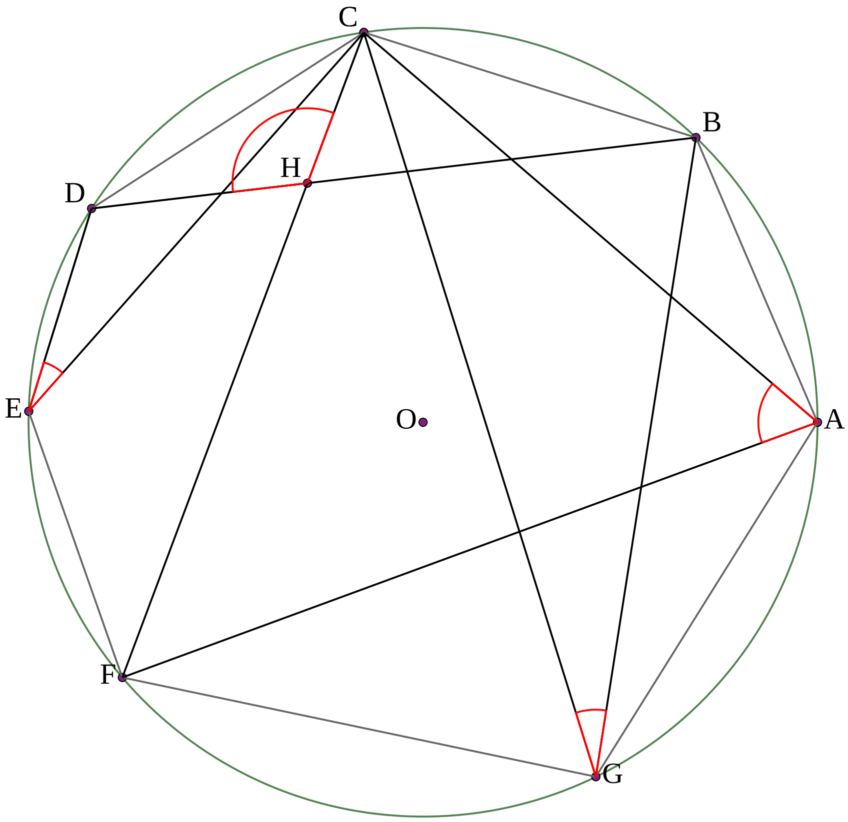
Solution to example 59



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AD and FO.
Angle FBC = 95°. Angle CAO = 60°. Angle DGF = 84°.
Find angle DEF.

As CBF and CAF stand on the same chord, CAF=CBF, so CAF=95.
As CAO=60, OAF=35.
As triangle FAO is isosceles, AFO=35.
As DGF=84, FGA=96.
As AFG=35, FAG=49.
As DAFE is a cyclic quadrilateral, DEF=180-DAF, so DEF=131.

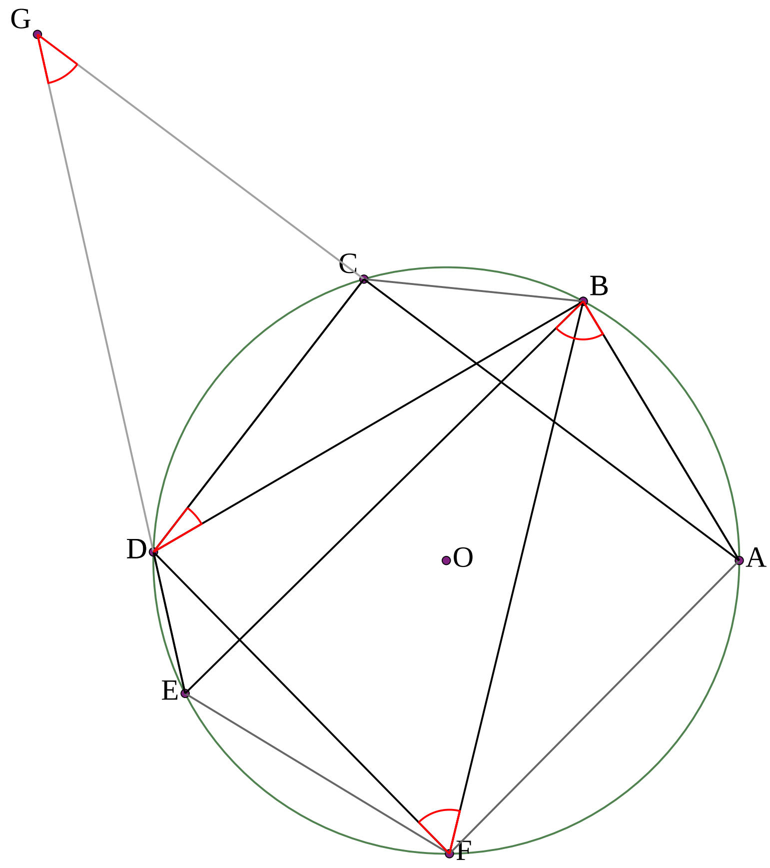
Solution to example 61



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FC and BD.
Angle DEC = 24°. Angle CGB = 26°. Angle CHD = 117°.
Find angle CAF.

As CED and CBD stand on the same chord, CBD=CED, so CBD=24.
As CHD=117, CHB=63.
As CBH=24, BCH=93.
As BCFA is a cyclic quadrilateral, BAF=180-BCF, so BAF=87.
As BGC and BAC stand on the same chord, BAC=BGC, so BAC=26.
As BAF=87, FAC=61.

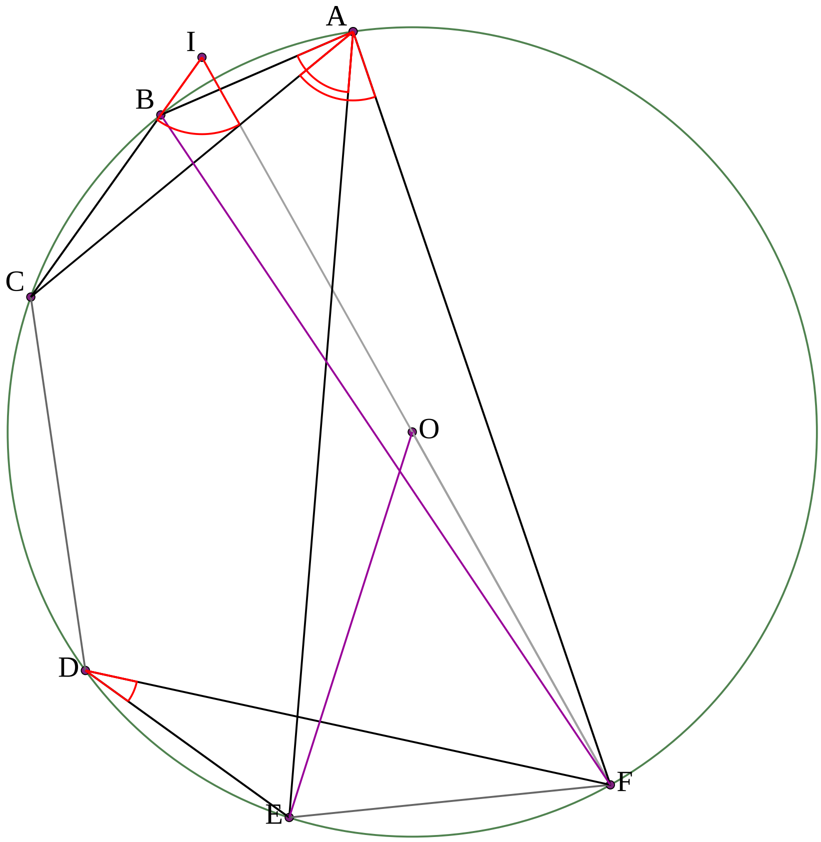
Solution to example 63



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and CA.
Angle BFD = 58°. Angle DGC = 41°. Angle ABE = 77°.
Find angle BDC.

Let CDG=u.
As CDG=u, CDE=180-u.
As CDEB is a cyclic quadrilateral, CBE=180-CDE, so CBE=u.
As CBE=u, CBA=u+77.
As BFDC is a cyclic quadrilateral, BCD=180-BFD, so BCD=122.
As CGD=41, DCG=139-u.
As BCD=122, BCG=u+99.
As BCG=u+99, BCA=81-u.
As ABC=u+77, BAC=22.
As BAC and BDC stand on the same chord, BDC=BAC, so BDC=22.

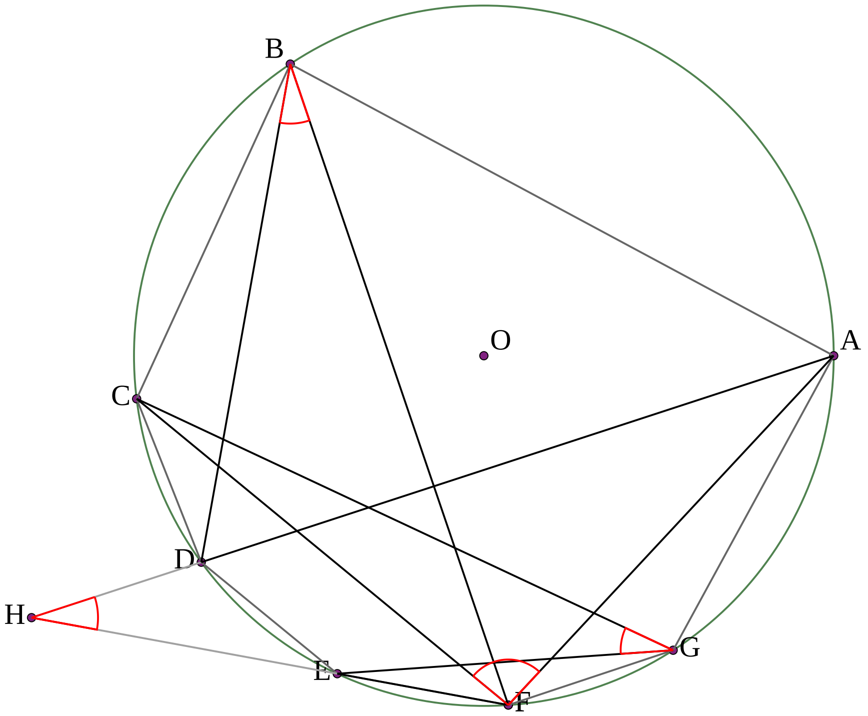
Solution to example 65



Let ABCDEF be a cyclic hexagon with center O. Let I be the intersection of CB and FO.
Angle FDE = x. Angle EAB = y. Angle CAF = z.
Find angle BIF.

Draw lines BF and EO.
As BAE and BFE stand on the same chord, BFE=BAE, so BFE=y.
As EOF is at the center of a circle on the same chord as EDF, EOF=2EDF, so EOF=2x.
As triangle EOF is isosceles, EFO=90-x.
As BFE=y, BFO=90-x-y.
As CAF and CBF stand on the same chord, CBF=CAF, so CBF=z.
As CBF=z, FBI=180-z.
As BFI=90-x-y, BIF=x+y+z-90.

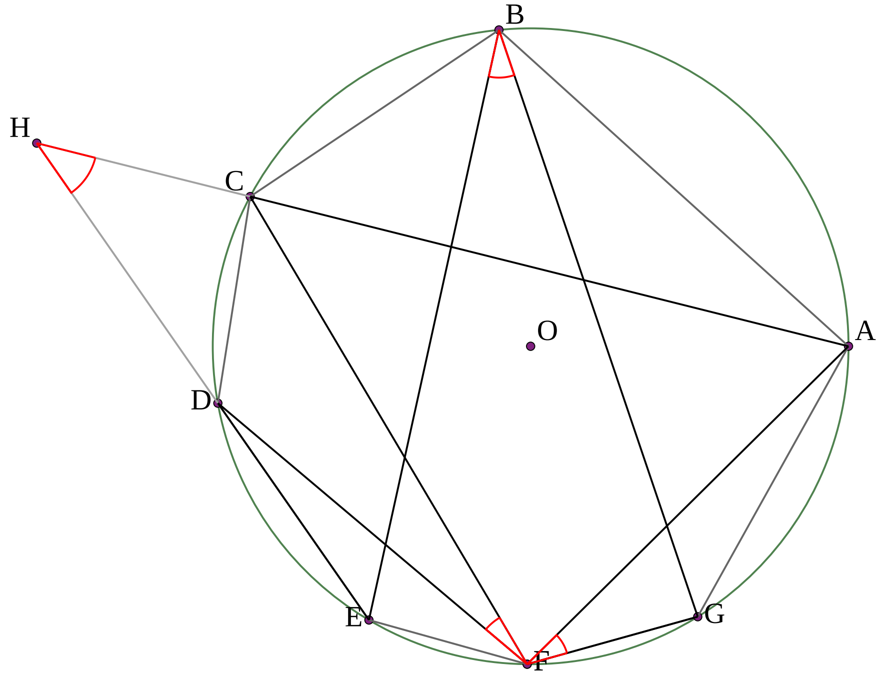
Solution to example 67



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and DA.
Angle AFC = 94°. Angle CGE = 29°. Angle EHD = 28°.
Find angle FBD.

As CGE and CFE stand on the same chord, CFE=CGE, so CFE=29.
As AFC=94, AFE=123.
As AFH=123, FAH=29.
As DAF and DBF stand on the same chord, DBF=DAF, so DBF=29.

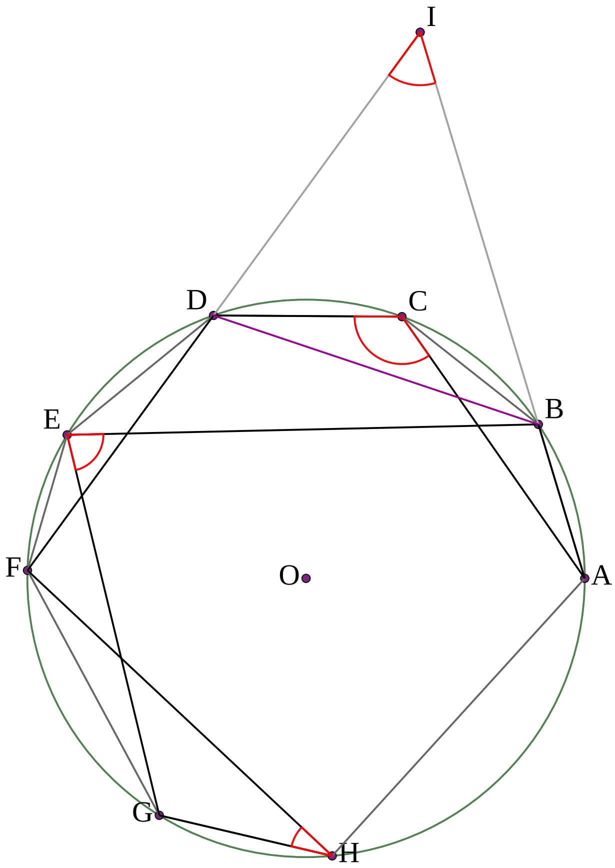
Solution to example 69



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of ED and CA.
Angle AFG = x. Angle GBE = y. Angle DHC = z.
Find angle DFC.

As EBGF is a cyclic quadrilateral, EFG=180-EBG, so EFG=180-y.
As AFG=x, AFE=180-x-y.
Let CDH=u.
As CDH=u, CDE=180-u.
As CDEF is a cyclic quadrilateral, CFE=180-CDE, so CFE=u.
As AFE=180-x-y, AFC=180-x-y-u.
As CHD=z, DCH=180-z-u.
As DCH=180-z-u, DCA=z+u.
As ACDF is a cyclic quadrilateral, AFD=180-ACD, so AFD=180-z-u.
As AFC=180-x-y-u, CFD=x+y-z.

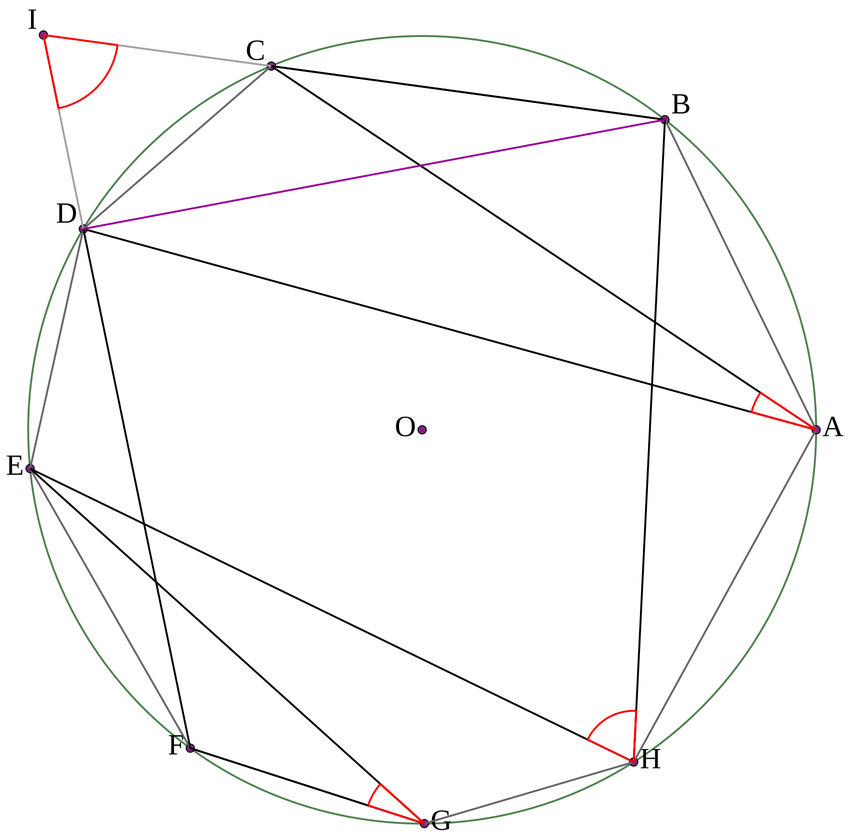
Solution to example 71



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FD and AB.
Prove that BEG+FHG+ACD = BID+180

Draw line BD.
Let BEG=x. Let FHG=y. Let ACD=z. Let BID=w.
As ACD and ABD stand on the same chord, ABD=ACD, so ABD=z.
As ABD=z, DBI=180-z.
As DBI=180-z, BDI=z-w.
As FHG and FEG stand on the same chord, FEG=FHG, so FEG=y.
As BEG=x, BEF=x+y.
As BEF and BDF stand on the same chord, BDF=BEF, so BDF=x+y.
As BDF=x+y, BDI=180-x-y.
But BDI=z-w, so 180-x-y=z-w, or x+y+z=w+180, or BEG+FHG+ACD=BID+180.

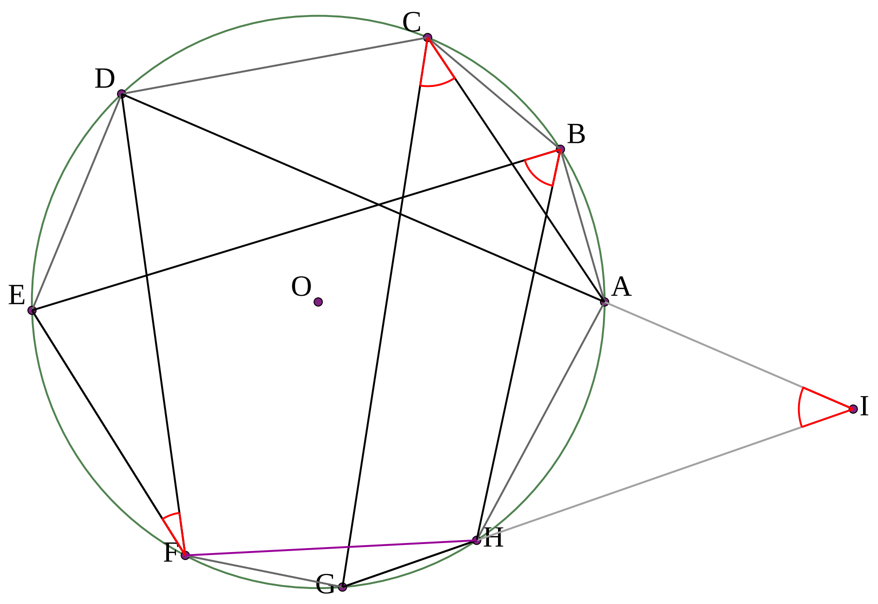
Solution to example 73



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FD and CB.
Angle EGF = 24°. Angle DAC = 18°. Angle DIC = 71°.
Find angle BHE.

Draw line BD.
As EGF and EDF stand on the same chord, EDF=EGF, so EDF=24.
As EDF=24, EDI=156.
As CAD and CBD stand on the same chord, CBD=CAD, so CBD=18.
As DBI=18, BDI=91.
As EDI=156, EDB=113.
As BDEH is a cyclic quadrilateral, BHE=180-BDE, so BHE=67.

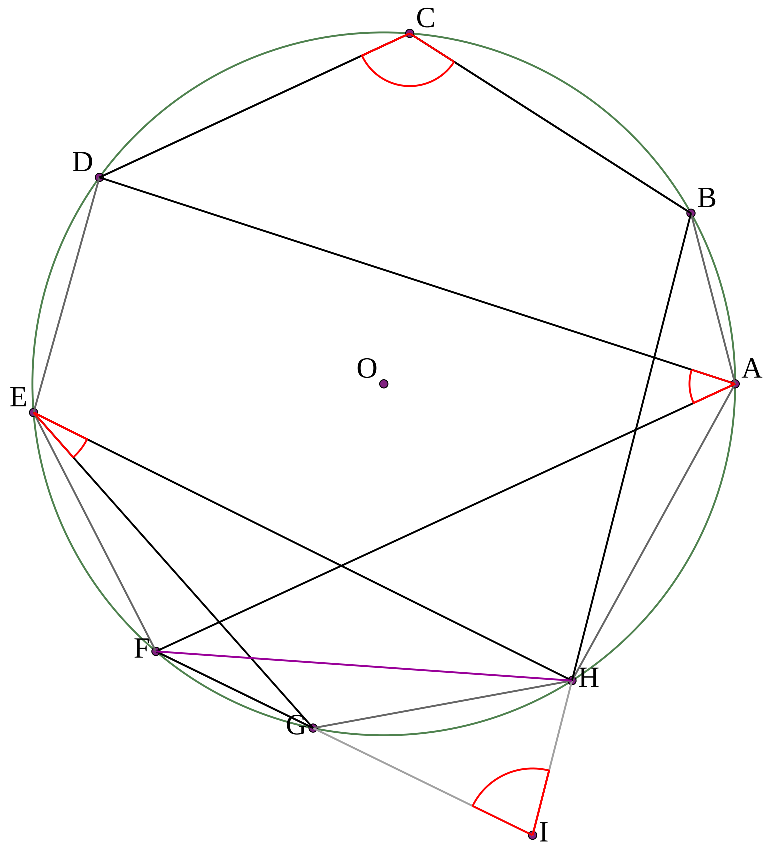
Solution to example 75



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DA and GH.
Prove that EBH+DFE = ACG+AIH

Draw line FH.
Let EBH=x. Let DFE=y. Let ACG=z. Let AIH=w.
As ACGH is a cyclic quadrilateral, AHG=180-ACG, so AHG=180-z.
As AHG=180-z, AHI=z.
As AHI=z, HAI=180-z-w.
As EBHF is a cyclic quadrilateral, EFH=180-EBH, so EFH=180-x.
As DFE=y, DFH=180-x-y.
As DFHA is a cyclic quadrilateral, DAH=180-DFH, so DAH=x+y.
As DAH=x+y, HAI=180-x-y.
But HAI=180-z-w, so 180-x-y=180-z-w, or z+w=x+y, or ACG+AIH=EBH+DFE.

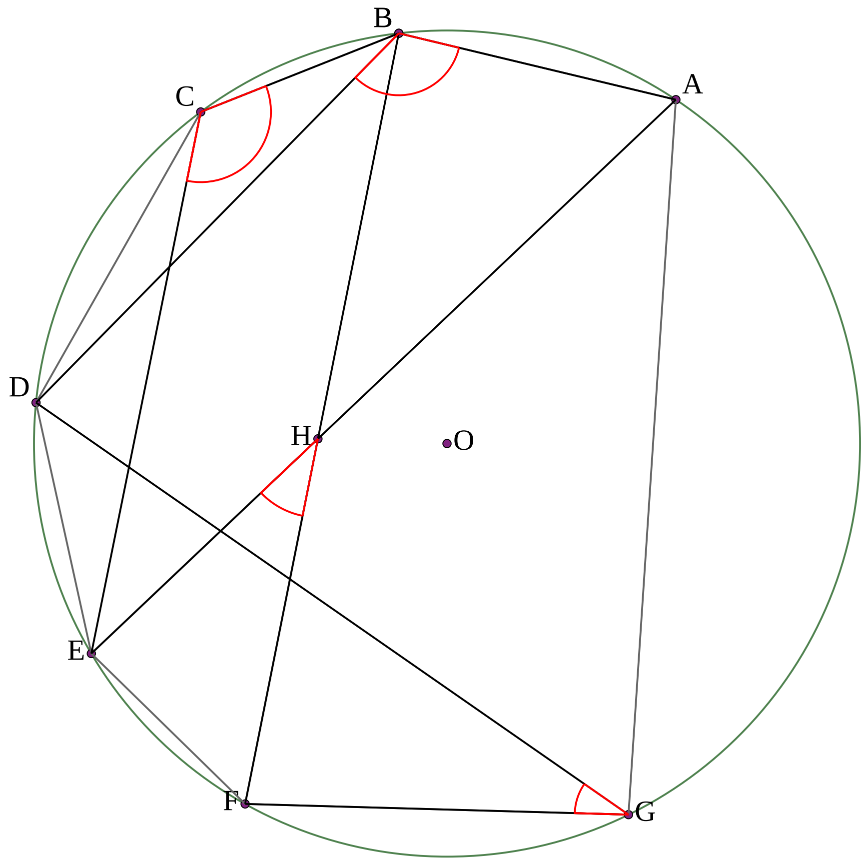
Solution to example 77



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of BH and GF.
Angle FAD = x. Angle HEG = y. Angle HIG = z.
Find angle DCB.

Draw line FH.
As GEH and GFH stand on the same chord, GFH=GEH, so GFH=y.
As HFI=y, FHI=180-y-z.
As FHI=180-y-z, FHB=y+z.
As BHF and BAF stand on the same chord, BAF=BHF, so BAF=y+z.
As BAF=y+z, BAD=y+z-x.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=x-y-z+180.

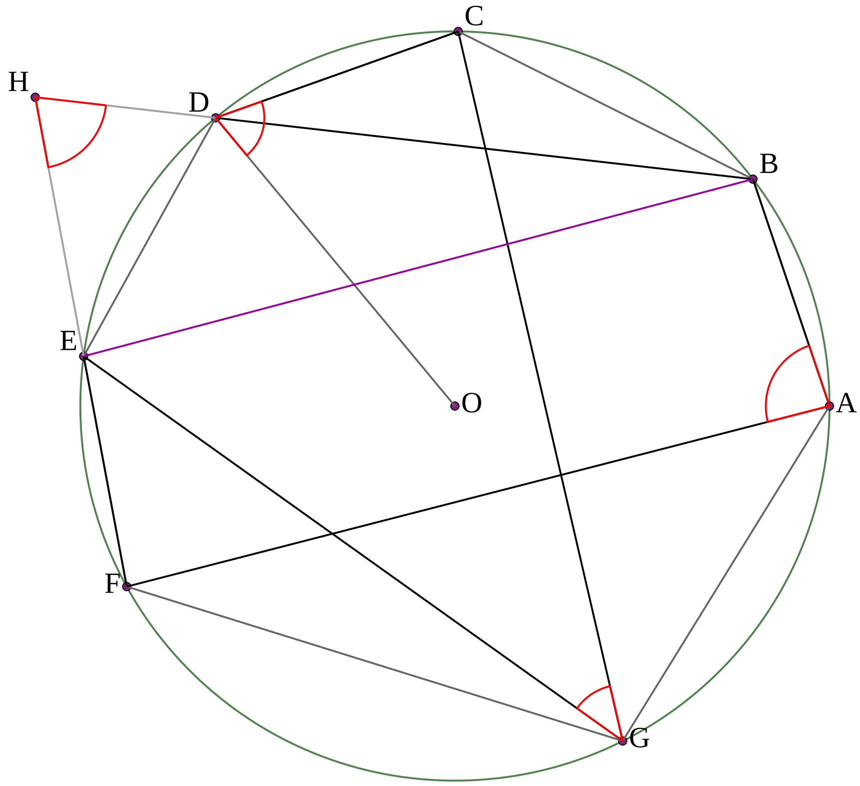
Solution to example 79



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AE and BF.
Prove that DGF+BCE = ABD+EHF

Let DGF=x. Let ABD=y. Let BCE=z. Let EHF=w.
Let FEH=u.
As EHF=w, EFH=180-w-u.
As BCEF is a cyclic quadrilateral, BFE=180-BCE, so BFE=180-z.
But EFH=180-w-u, so 180-z=180-w-u, or w+u=z.
As ABDG is a cyclic quadrilateral, AGD=180-ABD, so AGD=180-y.
As DGF=x, FGA=x-y+180.
As AGFE is a cyclic quadrilateral, AEF=180-AGF, so AEF=y-x.
But AEF=u, so y-x=u, or y=x+u.
We have these equations: z-w-u=0 (E1), x+u-y=0 (E2).
Hence x+z-y-w=0 (E1+E2), or x+z=y+w, or DGF+BCE=ABD+EHF.

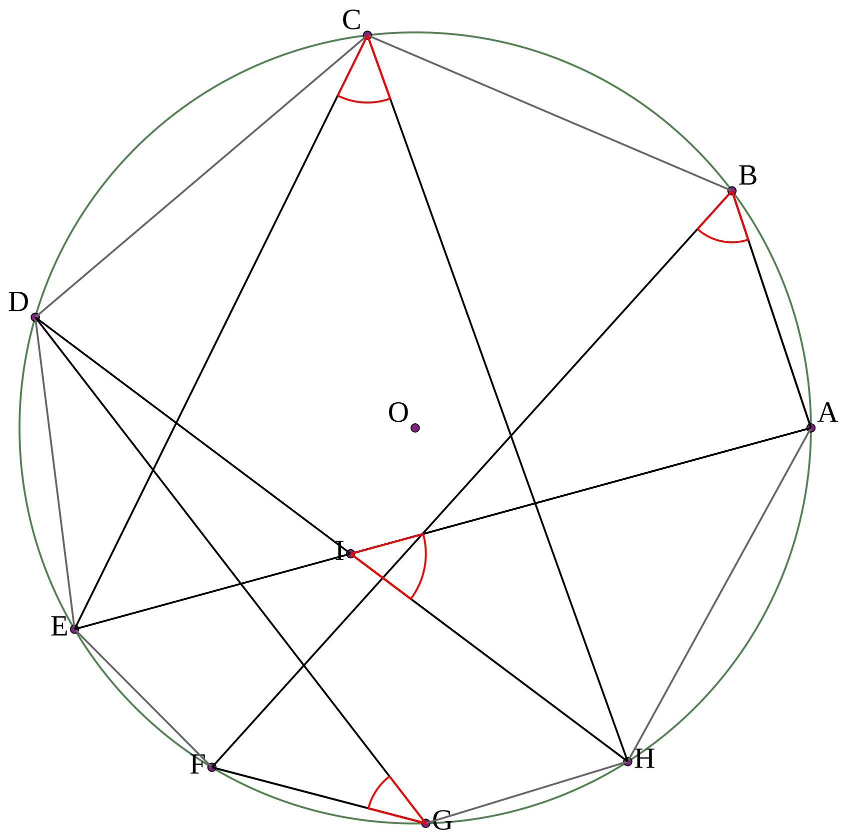
Solution to example 81



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and BD.
Angle ODC = x. Angle FAB = y. Angle EHD = z.
Find angle CGE.

Draw line BE.
As BAFE is a cyclic quadrilateral, BEF=180-BAF, so BEF=180-y.
As BEF=180-y, BEH=y.
As BEH=y, EBH=180-y-z.
As triangle CDO is isosceles, COD=180-2x.
As COD is at the center of a circle on the same chord as CBD, COD=2CBD, so CBD=90-x.
As EBH=180-y-z, EBC=270-x-y-z.
As CBE and CGE stand on the same chord, CGE=CBE, so CGE=270-x-y-z.

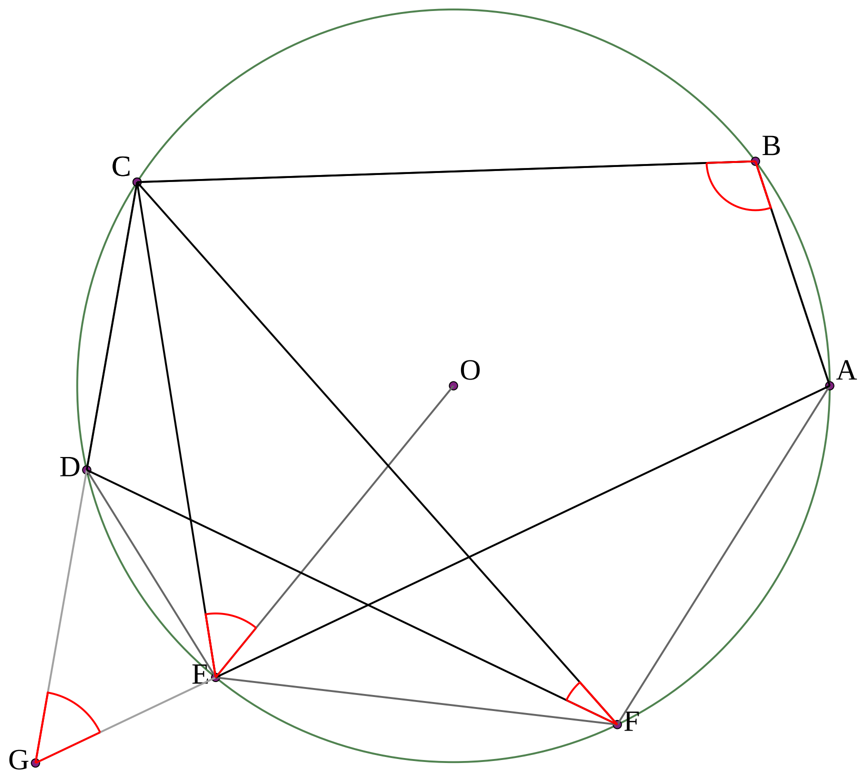
Solution to example 83



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DH and EA.
Angle HCE = 46°. Angle HIA = 52°. Angle FGD = 38°.
Find angle ABF.

As ECH and EDH stand on the same chord, EDH=ECH, so EDH=46.
As AIH=52, AID=128.
As AID=128, DIE=52.
As EDI=46, DEI=82.
As DGFE is a cyclic quadrilateral, DEF=180-DGF, so DEF=142.
As AED=82, AEF=60.
As AEF and ABF stand on the same chord, ABF=AEF, so ABF=60.

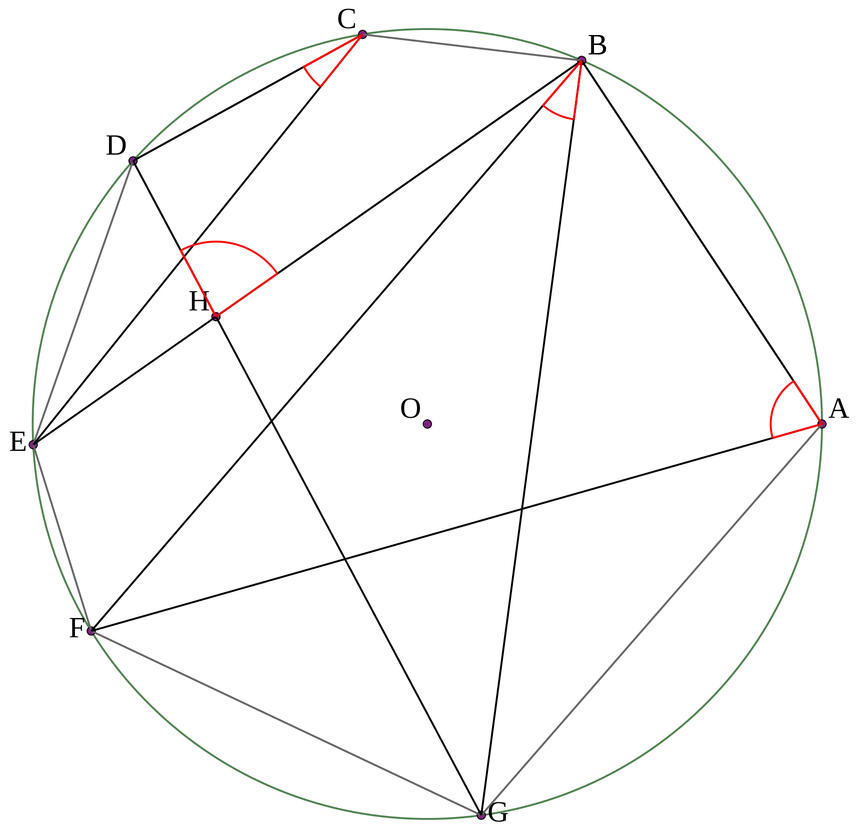
Solution to example 85



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DC and EA.
Prove that ABC+DGE = CFD+CEO+90

Let ABC=x. Let CFD=y. Let CEO=z. Let DGE=w.
As ABCE is a cyclic quadrilateral, AEC=180-ABC, so AEC=180-x.
As AEC=180-x, CEG=x.
As CEG=x, ECG=180-x-w.
As CFD and CED stand on the same chord, CED=CFD, so CED=y.
As triangle CEO is isosceles, COE=180-2z.
As COE is at the center of a circle on the same chord, but in the opposite direction to CDE, COE=360-2CDE, so CDE=z+90.
As CED=y, DCE=90-y-z.
But ECG=180-x-w, so 90-y-z=180-x-w, or y+z+90=x+w, or CFD+CEO+90=ABC+DGE.

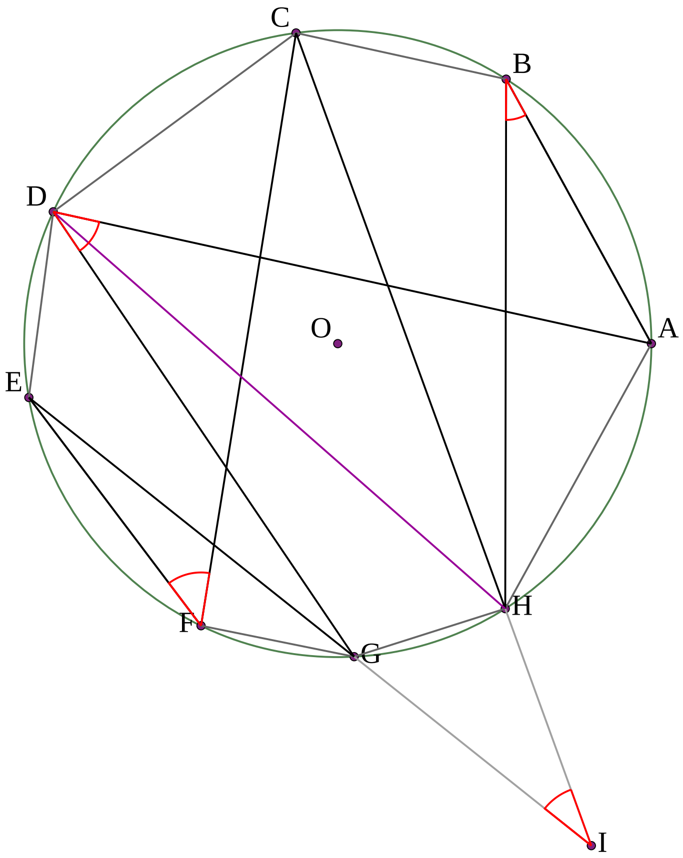
Solution to example 87



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GD and EB.
Angle DCE = x. Angle DHB = y. Angle FBG = z.
Find angle BAF.

Let BGH=u.
As BGDC is a cyclic quadrilateral, BCD=180-BGD, so BCD=180-u.
As BCD=180-u, BCE=180-x-u.
As BCEF is a cyclic quadrilateral, BFE=180-BCE, so BFE=x+u.
As BHD=y, BHG=180-y.
As BHG=180-y, GBH=y-u.
As GBH=y-u, HBF=y-z-u.
As BFE=x+u, BEF=z-x-y+180.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=x+y-z.

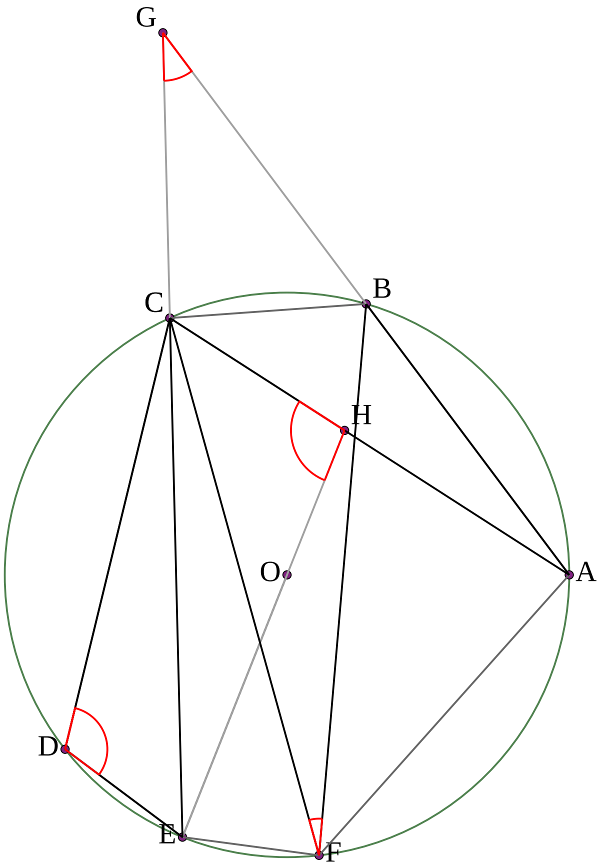
Solution to example 89



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GE and CH.
Angle HBA = 29°. Angle EFC = 46°. Angle GIH = 31°.
Find angle ADG.

Draw line DH.
Let HGI=u.
As GIH=31, GHI=149-u.
As GHI=149-u, GHC=u+31.
As CHGD is a cyclic quadrilateral, CDG=180-CHG, so CDG=149-u.
As ABH and ADH stand on the same chord, ADH=ABH, so ADH=29.
As HGI=u, HGE=180-u.
As EGHD is a cyclic quadrilateral, EDH=180-EGH, so EDH=u.
As ADH=29, ADE=u+29.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=134.
As ADE=u+29, ADC=105-u.
As CDG=149-u, GDA=44.

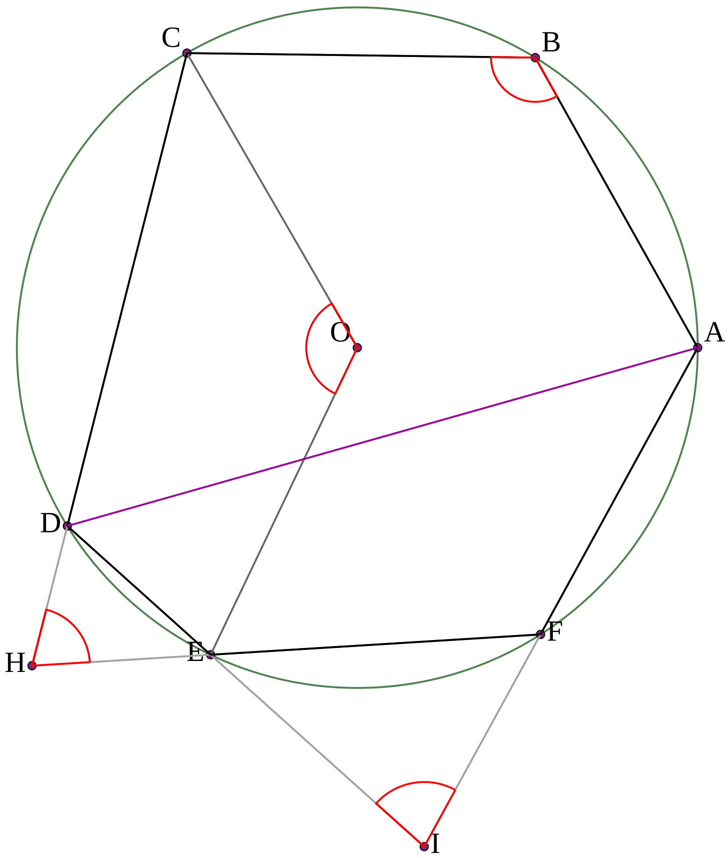
Solution to example 91



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BA and EC. Let H be the intersection of AC and EO.
Angle CFB = x. Angle BGC = y. Angle CHE = z.
Find angle CDE.

As BFC and BAC stand on the same chord, BAC=BFC, so BAC=x.
As CAG=x, ACG=180-x-y.
As ACG=180-x-y, ACE=x+y.
As ECH=x+y, CEH=180-x-y-z.
As triangle CEO is isosceles, COE=2x+2y+2z-180.
As COE is at the center of a circle on the same chord, but in the opposite direction to CDE, COE=360-2CDE, so CDE=270-x-y-z.

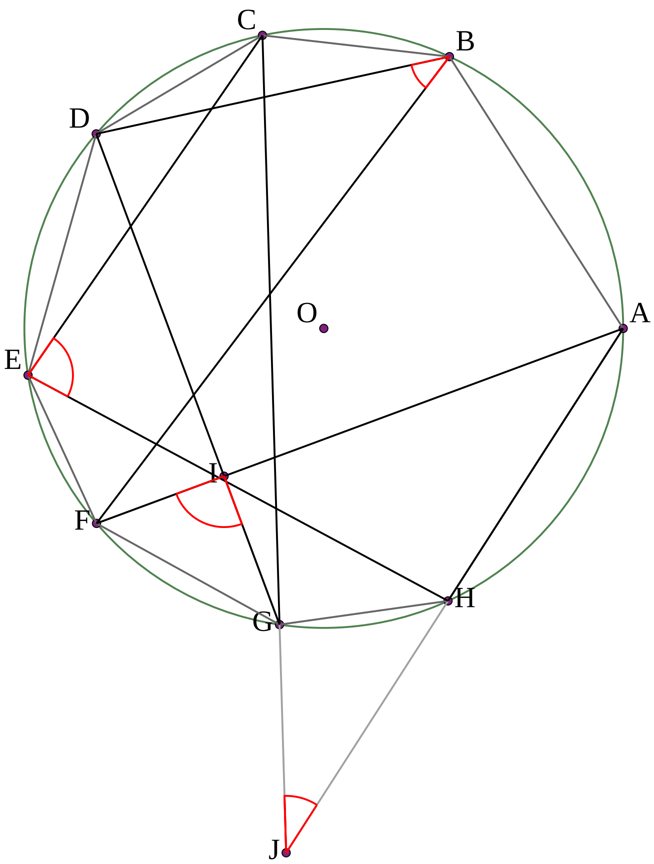
Solution to example 93



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of CD and EF. Let I be the intersection of DE and FA.
Angle COE = 124°. Angle DHE = 72°. Angle ABC = 120°.
Find angle EIF.

Draw line AD.
As COE is at the center of a circle on the same chord, but in the opposite direction to CDE, COE=360-2CDE, so CDE=118.
As CDE=118, EDH=62.
As EDH=62, DEH=46.
As DEH=46, DEF=134.
As DEF=134, FEI=46.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=60.
As ADC=60, ADE=58.
As ADEF is a cyclic quadrilateral, AFE=180-ADE, so AFE=122.
As AFE=122, EFI=58.
As FEI=46, EIF=76.

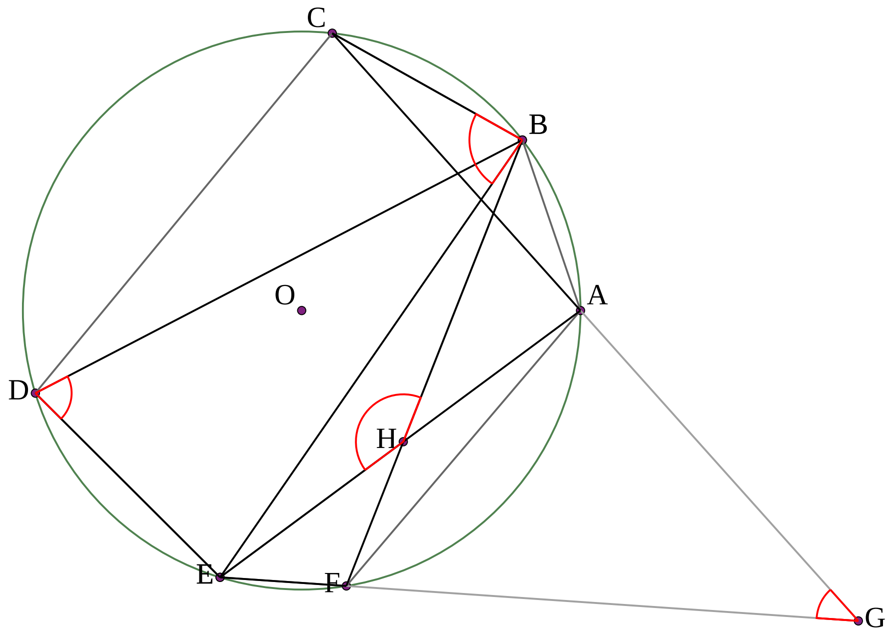
Solution to example 95



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DG and AF. Let J be the intersection of GC and HA.
Prove that DBF+CEH+FIG = GJH+180

Let DBF=x. Let CEH=y. Let FIG=z. Let GJH=w.
As CEH and CGH stand on the same chord, CGH=CEH, so CGH=y.
As CGH=y, HGJ=180-y.
As HGJ=180-y, GHJ=y-w.
As GHJ=y-w, GHA=w-y+180.
As DBF and DGF stand on the same chord, DGF=DBF, so DGF=x.
As FGI=x, GFI=180-x-z.
As AFGH is a cyclic quadrilateral, AHG=180-AFG, so AHG=x+z.
But AHG=w-y+180, so x+z=w-y+180, or x+y+z=w+180, or DBF+CEH+FIG=GJH+180.

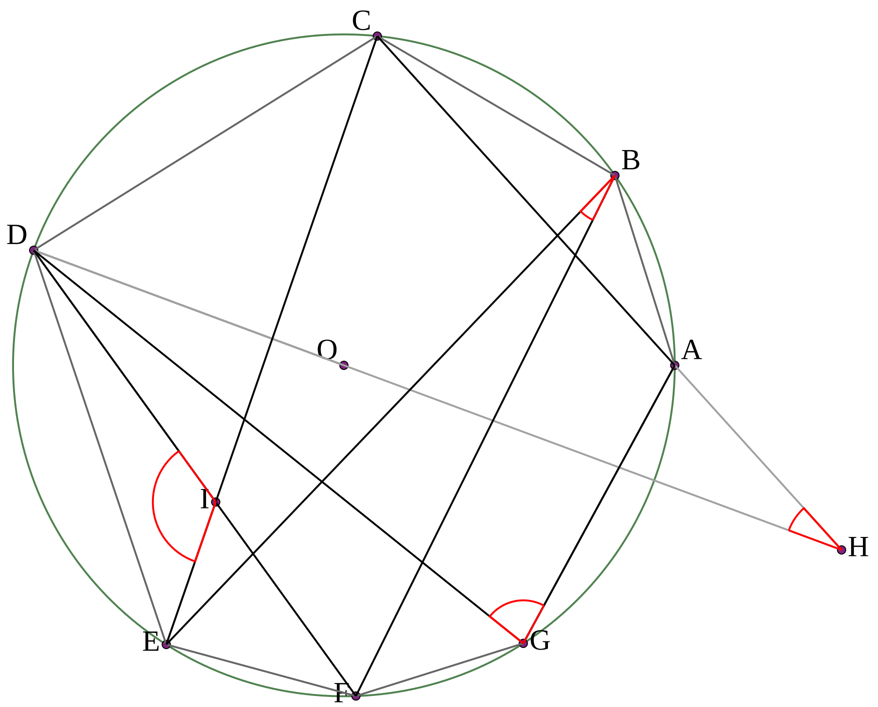
Solution to example 97



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EF and CA. Let H be the intersection of FB and AE.
Angle CBE = x. Angle BDE = y. Angle FGA = z.
Find angle BHE.

As CBE and CAE stand on the same chord, CAE=CBE, so CAE=x.
As CAE=x, EAG=180-x.
As EAG=180-x, AEG=x-z.
As BDEF is a cyclic quadrilateral, BFE=180-BDE, so BFE=180-y.
As FEH=x-z, EHF=y+z-x.
As EHF=y+z-x, EHB=x-y-z+180.

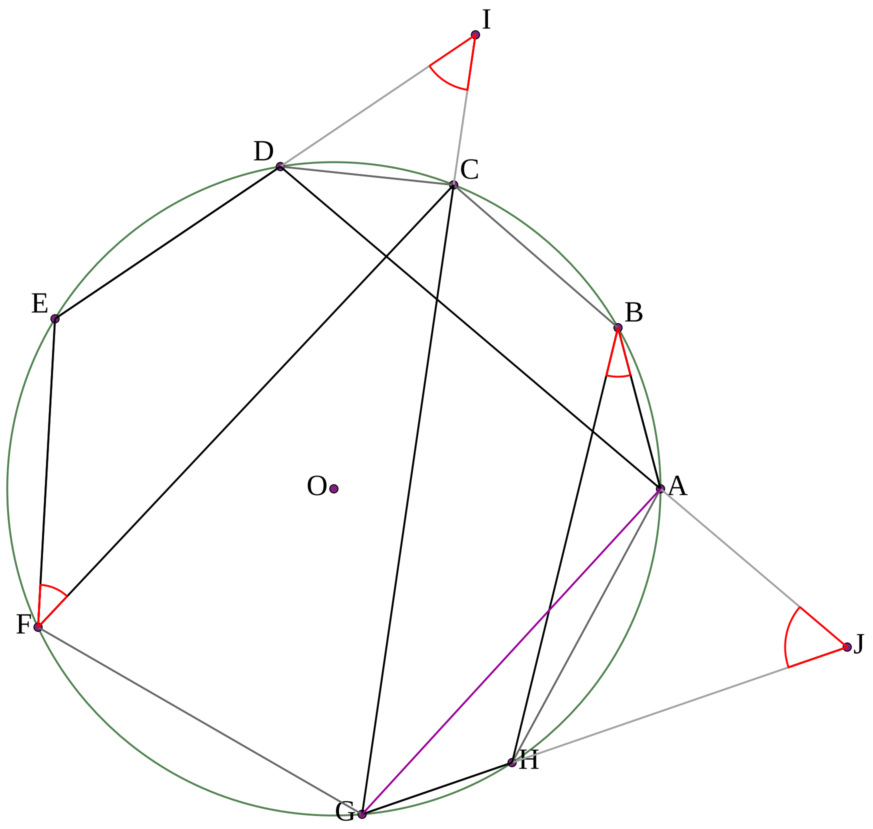
Solution to example 99



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AC and DO. Let I be the intersection of CE and FD.
Angle DGA = x. Angle AHD = y. Angle EID = z.
Find angle EBF.

As AGDC is a cyclic quadrilateral, ACD=180-AGD, so ACD=180-x.
As DCH=180-x, CDH=x-y.
As triangle CDO is isosceles, COD=2y-2x+180.
As COD is at the center of a circle on the same chord as CED, COD=2CED, so CED=y-x+90.
As DEI=y-x+90, EDI=x-y-z+90.
As EDF and EBF stand on the same chord, EBF=EDF, so EBF=x-y-z+90.

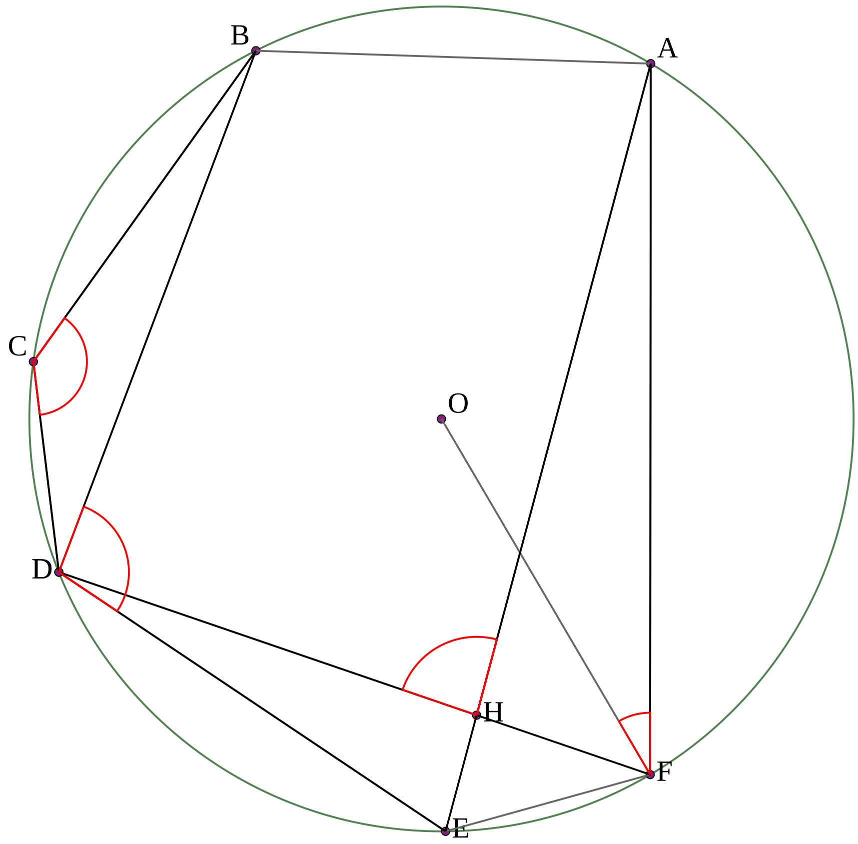
Solution to example 101



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CG and DE. Let J be the intersection of GH and AD.
Angle HBA = 28°. Angle HJA = 59°. Angle EFC = 40°.
Find angle CID.

Draw line AG.
As ABH and AGH stand on the same chord, AGH=ABH, so AGH=28.
As AGJ=28, GAJ=93.
As GAJ=93, GAD=87.
As DAG and DCG stand on the same chord, DCG=DAG, so DCG=87.
As DCG=87, DCI=93.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=140.
As CDE=140, CDI=40.
As DCI=93, CID=47.

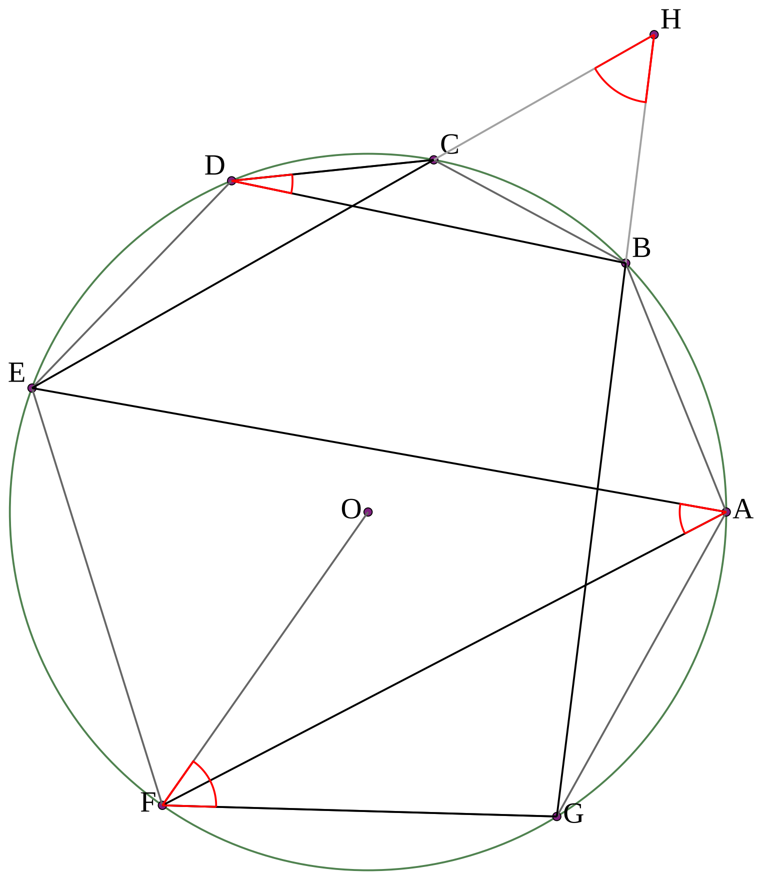
Solution to example 103



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of EA and FD.
Prove that BCD+AHD = AFO+BDE+90

Let BCD=x. Let AFO=y. Let BDE=z. Let AHD=w.
Let DEH=u.
As AED and AFD stand on the same chord, AFD=AED, so AFD=u.
As AFD=u, DFO=u-y.
As triangle DFO is isosceles, FDO=u-y.
As AHD=w, DHE=180-w.
As DHE=180-w, EDH=w-u.
As EDH=w-u, HDB=z+u-w.
As FDO=u-y, ODB=y+z-w.
As triangle BDO is isosceles, BOD=2w-2y-2z+180.
As BOD is at the center of a circle on the same chord, but in the opposite direction to BCD, BOD=360-2BCD, so BCD=y+z-w+90.
But BCD=x, so y+z-w+90=x, or y+z+90=x+w, or AFO+BDE+90=BCD+AHD.

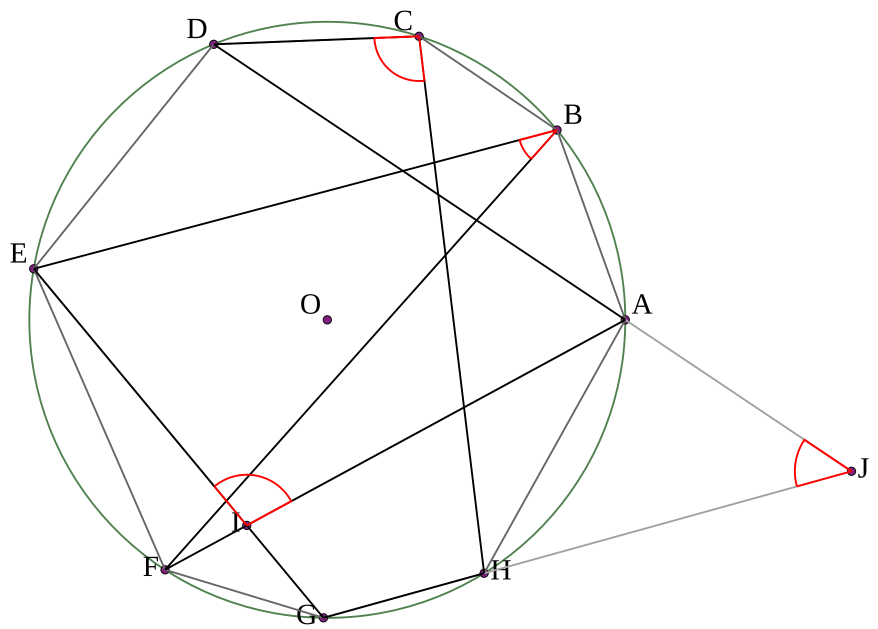
Solution to example 105



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GB and CE.
Prove that GFO+BDC+BHC = EAF+90

Let EAF=x. Let GFO=y. Let BDC=z. Let BHC=w.
Let CBH=u.
As BHC=w, BCH=180-w-u.
As BCH=180-w-u, BCE=w+u.
As BCEA is a cyclic quadrilateral, BAE=180-BCE, so BAE=180-w-u.
As BAE=180-w-u, BAF=x-w-u+180.
As BAF and BGF stand on the same chord, BGF=BAF, so BGF=x-w-u+180.
As triangle GFO is isosceles, FGO=y.
As BGF=x-w-u+180, BGO=x-y-w-u+180.
As triangle BGO is isosceles, GBO=x-y-w-u+180.
As CBH=u, CBG=180-u.
As GBO=x-y-w-u+180, OBC=y+w-x.
As triangle CBO is isosceles, BOC=2x-2y-2w+180.
As BOC is at the center of a circle on the same chord as BDC, BOC=2BDC, so BDC=x-y-w+90.
But BDC=z, so x-y-w+90=z, or x+90=y+z+w, or EAF+90=GFO+BDC+BHC.

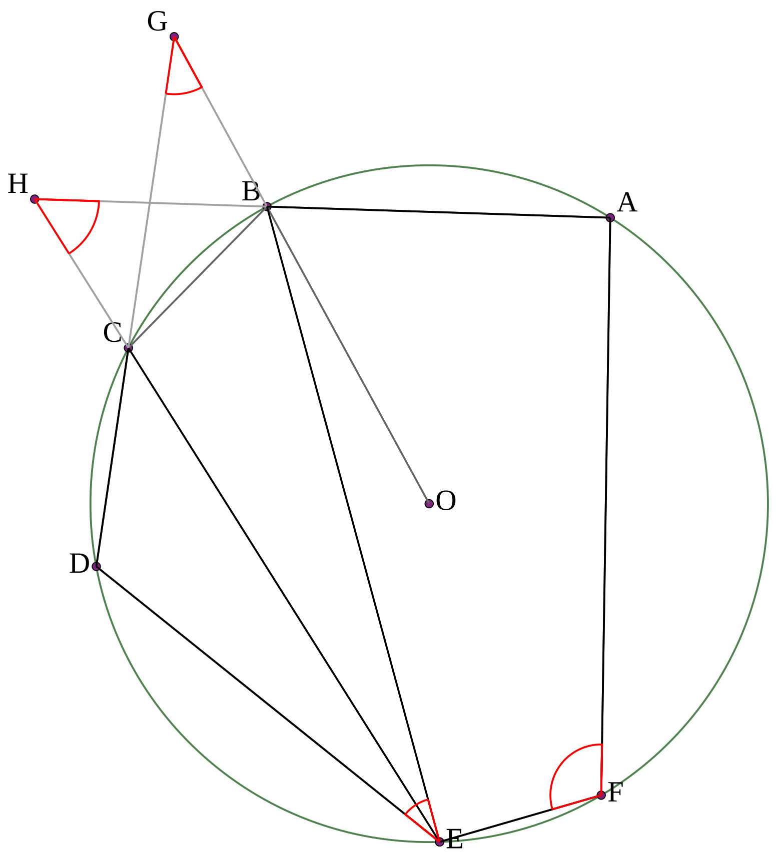
Solution to example 107



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FA and GE. Let J be the intersection of AD and HG.
Angle AJH = 49°. Angle EBF = 33°. Angle AIE = 101°.
Find angle DCH.

As EBF and EGF stand on the same chord, EGF=EBF, so EGF=33.
As AIE=101, EIF=79.
As EIF=79, FIG=101.
As FGI=33, GFI=46.
As AFGH is a cyclic quadrilateral, AHG=180-AFG, so AHG=134.
As AHG=134, AHJ=46.
As AHJ=46, HAJ=85.
As HAJ=85, HAD=95.
As DAH and DCH stand on the same chord, DCH=DAH, so DCH=95.

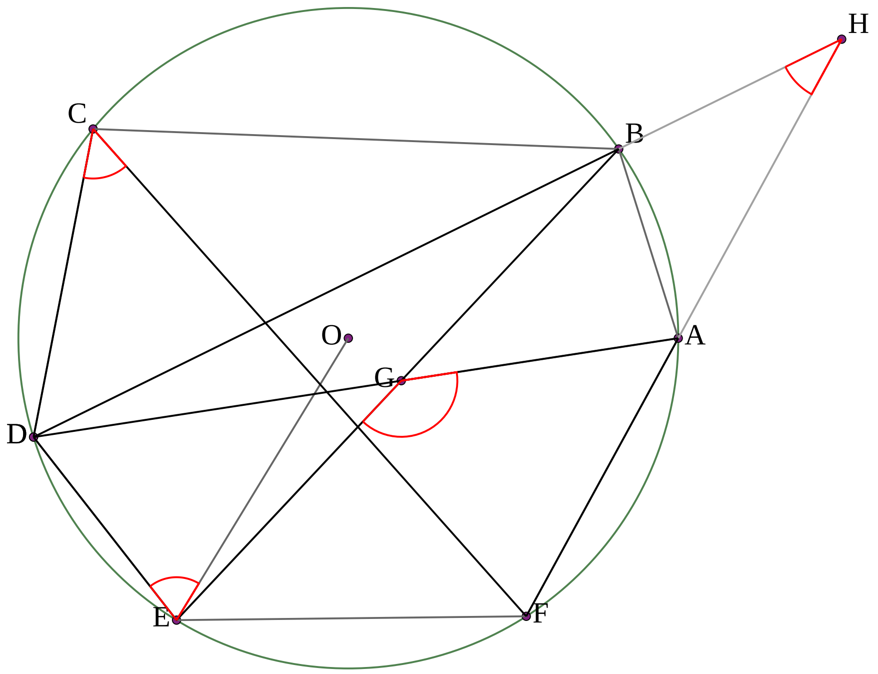
Solution to example 109



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of OB and CD. Let H be the intersection of BA and EC.
Prove that AFE+BHC = BED+BGC+90

Let BED=x. Let AFE=y. Let BGC=z. Let BHC=w.
As AFEB is a cyclic quadrilateral, ABE=180-AFE, so ABE=180-y.
As ABE=180-y, EBH=y.
As EBH=y, BEH=180-y-w.
As BEDC is a cyclic quadrilateral, BCD=180-BED, so BCD=180-x.
As BCD=180-x, BCG=x.
As BCG=x, CBG=180-x-z.
As CBG=180-x-z, CBO=x+z.
As triangle CBO is isosceles, BOC=180-2x-2z.
As BOC is at the center of a circle on the same chord as BEC, BOC=2BEC, so BEC=90-x-z.
But BEC=180-y-w, so 90-x-z=180-y-w, or x+z+90=y+w, or BED+BGC+90=AFE+BHC.

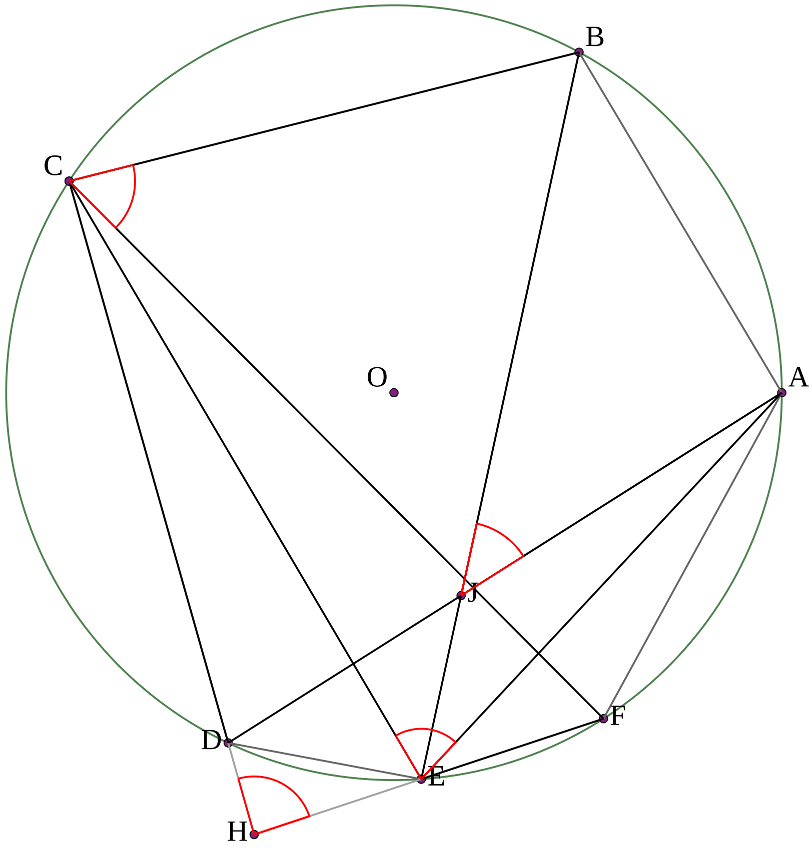
Solution to example 111



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DA and BE. Let H be the intersection of AF and DB.
Angle OED = 69°. Angle AHB = 35°. Angle FCD = 53°.
Find angle AGE.

As triangle DEO is isosceles, DOE=42.
As DOE is at the center of a circle on the same chord as DBE, DOE=2DBE, so DBE=21.
Let BAH=u.
As AHB=35, ABH=145-u.
As ABH=145-u, ABD=u+35.
As DBG=21, GBA=u+14.
As DCF and DAF stand on the same chord, DAF=DCF, so DAF=53.
As BAH=u, BAF=180-u.
As FAG=53, GAB=127-u.
As ABG=u+14, AGB=39.
As AGB=39, AGE=141.

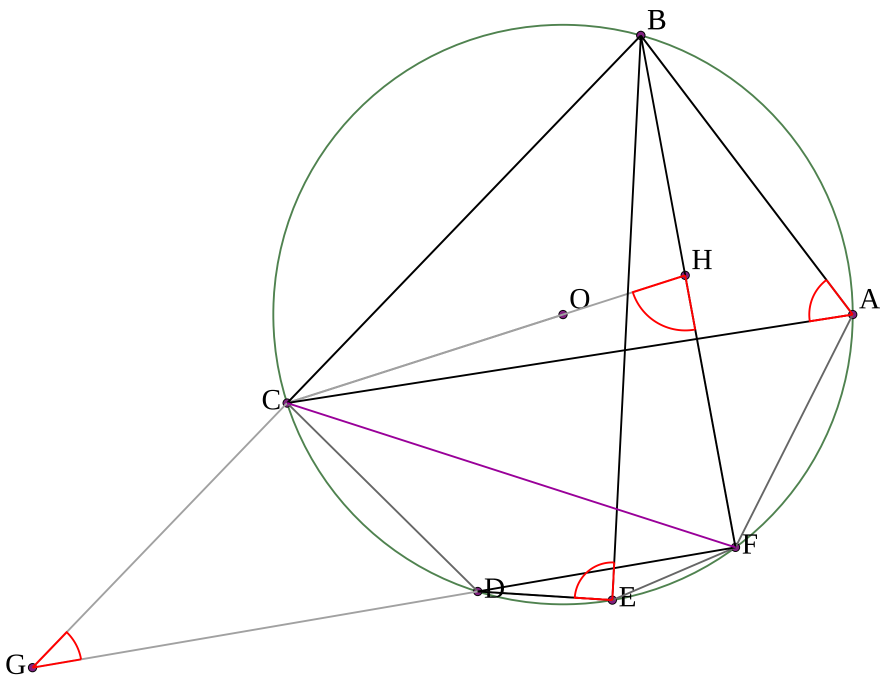
Solution to example 113



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of EF and CD. Let J be the intersection of EB and DA.
Prove that DHE+AJB = AEC+BCF

Let AEC=x. Let DHE=y. Let BCF=z. Let AJB=w.
Let ABJ=u.
As ABE and ADE stand on the same chord, ADE=ABE, so ADE=u.
As AEC and ADC stand on the same chord, ADC=AEC, so ADC=x.
As ADC=x, ADH=180-x.
As ADE=u, EDH=180-x-u.
As BCF and BEF stand on the same chord, BEF=BCF, so BEF=z.
As BEF=z, BEH=180-z.
As AJB=w, BAJ=180-w-u.
As BAD and BED stand on the same chord, BED=BAD, so BED=180-w-u.
As BEH=180-z, HED=w+u-z.
As EDH=180-x-u, DHE=x+z-w.
But DHE=y, so x+z-w=y, or x+z=y+w, or AEC+BCF=DHE+AJB.

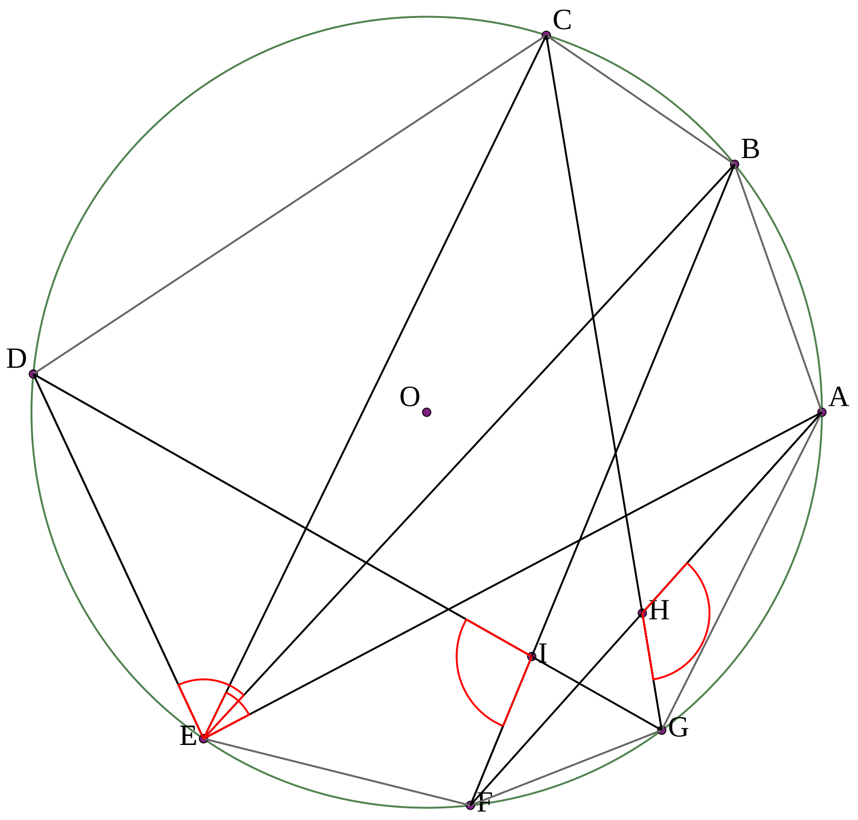
Solution to example 115



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of BC and FD. Let H be the intersection of OC and BF.
Angle CAB = x. Angle CGD = y. Angle CHF = z.
Find angle DEB.

Draw line CF.
As BAC and BFC stand on the same chord, BFC=BAC, so BFC=x.
As CFH=x, FCH=180-x-z.
As triangle FCO is isosceles, COF=2x+2z-180.
As COF is at the center of a circle on the same chord as CBF, COF=2CBF, so CBF=x+z-90.
As FBG=x+z-90, BFG=270-x-y-z.
As BFD and BED stand on the same chord, BED=BFD, so BED=270-x-y-z.

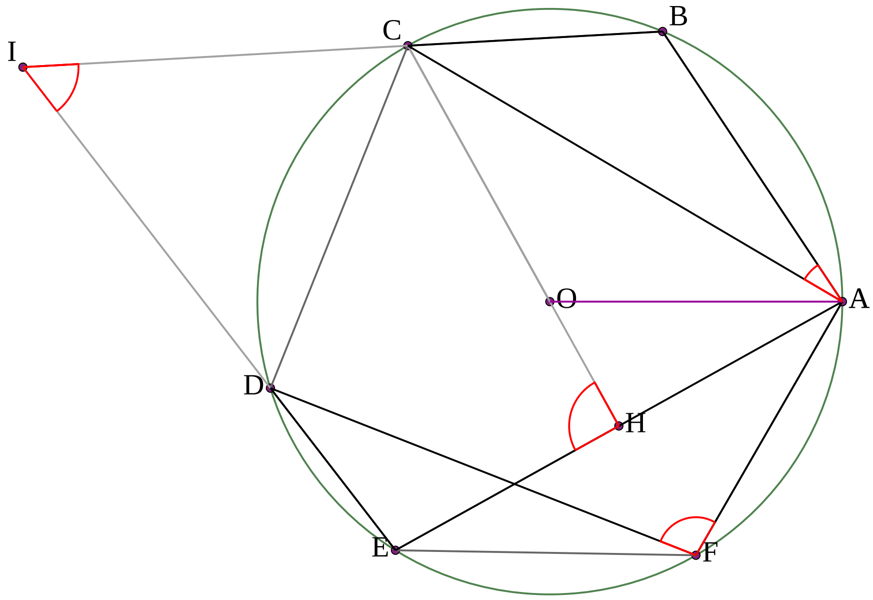
Solution to example 117



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CG and FA. Let I be the intersection of GD and BF.
Angle DEB = 68°. Angle DIF = 97°. Angle GHA = 129°.
Find angle AEC.

Let FGI=u.
As DGFE is a cyclic quadrilateral, DEF=180-DGF, so DEF=180-u.
As DEF=180-u, FEB=112-u.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=u+68.
As DIF=97, FIG=83.
As FIG=83, GFI=97-u.
As BFGA is a cyclic quadrilateral, BAG=180-BFG, so BAG=u+83.
As BAH=u+68, HAG=15.
As GAH=15, AGH=36.
As AGC and AEC stand on the same chord, AEC=AGC, so AEC=36.

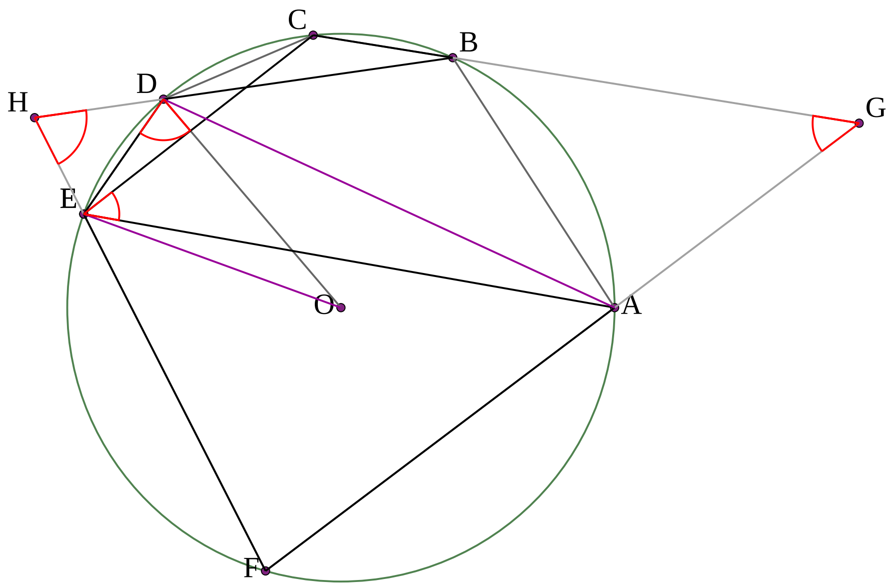
Solution to example 119



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of OC and AE. Let I be the intersection of CB and ED.
Angle DFA = 99°. Angle CAB = 26°. Angle CID = 56°.
Find angle CHE.

Draw line AO.
As AFDC is a cyclic quadrilateral, ACD=180-AFD, so ACD=81.
Let DCI=u.
As DCI=u, DCB=180-u.
As ACD=81, ACB=99-u.
As ACB=99-u, ABC=u+55.
As AOC is at the center of a circle on the same chord, but in the opposite direction to ABC, AOC=360-2ABC, so AOC=250-2u.
As triangle AOC is isosceles, ACO=u-35.
As CID=56, CDI=124-u.
As CDI=124-u, CDE=u+56.
As CDEA is a cyclic quadrilateral, CAE=180-CDE, so CAE=124-u.
As ACH=u-35, AHC=91.
As AHC=91, CHE=89.

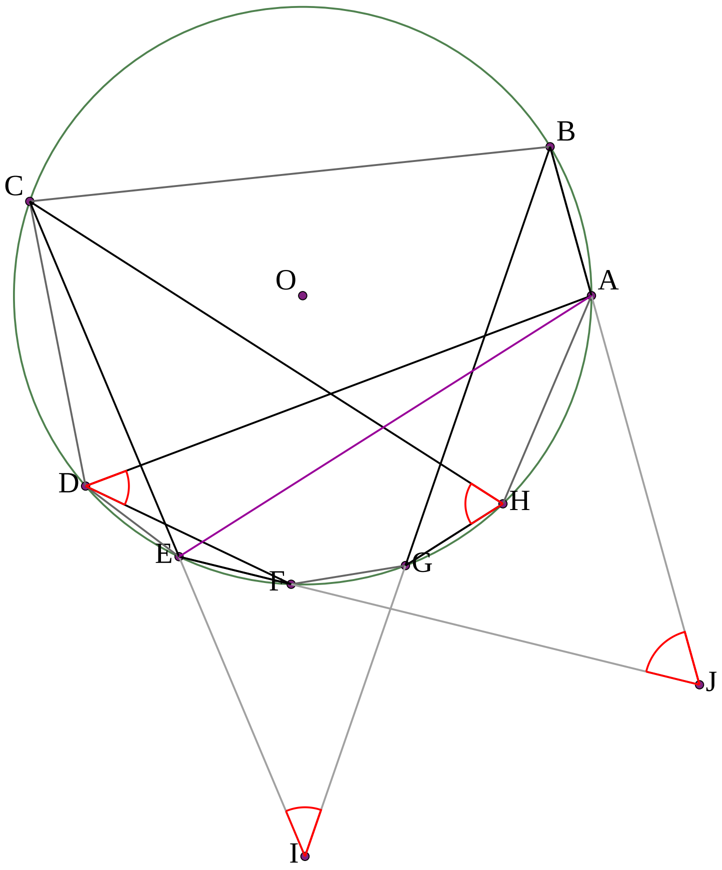
Solution to example 121



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CB and FA. Let H be the intersection of BD and EF.
Angle AEC = 48°. Angle BGA = 46°. Angle DHE = 71°.
Find angle ODE.

Draw lines AD and EO.
Let EDH=u.
As DHE=71, DEH=109-u.
As DEH=109-u, DEF=u+71.
As DEFA is a cyclic quadrilateral, DAF=180-DEF, so DAF=109-u.
As DAF=109-u, DAG=u+71.
As EDH=u, EDB=180-u.
As BDEA is a cyclic quadrilateral, BAE=180-BDE, so BAE=u.
As AECB is a cyclic quadrilateral, ABC=180-AEC, so ABC=132.
As ABC=132, ABG=48.
As ABG=48, BAG=86.
As BAE=u, EAG=u+86.
As DAG=u+71, DAE=15.
As DOE is at the center of a circle on the same chord as DAE, DOE=2DAE, so DOE=30.
As triangle DOE is isosceles, EDO=75.

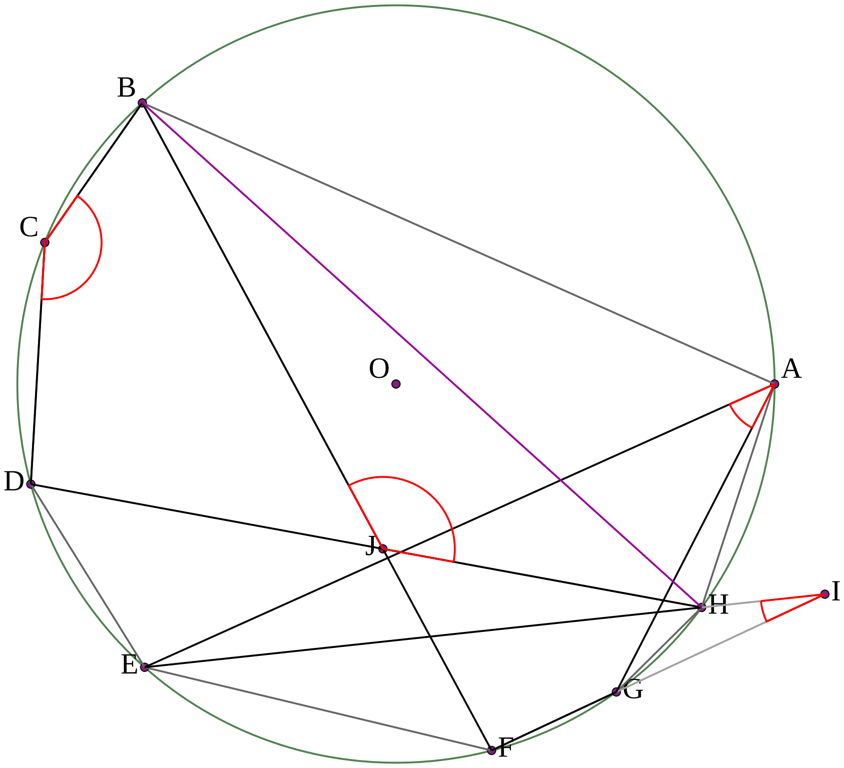
Solution to example 123



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GB and EC. Let J be the intersection of BA and FE.
Angle CHG = 65°. Angle GIE = 42°. Angle ADF = 46°.
Find angle AJF.

Draw line AE.
As CHG and CBG stand on the same chord, CBG=CHG, so CBG=65.
As CBI=65, BCI=73.
As BCEA is a cyclic quadrilateral, BAE=180-BCE, so BAE=107.
As BAE=107, EAJ=73.
As ADF and AEF stand on the same chord, AEF=ADF, so AEF=46.
As EAJ=73, AJE=61.

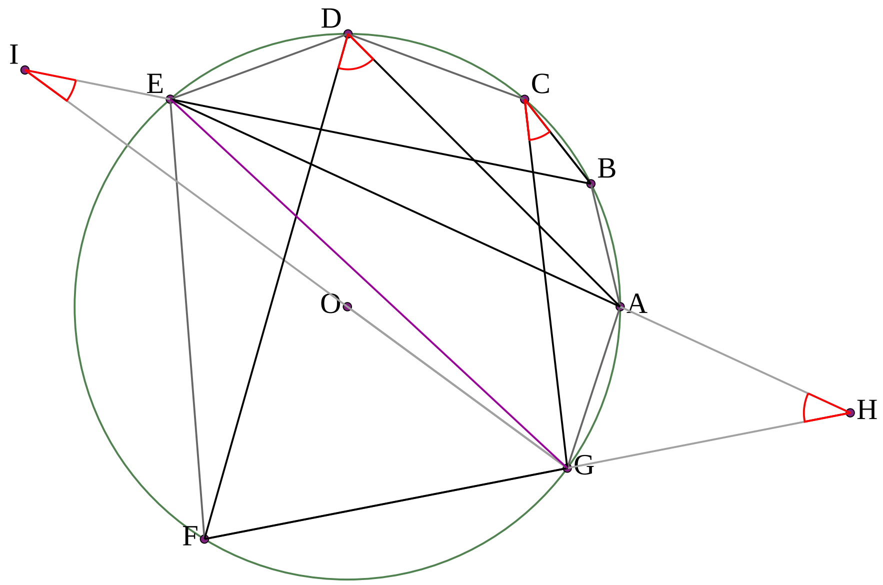
Solution to example 125



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GF and HE. Let J be the intersection of FB and DH.
Prove that BCD+GIH = EAG+BJH

Draw line BH.
Let EAG=x. Let BCD=y. Let GIH=z. Let BJH=w.
As BCDH is a cyclic quadrilateral, BHD=180-BCD, so BHD=180-y.
As BHJ=180-y, HBJ=y-w.
As EAGF is a cyclic quadrilateral, EFG=180-EAG, so EFG=180-x.
As EFI=180-x, FEI=x-z.
As FEH and FBH stand on the same chord, FBH=FEH, so FBH=x-z.
But FBH=y-w, so x-z=y-w, or x+w=y+z, or EAG+BJH=BCD+GIH.

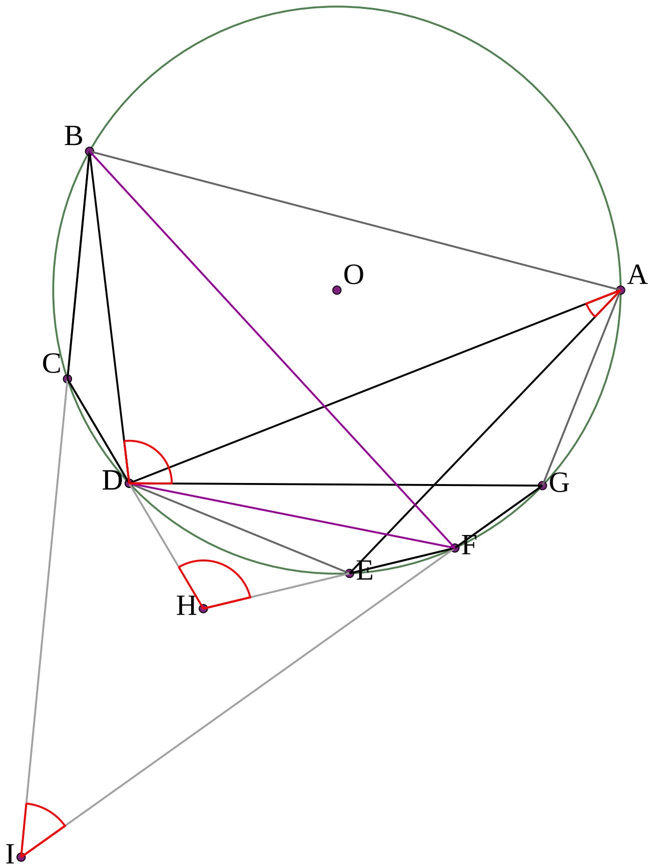
Solution to example 127



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and EA. Let I be the intersection of OG and BE.
Prove that ADF+AHG+EIG = BCG+90

Draw line EG.
Let ADF=x. Let BCG=y. Let AHG=z. Let EIG=w.
As ADF and AEF stand on the same chord, AEF=ADF, so AEF=x.
As FEH=x, EFH=180-x-z.
As BCG and BEG stand on the same chord, BEG=BCG, so BEG=y.
As BEG=y, GEI=180-y.
As GEI=180-y, EGI=y-w.
As triangle EGO is isosceles, EOG=2w-2y+180.
As EOG is at the center of a circle on the same chord as EFG, EOG=2EFG, so EFG=w-y+90.
But EFG=180-x-z, so w-y+90=180-x-z, or x+z+w=y+90, or ADF+AHG+EIG=BCG+90.

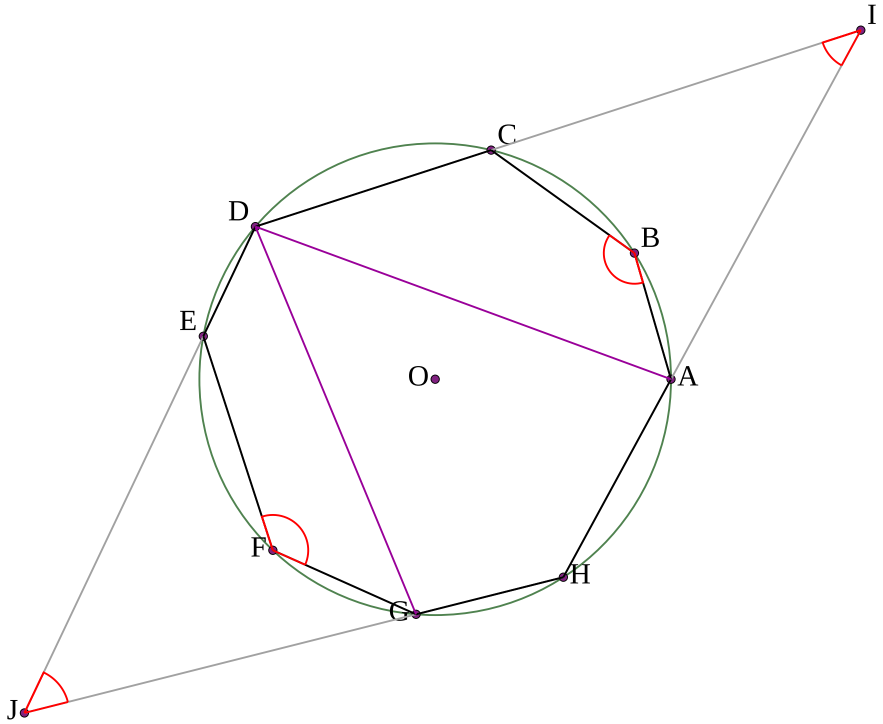
Solution to example 129



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EF and CD. Let I be the intersection of FG and BC.
Prove that DAE+BDG+DHE = CIF+180

Draw lines BF and DF.
Let DAE=x. Let BDG=y. Let DHE=z. Let CIF=w.
As BDG and BFG stand on the same chord, BFG=BDG, so BFG=y.
As BFG=y, BFI=180-y.
As BFI=180-y, FBI=y-w.
As DAE and DFE stand on the same chord, DFE=DAE, so DFE=x.
As DFH=x, FDH=180-x-z.
As FDH=180-x-z, FDC=x+z.
As CDFB is a cyclic quadrilateral, CBF=180-CDF, so CBF=180-x-z.
But CBF=y-w, so 180-x-z=y-w, or x+y+z=w+180, or DAE+BDG+DHE=CIF+180.

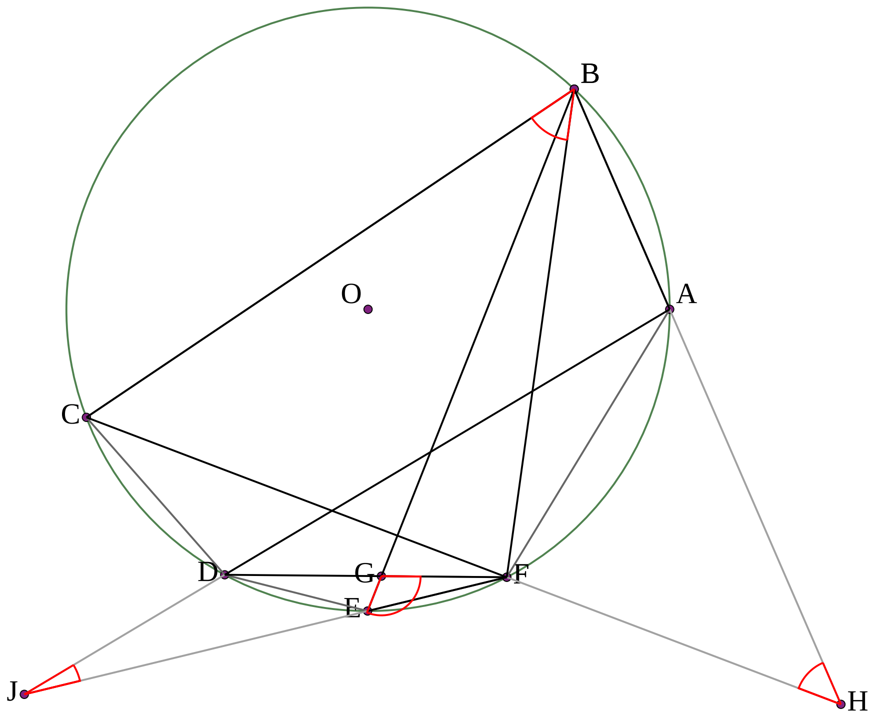
Solution to example 131



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CD and HA. Let J be the intersection of DE and GH.
Prove that ABC+EFG = AIC+EJG+180

Draw lines DG and AD.
Let ABC=x. Let EFG=y. Let AIC=z. Let EJG=w.
As EFGD is a cyclic quadrilateral, EDG=180-EFG, so EDG=180-y.
As GDJ=180-y, DGJ=y-w.
As DGJ=y-w, DGH=w-y+180.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-x.
As ADI=180-x, DAI=x-z.
As DAI=x-z, DAH=z-x+180.
As DAHG is a cyclic quadrilateral, DGH=180-DAH, so DGH=x-z.
But DGH=w-y+180, so x-z=w-y+180, or z+w+180=x+y, or AIC+EJG+180=ABC+EFG.

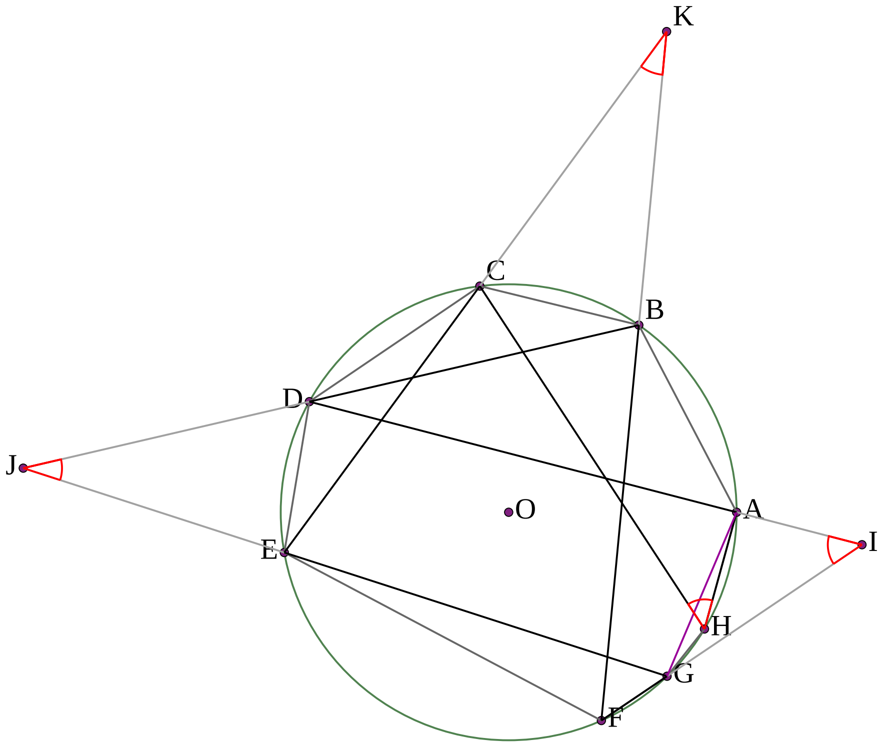
Solution to example 133



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DF and BE. Let H be the intersection of FC and BA. Let J be the intersection of EF and AD.
Angle FGE = x. Angle CBF = y. Angle EJD = z.
Find angle FHA.

Let DFJ=u.
As EFG=u, FEG=180-x-u.
As BEFA is a cyclic quadrilateral, BAF=180-BEF, so BAF=x+u.
As BAF=x+u, FAH=180-x-u.
As CBFD is a cyclic quadrilateral, CDF=180-CBF, so CDF=180-y.
As DJF=z, FDJ=180-z-u.
As CDF=180-y, CDJ=y+z+u.
As CDJ=y+z+u, CDA=180-y-z-u.
As ADC and AFC stand on the same chord, AFC=ADC, so AFC=180-y-z-u.
As AFC=180-y-z-u, AFH=y+z+u.
As FAH=180-x-u, AHF=x-y-z.

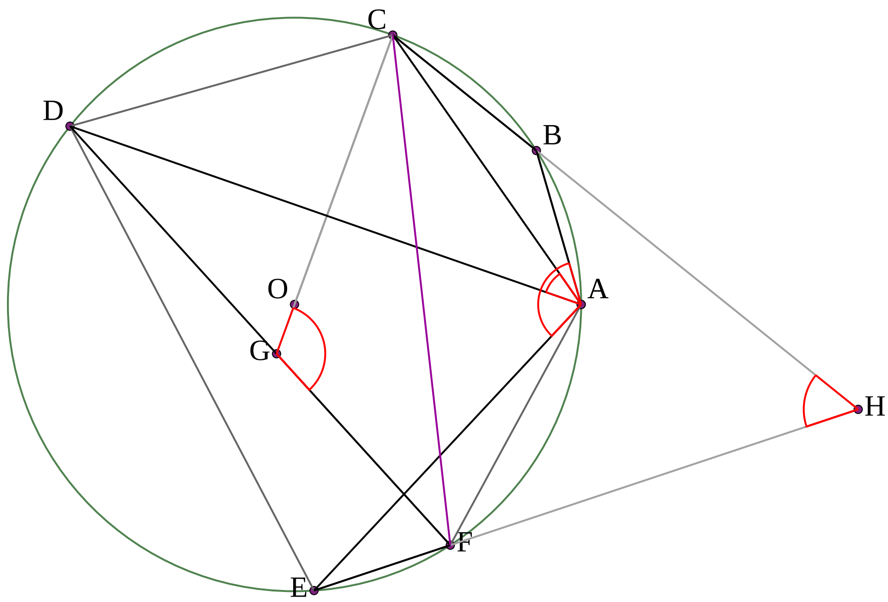
Solution to example 135



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of AD and FG. Let J be the intersection of DB and GE. Let K be the intersection of BF and EC.
Angle CHA = x. Angle BKC = y. Angle DJE = z.
Find angle AIG.

Draw line AG.
Let CBK=u.
As BKC=y, BCK=180-y-u.
As BCK=180-y-u, BCE=y+u.
As BCE and BDE stand on the same chord, BDE=BCE, so BDE=y+u.
As BDE=y+u, EDJ=180-y-u.
As EDJ=180-y-u, DEJ=y+u-z.
As DEJ=y+u-z, DEG=z-y-u+180.
As DEGA is a cyclic quadrilateral, DAG=180-DEG, so DAG=y+u-z.
As DAG=y+u-z, GAI=z-y-u+180.
As AHCB is a cyclic quadrilateral, ABC=180-AHC, so ABC=180-x.
As CBK=u, CBF=180-u.
As ABC=180-x, ABF=u-x.
As ABFG is a cyclic quadrilateral, AGF=180-ABF, so AGF=x-u+180.
As AGF=x-u+180, AGI=u-x.
As GAI=z-y-u+180, AIG=x+y-z.

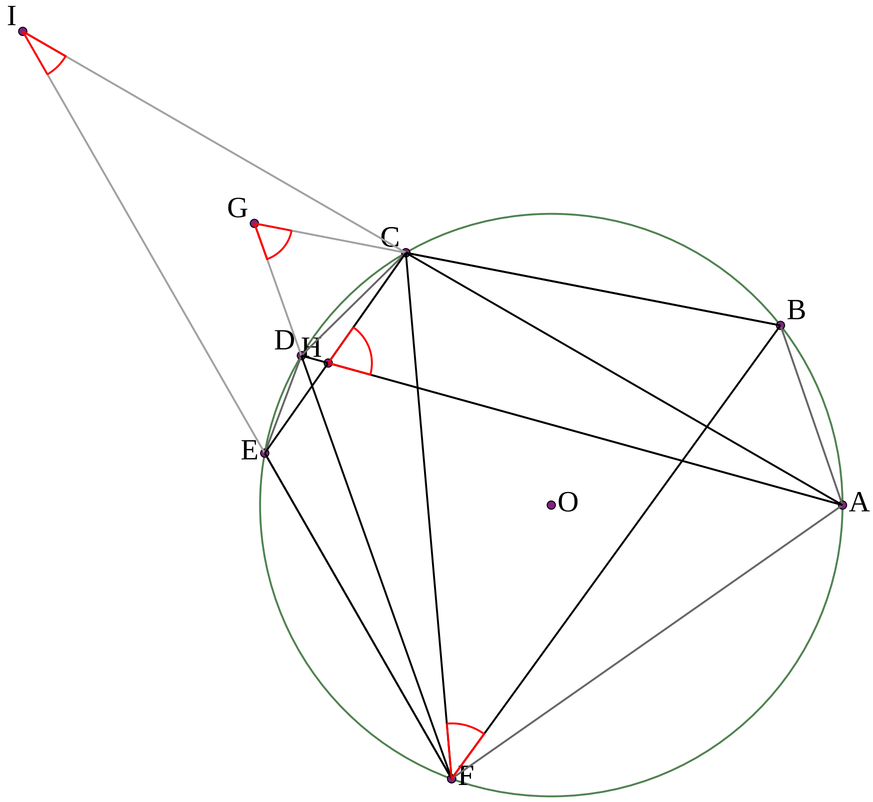
Solution to example 137



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of DF and CO. Let H be the intersection of FE and BC.
Angle EAB = x. Angle CAD = y. Angle FGC = z.
Find angle FHB.

Draw line CF.
As CAD and CFD stand on the same chord, CFD=CAD, so CFD=y.
Let CFH=u.
As CFH=u, CFE=180-u.
As CFD=y, DFE=180-y-u.
As DFE and DAE stand on the same chord, DAE=DFE, so DAE=180-y-u.
As DAE=180-y-u, DAB=y+x+u-180.
As BADC is a cyclic quadrilateral, BCD=180-BAD, so BCD=360-y-x-u.
As CFG=y, FCG=180-y-z.
As triangle FCO is isosceles, COF=2y+2z-180.
As COF is at the center of a circle on the same chord as CDF, COF=2CDF, so CDF=y+z-90.
As CDF=y+z-90, DCF=270-2y-z.
As DCH=360-y-x-u, HCF=y+z-x-u+90.
As FCH=y+z-x-u+90, CHF=x-y-z+90.

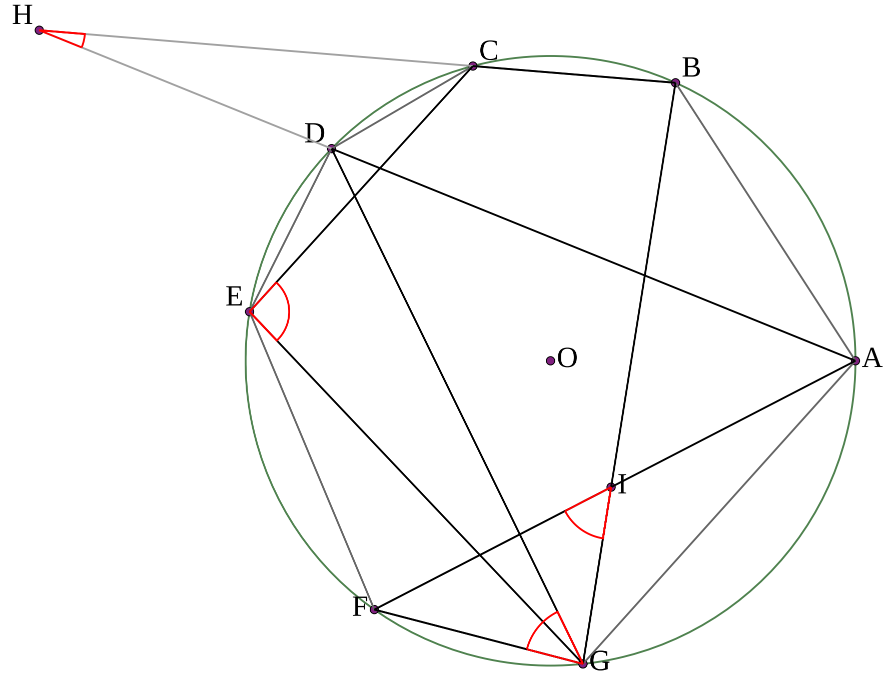
Solution to example 139



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of FD and CB. Let H be the intersection of DA and EC. Let I be the intersection of AC and FE.
Prove that AHC+CIE = CGD+BFC

Let CGD=x. Let AHC=y. Let CIE=z. Let BFC=w.
Let ECI=u.
As CIE=z, CEI=180-z-u.
As CEI=180-z-u, CEF=z+u.
As CEFB is a cyclic quadrilateral, CBF=180-CEF, so CBF=180-z-u.
As CBF=180-z-u, BCF=z+u-w.
As BCF=z+u-w, FCG=w-z-u+180.
As FCG=w-z-u+180, CFG=z+u-x-w.
As HCI=u, HCA=180-u.
As ACH=180-u, CAH=u-y.
As CAD and CFD stand on the same chord, CFD=CAD, so CFD=u-y.
But CFG=z+u-x-w, so u-y=z+u-x-w, or x+w=y+z, or CGD+BFC=AHC+CIE.

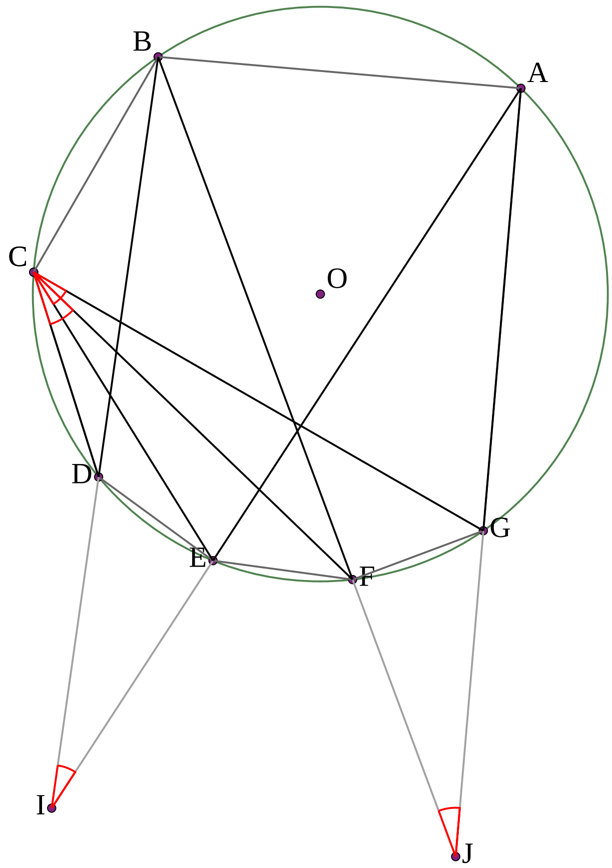
Solution to example 141



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CB and DA. Let I be the intersection of BG and AF.
Prove that CEG+FIG+DGF = CHD+180

Let CEG=x. Let CHD=y. Let FIG=z. Let DGF=w.
Let FGI=u.
As BGF and BAF stand on the same chord, BAF=BGF, so BAF=u.
As DGF and DAF stand on the same chord, DAF=DGF, so DAF=w.
As BAF=u, BAH=u-w.
As CEGB is a cyclic quadrilateral, CBG=180-CEG, so CBG=180-x.
As FIG=z, GFI=180-z-u.
As AFG and ABG stand on the same chord, ABG=AFG, so ABG=180-z-u.
As GBH=180-x, HBA=360-x-z-u.
As BAH=u-w, AHB=x+z+w-180.
But AHB=y, so x+z+w-180=y, or x+z+w=y+180, or CEG+FIG+DGF=CHD+180.

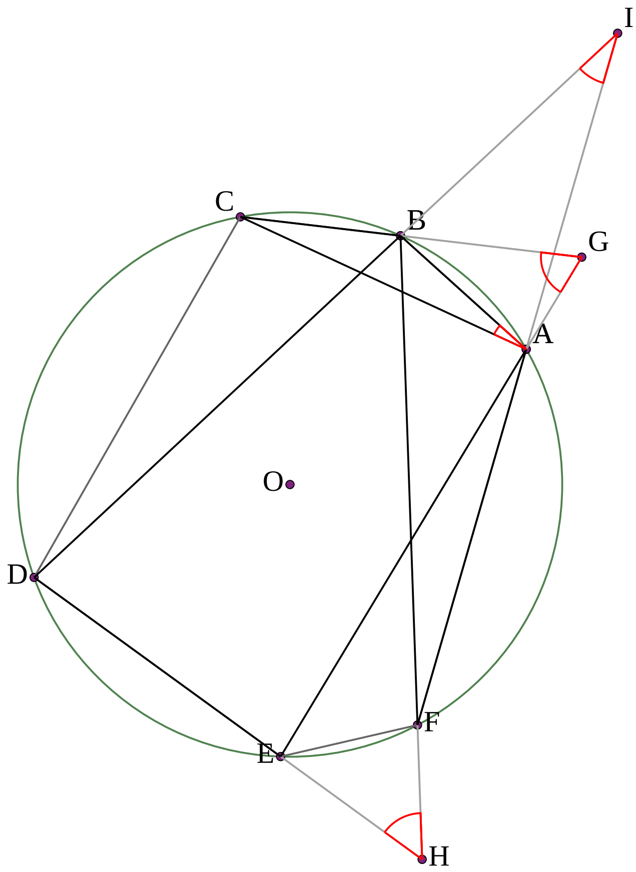
Solution to example 143



Let ABCDEFG be a cyclic heptagon with center O. Let I be the intersection of EA and BD. Let J be the intersection of AG and FB.
Prove that DIE+DCF = ECG+FJG

Let ECG=x. Let DIE=y. Let FJG=z. Let DCF=w.
Let FGJ=u.
As FGJ=u, FGA=180-u.
As AGFB is a cyclic quadrilateral, ABF=180-AGF, so ABF=u.
As DCF and DBF stand on the same chord, DBF=DCF, so DBF=w.
As ABF=u, ABI=w+u.
As ECG and EAG stand on the same chord, EAG=ECG, so EAG=x.
As FJG=z, GFJ=180-z-u.
As GFJ=180-z-u, GFB=z+u.
As BFGA is a cyclic quadrilateral, BAG=180-BFG, so BAG=180-z-u.
As GAI=x, IAB=180-x-z-u.
As ABI=w+u, AIB=x+z-w.
But AIB=y, so x+z-w=y, or x+z=y+w, or ECG+FJG=DIE+DCF.

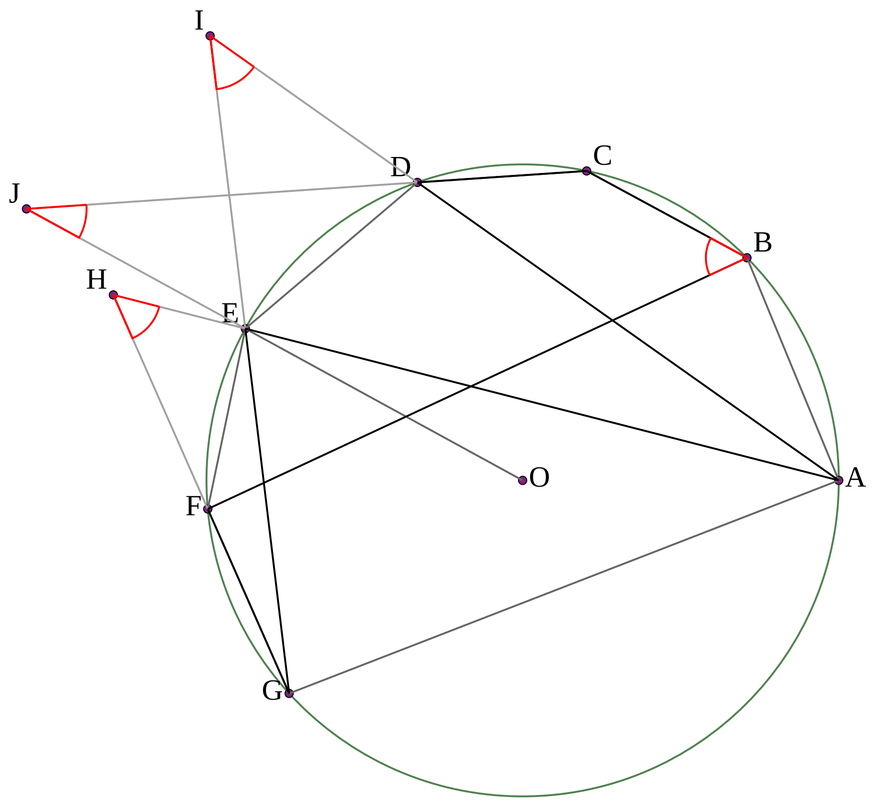
Solution to example 145



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CB and AE. Let H be the intersection of BF and ED. Let I be the intersection of FA and DB.
Prove that EHF+AIB = BAC+AGB

Let BAC=x. Let AGB=y. Let EHF=z. Let AIB=w.
Let EFH=u.
As EHF=z, FEH=180-z-u.
As FEH=180-z-u, FED=z+u.
As DEFB is a cyclic quadrilateral, DBF=180-DEF, so DBF=180-z-u.
As DBF=180-z-u, FBI=z+u.
As FBI=z+u, BFI=180-z-w-u.
As EFH=u, EFB=180-u.
As BFE and BAE stand on the same chord, BAE=BFE, so BAE=180-u.
As BAE=180-u, BAG=u.
As BAG=u, ABG=180-y-u.
As ABG=180-y-u, ABC=y+u.
As ABC=y+u, ACB=180-x-y-u.
As ACB and AFB stand on the same chord, AFB=ACB, so AFB=180-x-y-u.
But AFB=180-z-w-u, so 180-x-y-u=180-z-w-u, or z+w=x+y, or EHF+AIB=BAC+AGB.

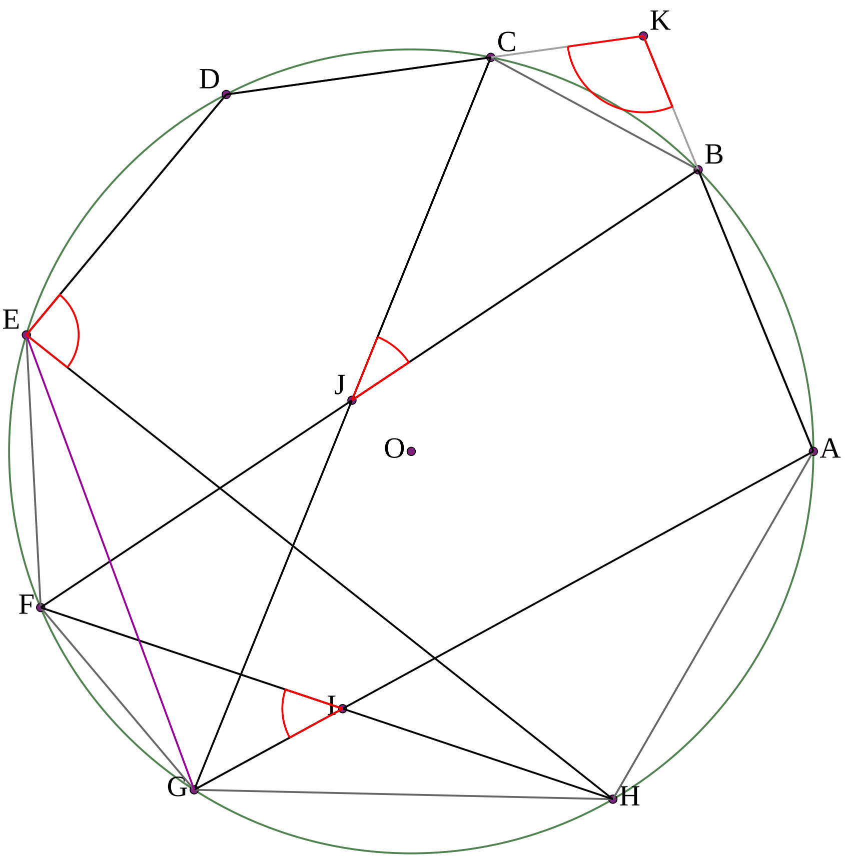
Solution to example 147



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and EA. Let I be the intersection of GE and AD. Let J be the intersection of OE and DC.
Angle CBF = x. Angle FHE = y. Angle EJD = z.
Find angle EID.

Let DEI=u.
As DEI=u, DEG=180-u.
Let CDE=v.
As CDE=v, EDJ=180-v.
As EDJ=180-v, DEJ=v-z.
As DEG=180-u, GEJ=z+u-v+180.
As GEJ=z+u-v+180, GEO=v-z-u.
As triangle GEO is isosceles,
EOG=2z+2u-2v+180.
As EOG is at the center of a circle on the same chord, but in the opposite direction to EFG,
EOG=360-2EFG, so EFG=v-z-u+90.
As EFG=v-z-u+90, EFH=z+u-v+90.
As EFH=z+u-v+90, FEH=v-y-z-u+90.
As FEH=v-y-z-u+90, FEA=y+z+u-v+90.
As AEF and ABF stand on the same chord,
ABF=AEF, so ABF=y+z+u-v+90.
As ABF=y+z+u-v+90, ABC=x+y+z+u-v+90.
As ABCD is a cyclic quadrilateral,
ADC=180-ABC, so ADC=v-x-y-z-u+90.
As ADC=v-x-y-z-u+90, ADE=x+y+z+u-90.
As ADE=x+y+z+u-90, EDI=270-x-y-z-u.
As EDI=270-x-y-z-u, DIE=x+y+z-90.

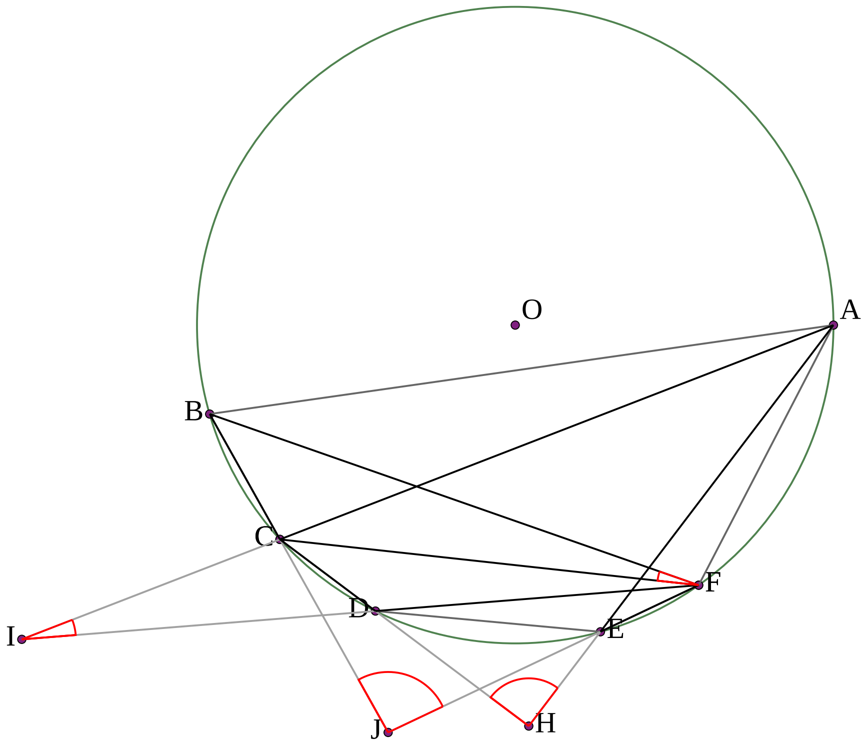
Solution to example 149



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of HF and AG. Let J be the intersection of FB and GC. Let K be the intersection of BA and CD.
Angle DEH = x. Angle BJC = y. Angle BKC = z.
Find angle FIG.

Draw line EG.
Let CBJ=u.
As BJC=y, BCJ=180-y-u.
Let CBK=v.
As BKC=z, BCK=180-z-v.
As BCK=180-z-v, BCD=z+v.
As BCG=180-y-u, GCD=y+z+u+v-180.
As DCGE is a cyclic quadrilateral, DEG=180-DCG, so DEG=360-y-z-u-v.
As DEG=360-y-z-u-v, GEH=360-x-y-z-u-v.
As GEH and GFH stand on the same chord, GFH=GEH, so GFH=360-x-y-z-u-v.
As CBK=v, CBA=180-v.
As CBF=u, FBA=180-u-v.
As ABFG is a cyclic quadrilateral, AGF=180-ABF, so AGF=u+v.
As GFI=360-x-y-z-u-v, FIG=x+y+z-180.

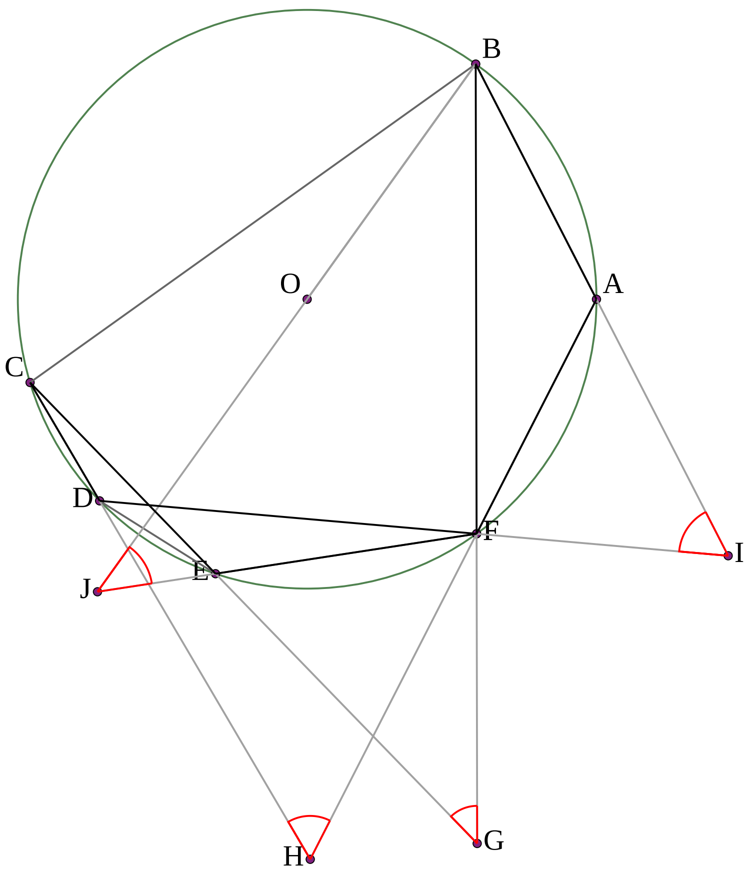
Solution to example 151



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of CD and AE. Let I be the intersection of DF and CA. Let J be the intersection of BC and EF.
Prove that BFC+CJE = DHE+CID

Let BFC=x. Let DHE=y. Let CID=z. Let CJE=w.
Let CDI=u.
As CID=z, DCI=180-z-u.
As DCI=180-z-u, DCA=z+u.
As ACDE is a cyclic quadrilateral, AED=180-ACD, so AED=180-z-u.
As AED=180-z-u, DEH=z+u.
As DEH=z+u, EDH=180-y-z-u.
Let FCJ=v.
As CJF=w, CFJ=180-w-v.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=w+v.
As CDE=w+v, EDH=180-w-v.
But EDH=180-y-z-u, so 180-w-v=180-y-z-u, or y+z+u=w+v.
As FCJ=v, FCB=180-v.
As CDI=u, CDF=180-u.
As CDFB is a cyclic quadrilateral, CBF=180-CDF, so CBF=u.
As BCF=180-v, BFC=v-u.
But BFC=x, so v-u=x, or v=x+u.
We have these equations: w+v-y-z-u=0 (E1), x+u-v=0 (E2).
Hence x+w-y-z=0 (E1+E2), or x+w=y+z, or BFC+CJE=DHE+CID.

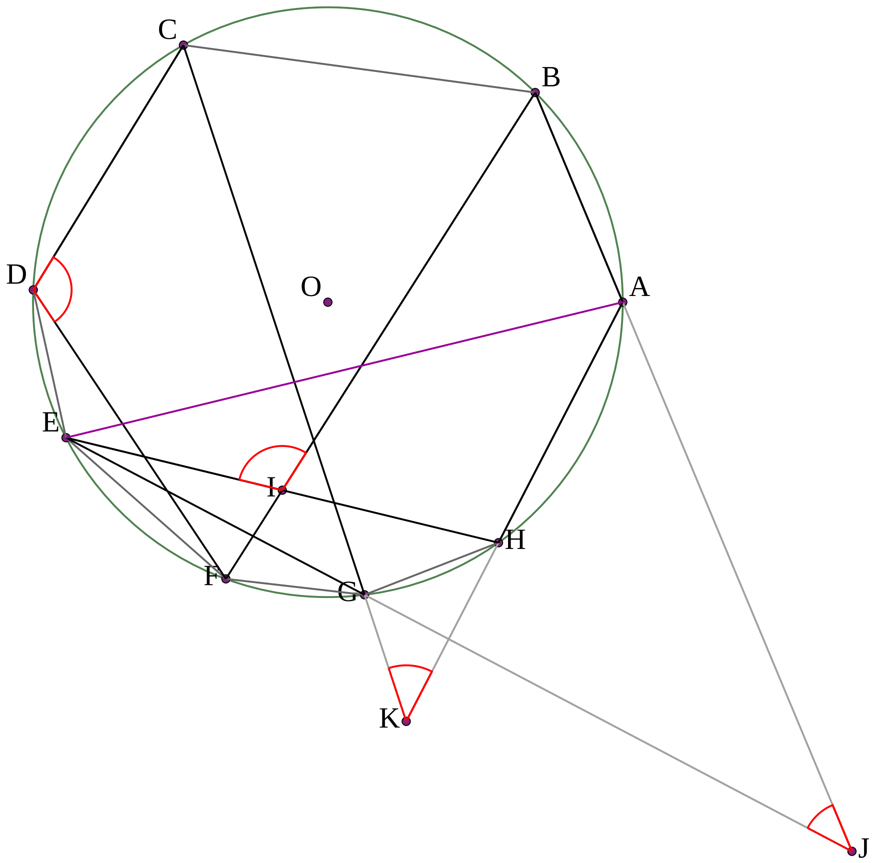
Solution to example 153



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and FB. Let H be the intersection of CD and AF. Let I be the intersection of DF and BA. Let J be the intersection of OB and FE.
Angle DHF = x. Angle EGF = y. Angle FIA = z.
Find angle BJE.

Let FBJ=u.
As triangle FBO is isosceles, BOF=180-2u.
As BOF is at the center of a circle on the same chord, but in the opposite direction to BAF, BOF=360-2BAF, so BAF=u+90.
As BAF=u+90, FAI=90-u.
As FAI=90-u, AFI=u-z+90.
As AFI=u-z+90, IFH=z-u+90.
As HFI=z-u+90, HFD=u-z+90.
As DFH=u-z+90, FDH=z-x-u+90.
As FDH=z-x-u+90, FDC=x+u-z+90.
As CDF and CEF stand on the same chord, CEF=CDF, so CEF=x+u-z+90.
As CEF=x+u-z+90, FEG=z-x-u+90.
As FEG=z-x-u+90, EFG=x+u-y-z+90.
As EFG=x+u-y-z+90, EFB=y+z-x-u+90.
As BFJ=y+z-x-u+90, BJF=x-y-z+90.

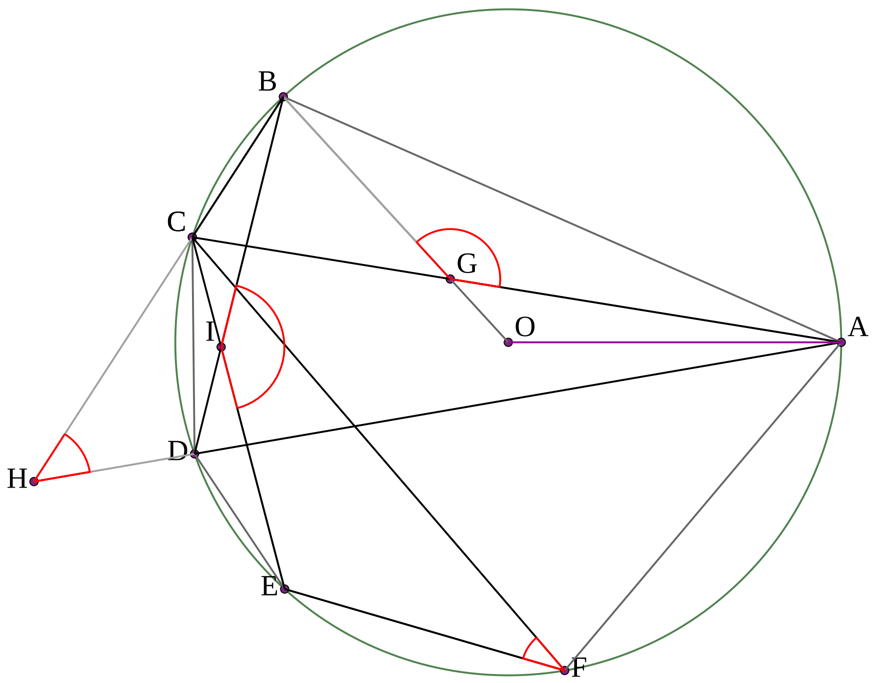
Solution to example 155



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FB and HE. Let J be the intersection of BA and EG. Let K be the intersection of AH and GC.
Angle BIE = 109°. Angle HKG = 46°. Angle CDF = 115°.
Find angle AJG.

Draw line AE.
As CDFG is a cyclic quadrilateral, CGF=180-CDF, so CGF=65.
As CGF=65, FGK=115.
Let GHK=u.
As GKH=46, HGK=134-u.
As FGK=115, FGH=u+111.
As FGHE is a cyclic quadrilateral, FEH=180-FGH, so FEH=69-u.
As BIE=109, EIF=71.
As FEI=69-u, EFI=u+40.
As BFE and BAE stand on the same chord, BAE=BFE, so BAE=u+40.
As BAE=u+40, EAJ=140-u.
As GHK=u, GHA=180-u.
As AHGE is a cyclic quadrilateral, AEG=180-AHG, so AEG=u.
As EAJ=140-u, AJE=40.

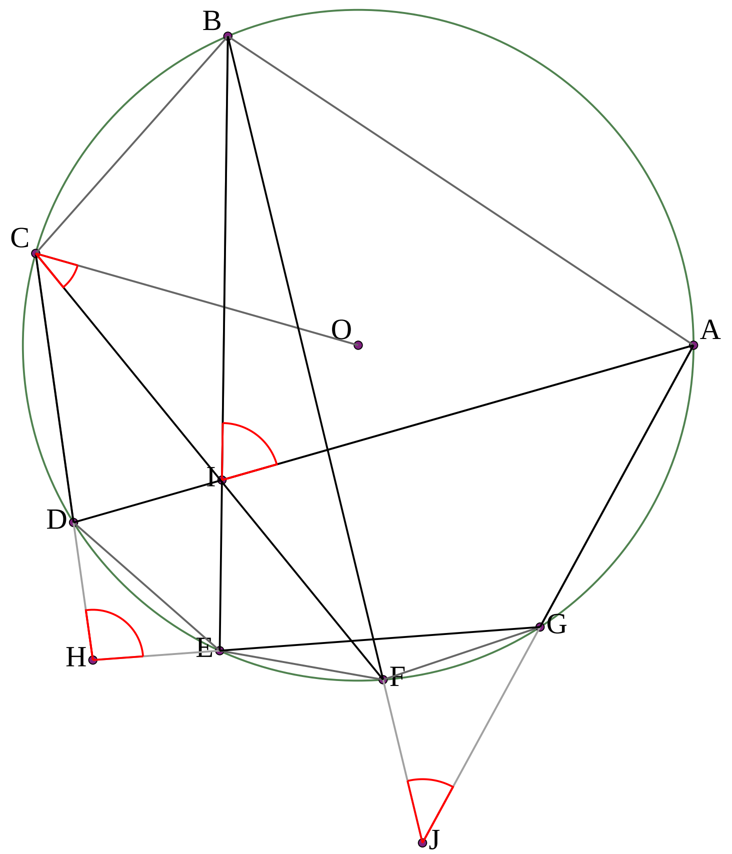
Solution to example 157



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of CA and BO. Let H be the intersection of AD and BC. Let I be the intersection of DB and CE.
Angle EFC = 33°. Angle DHC = 47°. Angle BIE = 151°.
Find angle AGB.

Draw line AO.
As BIE=151, EID=29.
Let CDI=u.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=147.
As CDI=u, IDE=147-u.
As DIE=29, DEI=u+4.
As CED and CBD stand on the same chord, CBD=CED, so CBD=u+4.
As DBH=u+4, BDH=129-u.
As BDH=129-u, BDA=u+51.
As AOB is at the center of a circle on the same chord as ADB, AOB=2ADB, so AOB=2u+102.
As triangle AOB is isosceles, ABO=39-u.
As BDC and BAC stand on the same chord, BAC=BDC, so BAC=u.
As ABG=39-u, AGB=141.

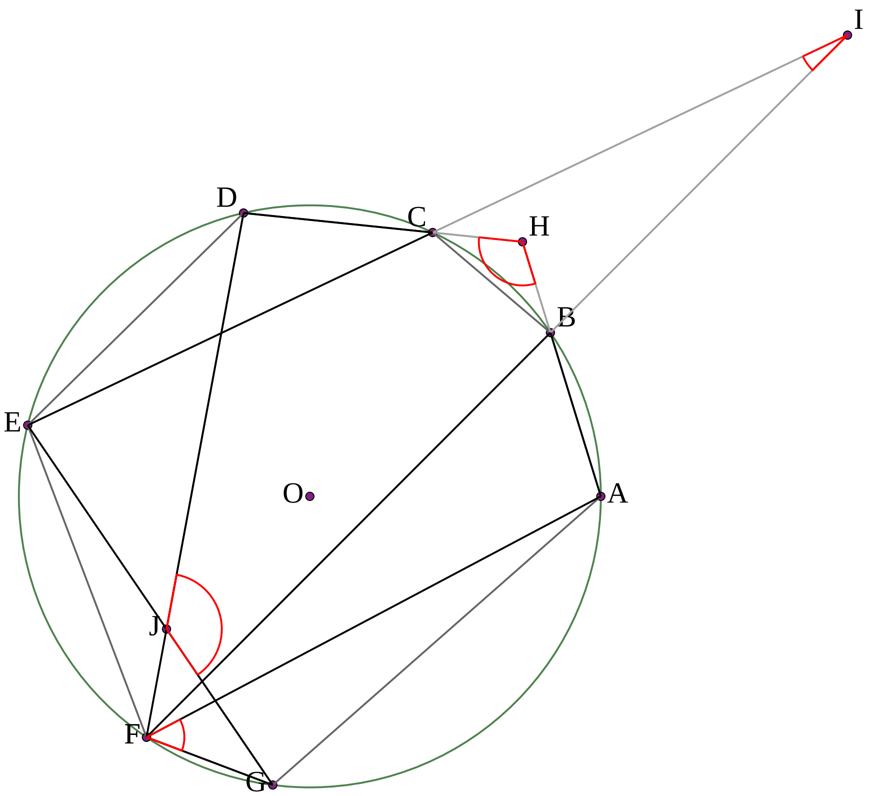
Solution to example 159



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of CD and GE. Let I be the intersection of DA and EB. Let J be the intersection of AG and BF.
Angle DHE = x. Angle AIB = y. Angle FCO = z.
Find angle GJF.

Let BAI=u.
As AIB=y, ABI=180-y-u.
As ABE and ADE stand on the same chord, ADE=ABE, so ADE=180-y-u.
As triangle FCO is isosceles, COF=180-2z.
As COF is at the center of a circle on the same chord as CBF, COF=2CBF, so CBF=90-z.
Let FGJ=v.
As FGJ=v, FGA=180-v.
As AGFB is a cyclic quadrilateral, ABF=180-AGF, so ABF=v.
As CBF=90-z, CBA=v-z+90.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=z-v+90.
As ADE=180-y-u, EDC=z-y-u-v+270.
As CDE=z-y-u-v+270, EDH=y+u+v-z-90.
As EDH=y+u+v-z-90, DEH=z-x-y-u-v+270.
As DEH=z-x-y-u-v+270, DEG=x+y+u+v-z-90.
As BAD and BED stand on the same chord, BED=BAD, so BED=u.
As DEG=x+y+u+v-z-90, GEB=x+y+v-z-90.
As BEG and BFG stand on the same chord, BFG=BEG, so BFG=x+y+v-z-90.
As BFG=x+y+v-z-90, GFJ=z-x-y-v+270.
As GFJ=z-x-y-v+270, FJG=x+y-z-90.

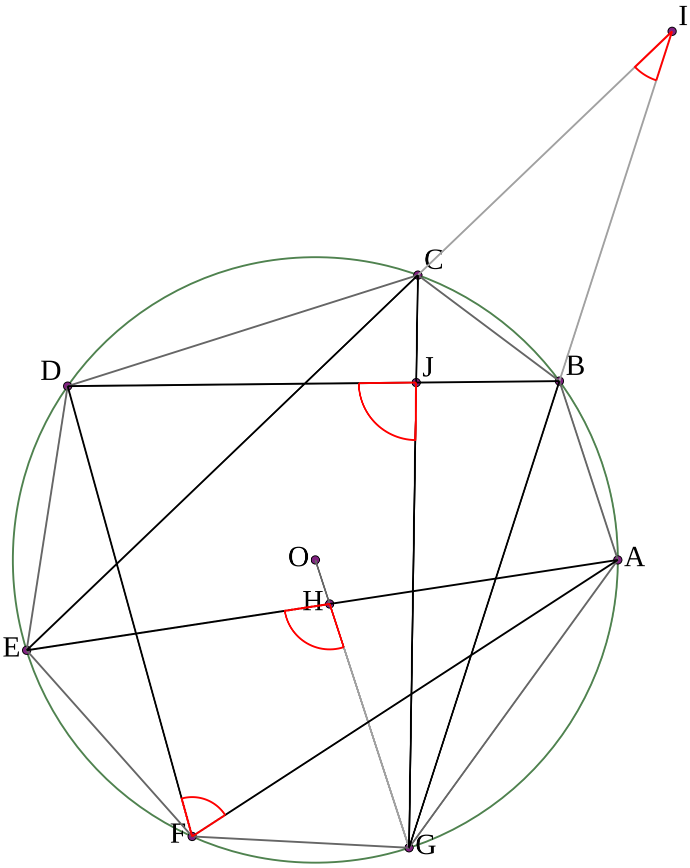
Solution to example 161



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AB and DC. Let I be the intersection of BF and CE. Let J be the intersection of FD and EG.
Angle GFA = 49°. Angle BIC = 20°. Angle BHC = 113°.
Find angle DJG.

Let EFJ=v.
As DFE and DCE stand on the same chord, DCE=DFE, so DCE=v.
As DCE=v, ECH=180-v.
Let CBI=u.
As BIC=20, BCI=160-u.
As BCI=160-u, BCE=u+20.
As ECH=180-v, HCB=160-u-v.
As BCH=160-u-v, CBH=u+v-93.
As CBH=u+v-93, CBA=273-u-v.
As CBI=u, CBF=180-u.
As ABC=273-u-v, ABF=93-v.
As ABFG is a cyclic quadrilateral, AGF=180-ABF, so AGF=v+87.
As AGF=v+87, FAG=44-v.
As FAG and FEG stand on the same chord, FEG=FAG, so FEG=44-v.
As FEJ=44-v, EJF=136.
As EJF=136, EJD=44.
As DJE=44, DJG=136.

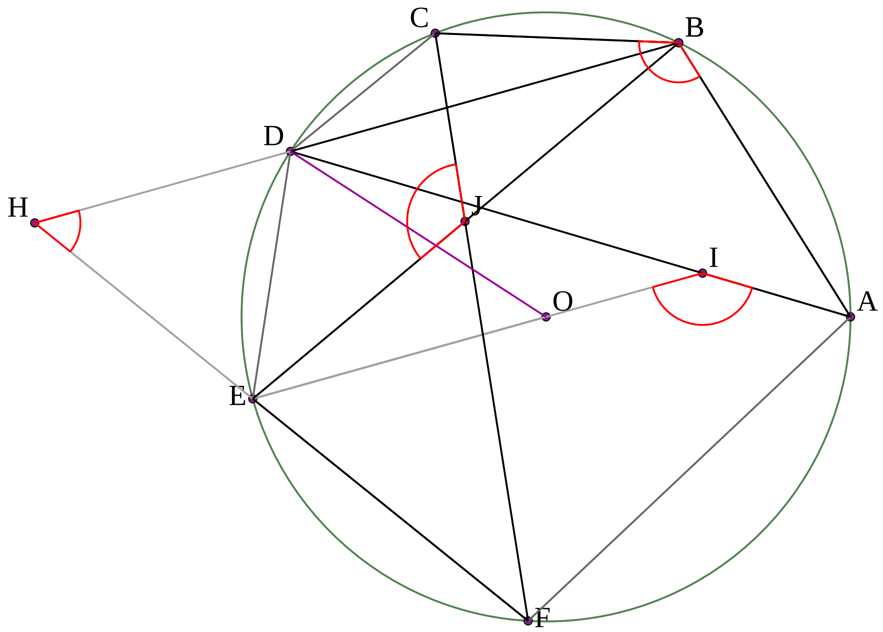
Solution to example 163



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of AE and GO. Let I be the intersection of EC and GB. Let J be the intersection of CG and BD.
Angle EHG = 99°. Angle CIB = 28°. Angle GJD = 89°.
Find angle DFA.

Let BGJ=u.
As CGI=u, GCI=152-u.
As GCI=152-u, GCE=u+28.
As ECG and EAG stand on the same chord, EAG=ECG, so EAG=u+28.
As EHG=99, GHA=81.
As GAH=u+28, AGH=71-u.
As triangle AGO is isosceles, AOG=2u+38.
As AOG is at the center of a circle on the same chord as AFG, AOG=2AFG, so AFG=u+19.
As DJG=89, GJB=91.
As BJG=91, GBJ=89-u.
As DBGF is a cyclic quadrilateral, DFG=180-DBG, so DFG=u+91.
As AFG=u+19, AFD=72.

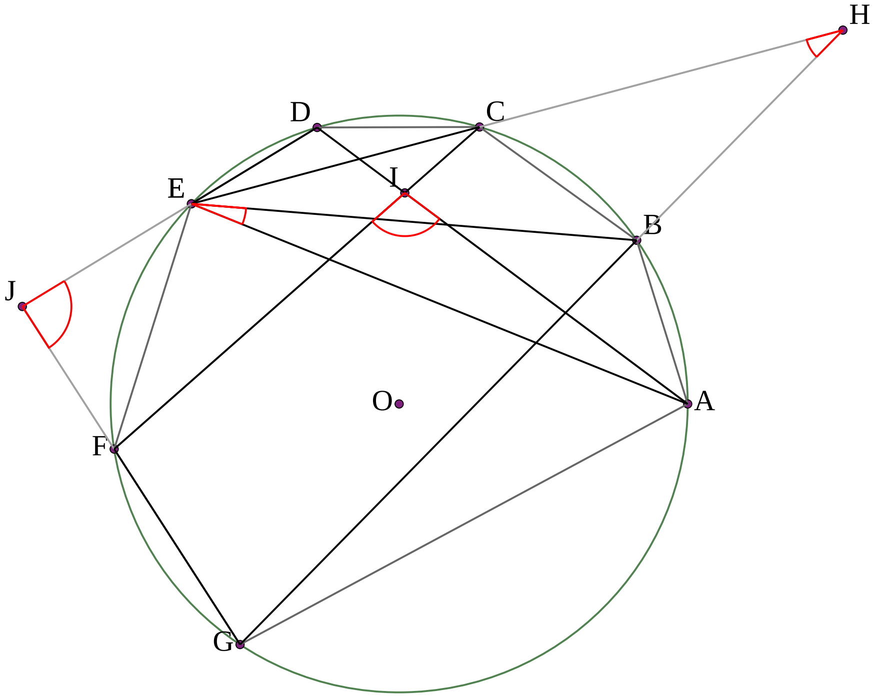
Solution to example 165



Let ABCDEF be a cyclic hexagon with center O. Let H be the intersection of BD and EF. Let I be the intersection of DA and EO. Let J be the intersection of BE and FC.
Angle CBA = x. Angle DHE = y. Angle EJC = z.
Find angle AIE.

Draw line DO.
Let FEJ=u.
As FEJ=u, JEH=180-u.
As BEH=180-u, EBH=u-y.
As DOE is at the center of a circle on the same chord as DBE, DOE=2DBE, so DOE=2u-2y.
As triangle DOE is isosceles, DEO=y-u+90.
As CJE=z, EJF=180-z.
As EJF=180-z, EFJ=z-u.
As CFED is a cyclic quadrilateral, CDE=180-CFE, so CDE=u-z+180.
As ABCD is a cyclic quadrilateral, ADC=180-ABC, so ADC=180-x.
As CDE=u-z+180, EDI=x+u-z.
As DEI=y-u+90, DIE=z-x-y+90.
As DIE=z-x-y+90, EIA=x+y-z+90.

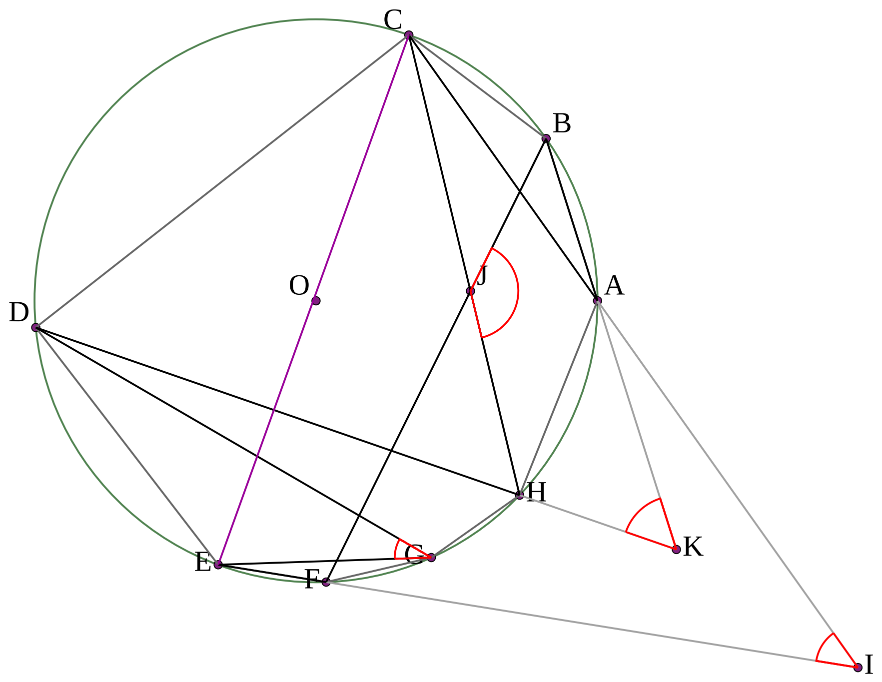
Solution to example 167



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EC and GB. Let I be the intersection of CF and DA. Let J be the intersection of FG and ED.
Prove that AIF+AEB = BHC+EJF

Let BHC=x. Let AIF=y. Let EJF=z. Let AEB=w.
Let EFJ=u.
As EFJ=u, EFG=180-u.
As EFGB is a cyclic quadrilateral, EBG=180-EFG, so EBG=u.
As EBG=u, EBH=180-u.
As EBH=180-u, BEH=u-x.
As BEH=u-x, HEA=w+u-x.
As EJF=z, FEJ=180-z-u.
As FEJ=180-z-u, FED=z+u.
As DEFC is a cyclic quadrilateral, DCF=180-DEF, so DCF=180-z-u.
As AIF=y, AIC=180-y.
As AIC=180-y, CID=y.
As DCI=180-z-u, CDI=z+u-y.
As ADC and AEC stand on the same chord, AEC=ADC, so AEC=z+u-y.
But AEC=w+u-x, so z+u-y=w+u-x, or x+z=y+w, or BHC+EJF=AIF+AEB.

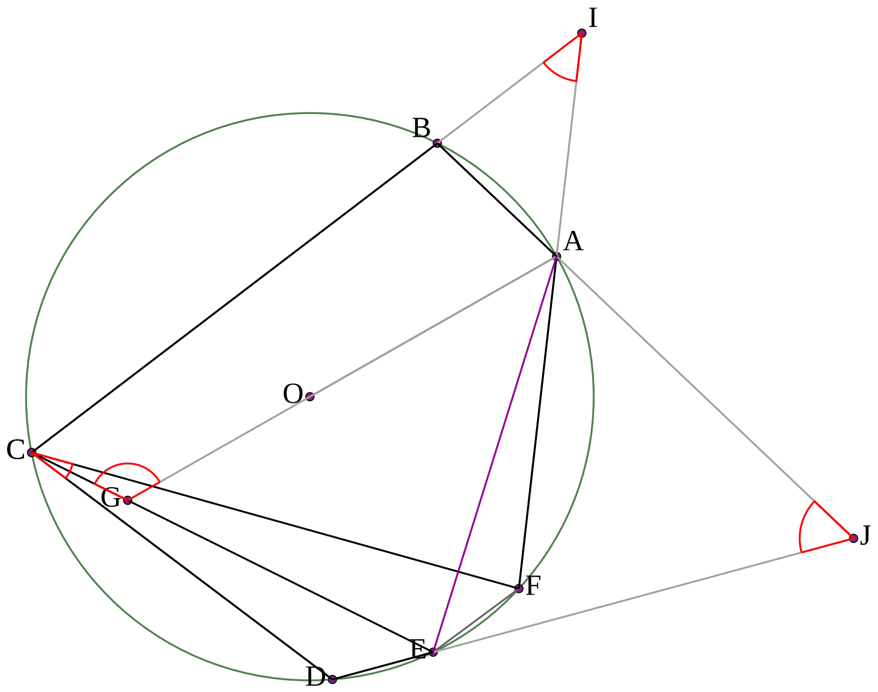
Solution to example 169



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of EF and AC. Let J be the intersection of FB and CH. Let K be the intersection of BA and HD.
Angle FIA = x. Angle DGE = y. Angle AKH = z.
Find angle BJH.

Draw line CE.
Let HAK=u.
As AKH=z, AHK=180-z-u.
As AHK=180-z-u, AHD=z+u.
As AHDC is a cyclic quadrilateral, ACD=180-AHD, so ACD=180-z-u.
As DGE and DCE stand on the same chord, DCE=DGE, so DCE=y.
As DCI=180-z-u, ICE=180-y-z-u.
As ECI=180-y-z-u, CEI=y+z+u-x.
As CEFB is a cyclic quadrilateral, CBF=180-CEF, so CBF=x-y-z-u+180.
As HAK=u, HAB=180-u.
As BAHC is a cyclic quadrilateral, BCH=180-BAH, so BCH=u.
As CBJ=x-y-z-u+180, BJC=y+z-x.
As BJC=y+z-x, BJH=x-y-z+180.

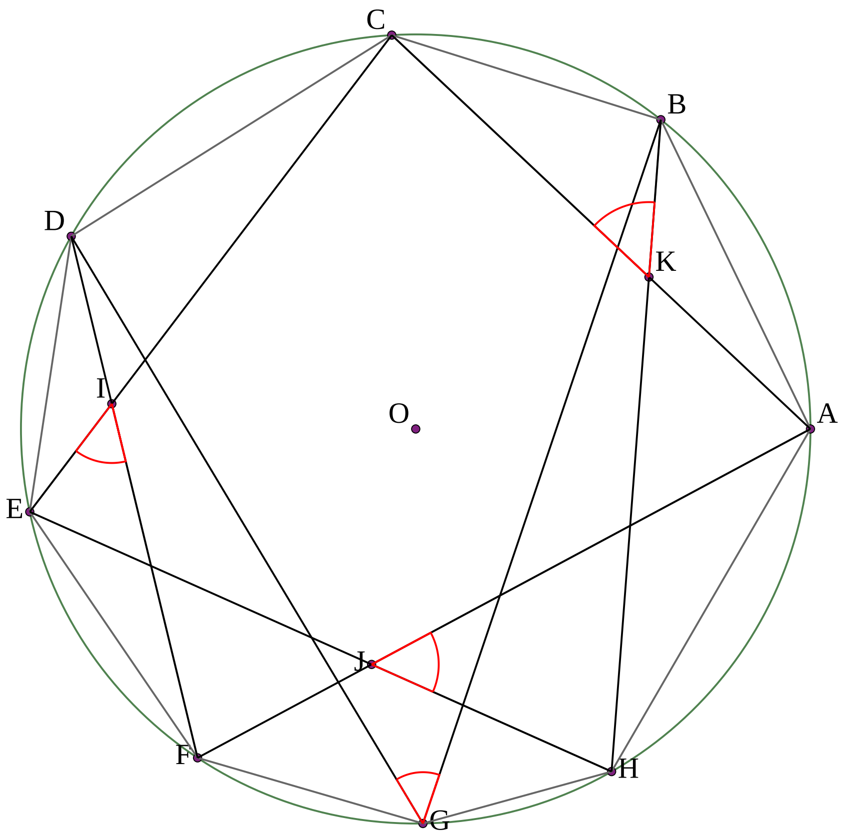
Solution to example 171



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of EC and AO. Let I be the intersection of FA and BC. Let J be the intersection of AB and DE.
Prove that AGC+AIB = DCF+AJE+90

Draw line AE.
Let AGC=x. Let DCF=y. Let AIB=z. Let AJE=w.
As DCFE is a cyclic quadrilateral, DEF=180-DCF, so DEF=180-y.
As DEF=180-y, FEJ=y.
Let EAJ=v.
As AJE=w, AEJ=180-w-v.
As FEJ=y, FEA=180-y-w-v.
Let BAI=u.
As BAI=u, BAF=180-u.
As EAJ=v, EAB=180-v.
As BAF=180-u, FAE=v-u.
As AEF=180-y-w-v, AFE=y+w+u.
As AIB=z, ABI=180-z-u.
As ABI=180-z-u, ABC=z+u.
As ABCE is a cyclic quadrilateral, AEC=180-ABC, so AEC=180-z-u.
As AGC=x, AGE=180-x.
As AEG=180-z-u, EAG=x+z+u-180.
As triangle EAO is isosceles, AOE=540-2x-2z-2u.
As AOE is at the center of a circle on the same chord, but in the opposite direction to AFE, AOE=360-2AFE, so AFE=x+z+u-90.
But AFE=y+w+u, so x+z+u-90=y+w+u, or x+z=y+w+90, or AGC+AIB=DCF+AJE+90.

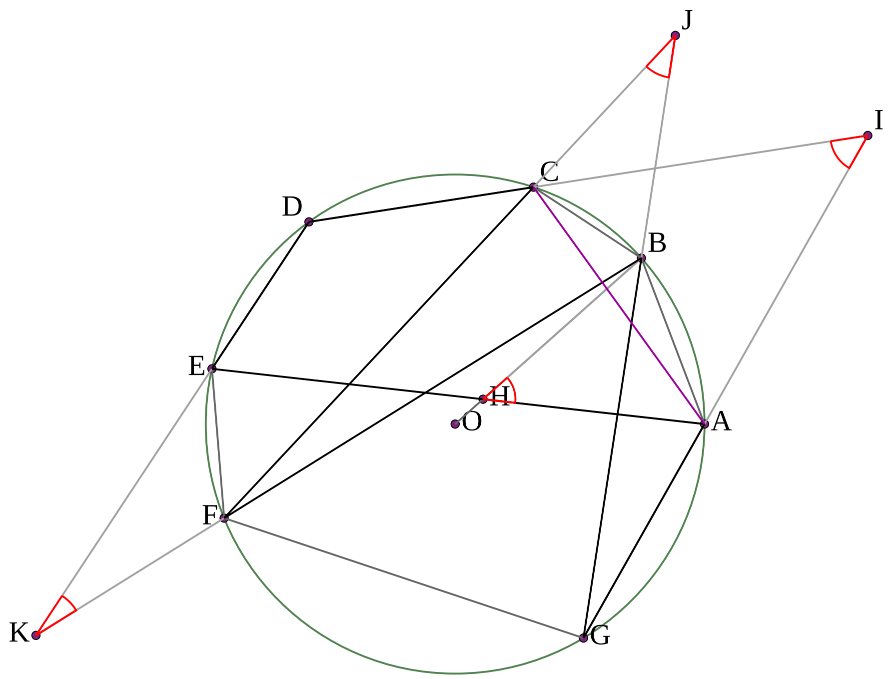
Solution to example 173



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of DF and CE. Let J be the intersection of FA and EH. Let K be the intersection of AC and HB.
Angle BGD = 50°. Angle AJH = 52°. Angle FIE = 51°.
Find angle CKB.

As BGDC is a cyclic quadrilateral, BCD=180-BGD, so BCD=130.
Let BCK=v.
As BCD=130, DCA=130-v.
Let HAJ=u.
As AJH=52, AHJ=128-u.
As AHEC is a cyclic quadrilateral, ACE=180-AHE, so ACE=u+52.
As ACD=130-v, DCE=78-u-v.
As EIF=51, EID=129.
As DIE=129, DIC=51.
As DCI=78-u-v, CDI=u+v+51.
As CDF and CEF stand on the same chord, CEF=CDF, so CEF=u+v+51.
As FAH and FEH stand on the same chord, FEH=FAH, so FEH=u.
As CEF=u+v+51, CEH=v+51.
As CEHB is a cyclic quadrilateral, CBH=180-CEH, so CBH=129-v.
As CBK=129-v, BKC=51.

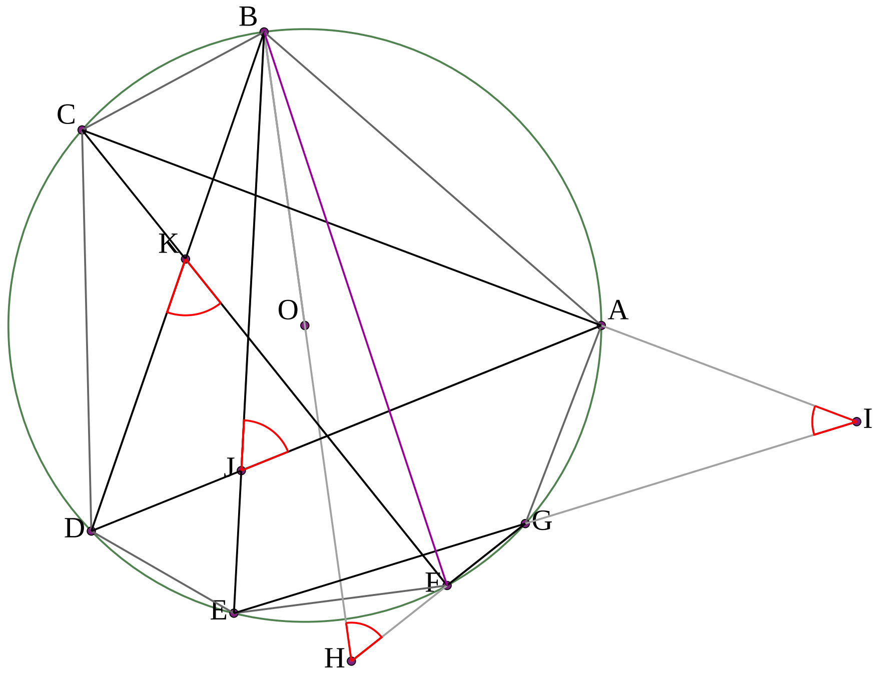
Solution to example 175



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of EA and BO. Let I be the intersection of AG and CD. Let J be the intersection of GB and FC. Let K be the intersection of BF and DE.
Angle AHB = 48°. Angle AIC = 52°. Angle FKE = 25°.
Find angle BJC.

Draw line AC.
Let CBJ=u.
As CBJ=u, CBG=180-u.
As CBG and CAG stand on the same chord, CAG=CBG, so CAG=180-u.
As CAG=180-u, CAI=u.
As CAI=u, ACI=128-u.
As ACI=128-u, ACD=u+52.
Let EFK=v.
As EKF=25, FEK=155-v.
As FEK=155-v, FED=v+25.
As DEFC is a cyclic quadrilateral, DCF=180-DEF, so DCF=155-v.
As DCF=155-v, DCJ=v+25.
As ACD=u+52, ACJ=283-u-v.
As EFK=v, EFB=180-v.
As BFE and BAE stand on the same chord, BAE=BFE, so BAE=180-v.
As BAH=180-v, ABH=v-48.
As triangle ABO is isosceles, AOB=276-2v.
As AOB is at the center of a circle on the same chord as ACB, AOB=2ACB, so ACB=138-v.
As ACJ=283-u-v, JCB=145-u.
As BCJ=145-u, BJC=35.

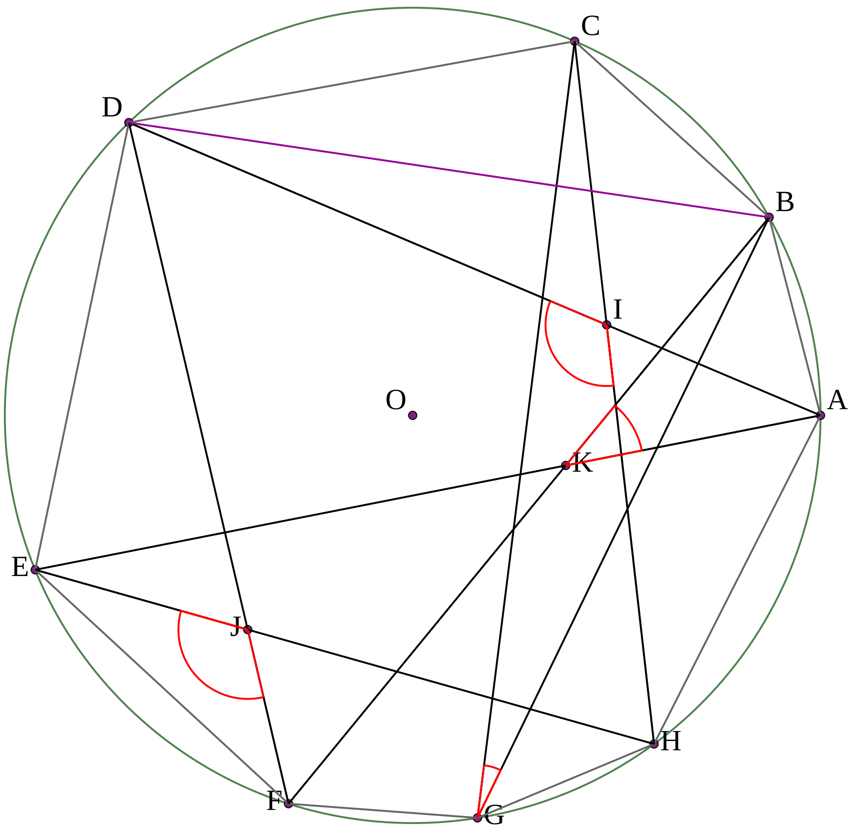
Solution to example 177



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of FG and BO. Let I be the intersection of GE and AC. Let J be the intersection of EB and DA. Let K be the intersection of BD and CF.
Angle FHB = 60°. Angle GIA = 38°. Angle DKF = 58°.
Find angle BJA.

Draw line BF.
Let AGI=u.
As AIG=38, GAI=142-u.
As GAI=142-u, GAC=u+38.
As CAGF is a cyclic quadrilateral, CFG=180-CAG, so CFG=142-u.
As CFG=142-u, CFH=u+38.
As DKF=58, FKB=122.
Let FBK=v.
As BKF=122, BFK=58-v.
As CFH=u+38, HFB=u-v+96.
As BFH=u-v+96, FBH=v-u+24.
As FBO=v-u+24, OBD=u-24.
As triangle DBO is isosceles, BOD=228-2u.
As BOD is at the center of a circle on the same chord as BAD, BOD=2BAD, so BAD=114-u.
As AGI=u, AGE=180-u.
As AGEB is a cyclic quadrilateral, ABE=180-AGE, so ABE=u.
As BAJ=114-u, AJB=66.

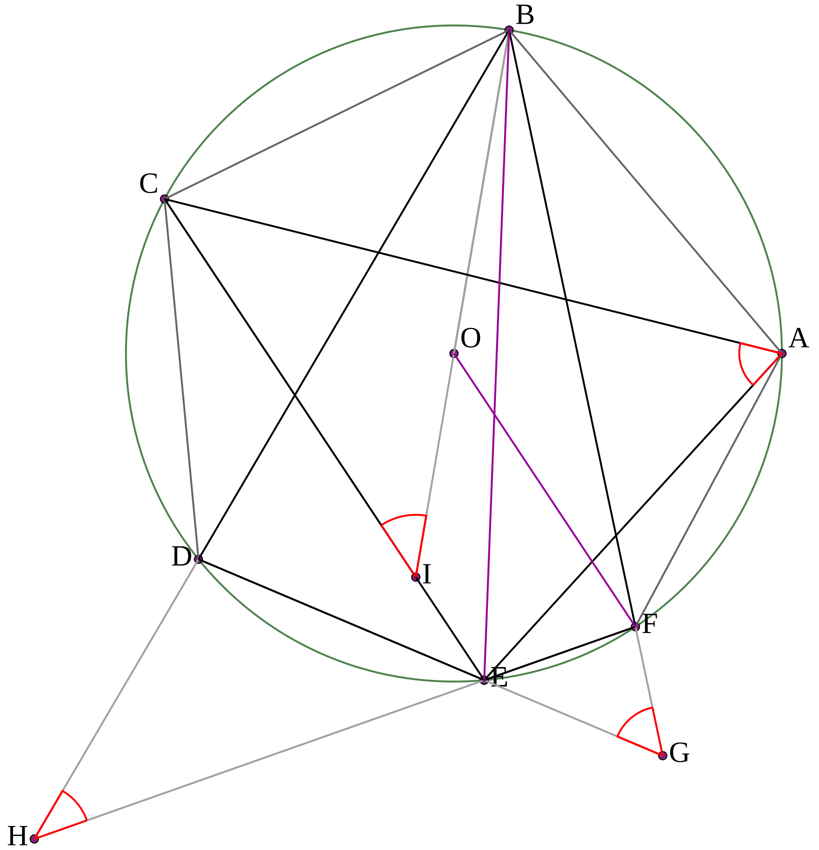
Solution to example 179



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CH and AD. Let J be the intersection of HE and DF. Let K be the intersection of EA and FB.
Prove that DIH+EJF = BGC+AKB+180

Draw line BD.
Let BGC=x. Let DIH=y. Let EJF=z. Let AKB=w.
As EJF=z, EJD=180-z.
Let DEJ=u.
As DJE=180-z, EDJ=z-u.
As BGC and BDC stand on the same chord, BDC=BGC, so BDC=x.
Let BAK=v.
As BAED is a cyclic quadrilateral, BDE=180-BAE, so BDE=180-v.
As BDC=x, CDE=x-v+180.
As EDF=z-u, FDC=x+u-z-v+180.
As DEHC is a cyclic quadrilateral, DCH=180-DEH, so DCH=180-u.
As DIH=y, DIC=180-y.
As DCI=180-u, CDI=y+u-180.
As AKB=w, ABK=180-w-v.
As ABF and ADF stand on the same chord, ADF=ABF, so ADF=180-w-v.
As CDI=y+u-180, CDF=y+u-w-v.
But CDF=x+u-z-v+180, so y+u-w-v=x+u-z-v+180, or y+z=x+w+180, or DIH+EJF=BGC+AKB+180.

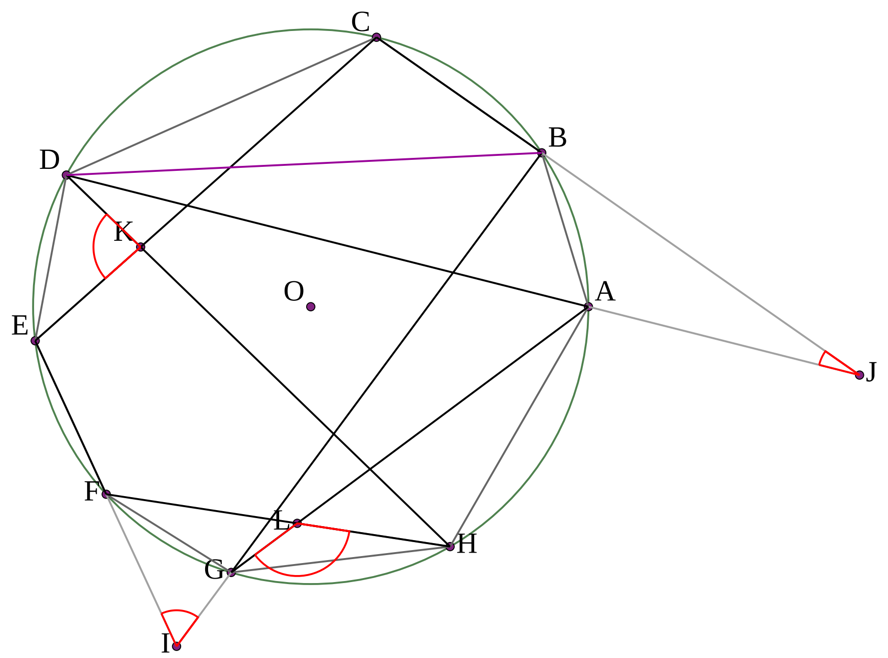
Solution to example 181



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of ED and BF. Let H be the intersection of DB and FE. Let I be the intersection of OB and EC.
Angle EGF = 55°. Angle CAE = 62°. Angle DHE = 40°.
Find angle BIC.

Draw lines BE and FO.
As CAED is a cyclic quadrilateral,
CDE=180-CAE, so CDE=118.
Let EBH=u.
As DBE and DCE stand on the same chord,
DCE=DBE, so DCE=u.
As CDE=118, CED=62-u.
As CED=62-u, CEG=u+118.
Let BEC=v.
As CAE and CBE stand on the same chord,
CBE=CAE, so CBE=62.
As BEC=v, BCE=118-v.
As BCEF is a cyclic quadrilateral, BFE=180-BCE,
so BFE=v+62.
As BFE=v+62, EFG=118-v.
As EFG=118-v, FEG=v+7.
As CEG=u+118, CEF=u-v+111.
As CEFB is a cyclic quadrilateral, CBF=180-CEF,
so CBF=v-u+69.
As BHE=40, BEH=140-u.
As BEH=140-u, BEF=u+40.
As BOF is at the center of a circle on the same
chord as BEF, BOF=2BEF, so BOF=2u+80.
As triangle BOF is isosceles, FBO=50-u.
As CBF=v-u+69, CBO=v+19.
As CBI=v+19, BIC=43.

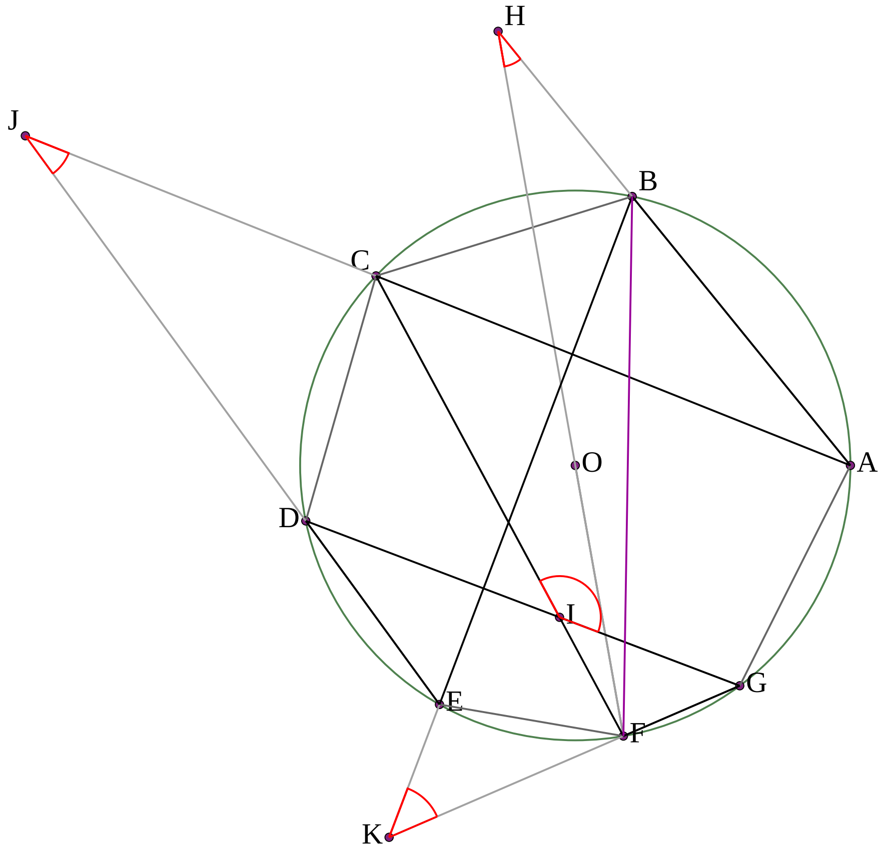
Solution to example 183



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of GB and EF. Let J be the intersection of BC and DA. Let K be the intersection of CE and HD. Let L be the intersection of FH and AG.
Angle GIF = 61°. Angle EKD = 86°. Angle HLG = 135°.
Find angle BJA.

Draw line BD.
Let DEK=u.
As DKE=86, EDK=94-u.
As EDHF is a cyclic quadrilateral, EFH=180-EDH, so EFH=u+86.
As EFH=u+86, HFI=94-u.
Let GFL=v.
As HFI=94-u, IFG=94-u-v.
As GFI=94-u-v, FGI=u+v+25.
As FGI=u+v+25, FGB=155-u-v.
As GLH=135, GLF=45.
As FLG=45, FGL=135-v.
As BGF=155-u-v, BGA=u-20.
As AGB and ADB stand on the same chord, ADB=AGB, so ADB=u-20.
As CED and CBD stand on the same chord, CBD=CED, so CBD=u.
As CBD=u, DBJ=180-u.
As BDJ=u-20, BJD=20.

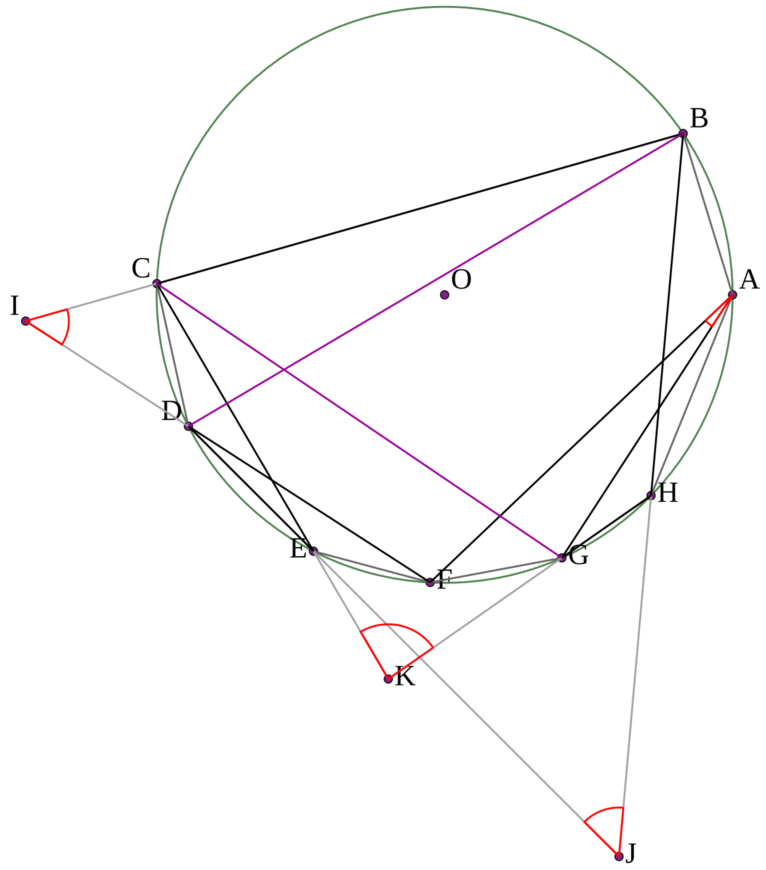
Solution to example 185



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of OF and AB. Let I be the intersection of FC and DG. Let J be the intersection of CA and ED. Let K be the intersection of BE and GF. Prove that BHF+CIG = CJD+EKF+90

Draw line BF.
Let BHF=x. Let CIG=y. Let CJD=z. Let EKF=w.
Let FEK=v.
As FEK=v, FEB=180-v.
As BEF and BCF stand on the same chord, BCF=BEF, so BCF=180-v.
As EKF=w, EFK=180-w-v.
As EFK=180-w-v, EFG=w+v.
As EFGD is a cyclic quadrilateral, EDG=180-EFG, so EDG=180-w-v.
As EDG=180-w-v, GDJ=w+v.
As CIG=y, CID=180-y.
Let DCI=u.
As CID=180-y, CDI=y-u.
As GDJ=w+v, JDC=w+u+v-y.
As CDJ=w+u+v-y, DCJ=y-z-w-u-v+180.
As DCJ=y-z-w-u-v+180, DCA=z+w+u+v-y.
As ACD=z+w+u+v-y, ACF=z+w+v-y.
As ACF and ABF stand on the same chord, ABF=ACF, so ABF=z+w+v-y.
As ABF=z+w+v-y, FBH=y-z-w-v+180.
As FBH=y-z-w-v+180, BFH=z+w+v-x-y.
As triangle BFO is isosceles, BOF=2x+2y-2z-2w-2v+180.
As BOF is at the center of a circle on the same chord as BCF, BOF=2BCF, so BCF=x+y-z-w-v+90.
But BCF=180-v, so x+y-z-w-v+90=180-v, or x+y=z+w+90, or BHF+CIG=CJD+EKF+90.

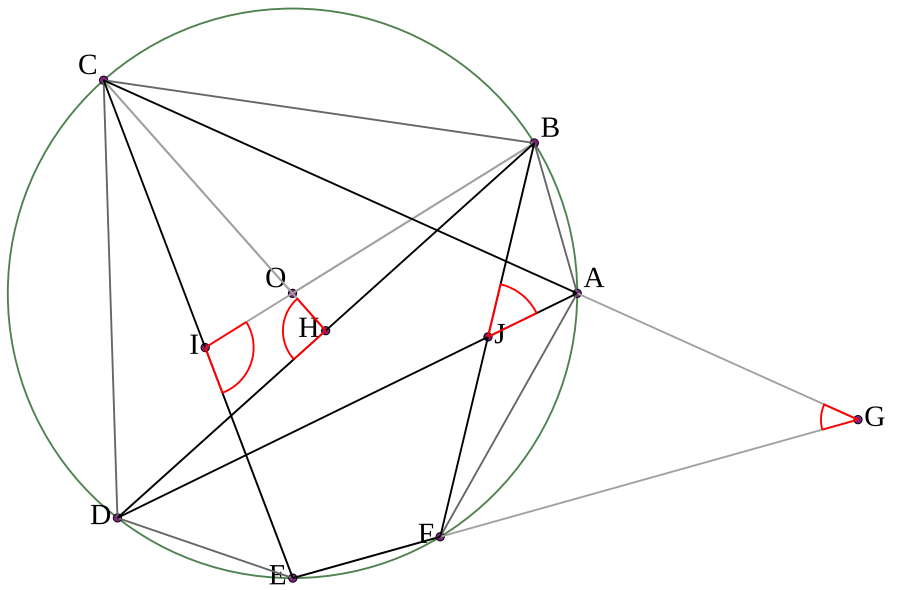
Solution to example 187



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FD and CB. Let J be the intersection of DE and BH. Let K be the intersection of EC and HG.
Angle GAF = x. Angle EKG = y. Angle DIC = z.
Find angle EJH.

Draw lines BD and CG.
Let GCK=v.
As CKG=y, CGK=180-y-v.
As CGK=180-y-v, CGH=y+v.
As CGHB is a cyclic quadrilateral, CBH=180-CGH, so CBH=180-y-v.
Let BDJ=u.
As BDE and BCE stand on the same chord, BCE=BDE, so BCE=u.
As ECG=v, GCB=u-v.
As BCGA is a cyclic quadrilateral, BAG=180-BCG, so BAG=v-u+180.
As BAG=v-u+180, BAF=v-x-u+180.
As BAFD is a cyclic quadrilateral, BDF=180-BAF, so BDF=x+u-v.
As BDF=x+u-v, BDI=v-x-u+180.
As BDI=v-x-u+180, DBI=x+u-z-v.
As IBJ=180-y-v, JBD=z-x-y-u+180.
As DBJ=z-x-y-u+180, BJD=x+y-z.

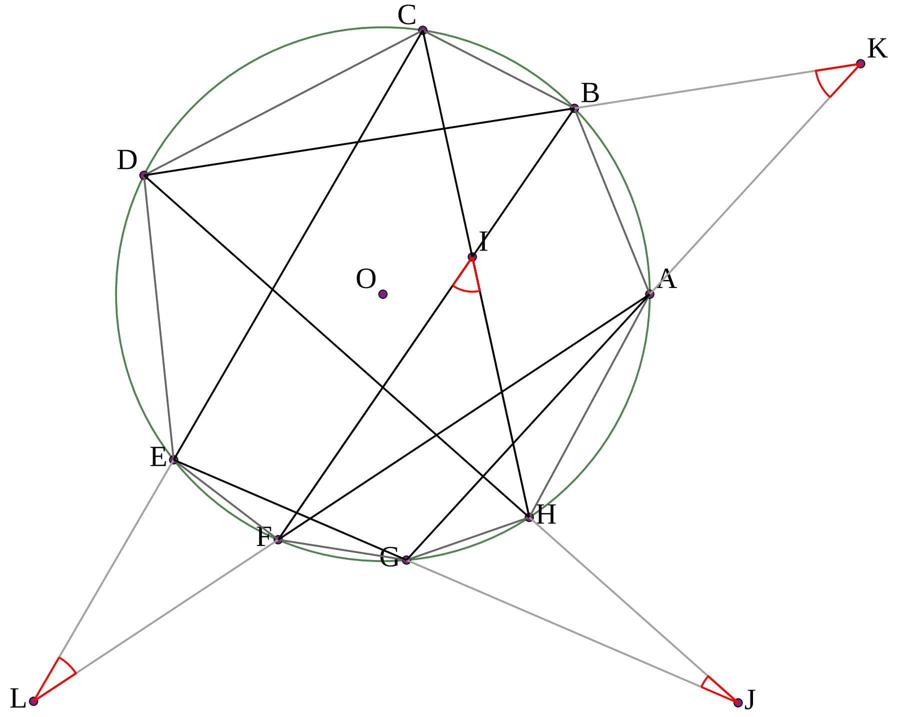
Solution to example 189



Let ABCDEF be a cyclic hexagon with center O. Let G be the intersection of AC and EF. Let H be the intersection of OC and BD. Let I be the intersection of CE and BO. Let J be the intersection of FB and DA.
Angle AGF = x. Angle EIB = y. Angle BJA = z.
Find angle CHD.

As BIE=y, BIC=180-y.
Let BCI=u.
As BIC=180-y, CBI=y-u.
As triangle CBO is isosceles, BOC=2u-2y+180.
As BOC is at the center of a circle on the same chord as BDC, BOC=2BDC, so BDC=u-y+90.
As triangle CBO is isosceles, BCO=y-u.
As AJB=z, AJF=180-z.
Let AFJ=v.
As AJF=180-z, FAJ=z-v.
As BCEF is a cyclic quadrilateral, BFE=180-BCE, so BFE=180-u.
As AFB=v, AFE=v-u+180.
As AFE=v-u+180, AFG=u-v.
As AFG=u-v, FAG=v-x-u+180.
As DAF=z-v, DAG=z-x-u+180.
As DAG=z-x-u+180, DAC=x+u-z.
As CAD and CBD stand on the same chord, CBD=CAD, so CBD=x+u-z.
As BDC=u-y+90, BCD=y+z-x-2u+90.
As BCH=y-u, HCD=z-x-u+90.
As CDH=u-y+90, CHD=x+y-z.

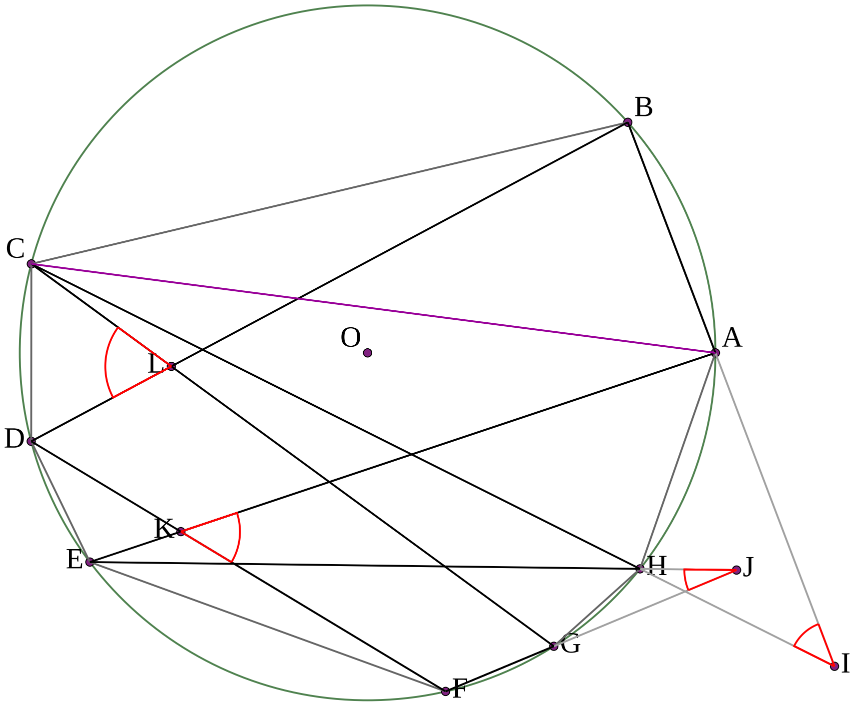
Solution to example 191



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CH and BF. Let J be the intersection of HD and GE. Let K be the intersection of DB and AG. Let L be the intersection of FA and EC.
Angle HIF = x. Angle HJG = y. Angle BKA = z.
Find angle FLE.

Let EFL=r.
As EFL=r, EFA=180-r.
As AFE and AGE stand on the same chord,
AGE=AFE, so AGE=180-r.
Let GHJ=u.
As GJH=y, HGJ=180-y-u.
As HGJ=180-y-u, HGE=y+u.
As AGE=180-r, AGH=y+u+r-180.
As GHJ=u, GHD=180-u.
Let ABK=v.
As ABK=v, ABD=180-v.
As ABDH is a cyclic quadrilateral,
AHD=180-ABD, so AHD=v.
As DHG=180-u, GHA=v-u+180.
As AGH=y+u+r-180, GAH=180-y-v-r.
As AKB=z, BAK=180-z-v.
As BAK=180-z-v, BAG=z+v.
As GAH=180-y-v-r, HAB=z-y-r+180.
As BAHC is a cyclic quadrilateral,
BCH=180-BAH, so BCH=y+r-z.
As FIH=x, FIC=180-x.
As CIF=180-x, CIB=x.
As BCI=y+r-z, CBI=z-x-y-r+180.
As CBFE is a cyclic quadrilateral, CEF=180-CBF,
so CEF=x+y+r-z.
As CEF=x+y+r-z, FEL=z-x-y-r+180.
As FEL=z-x-y-r+180, ELF=x+y-z.

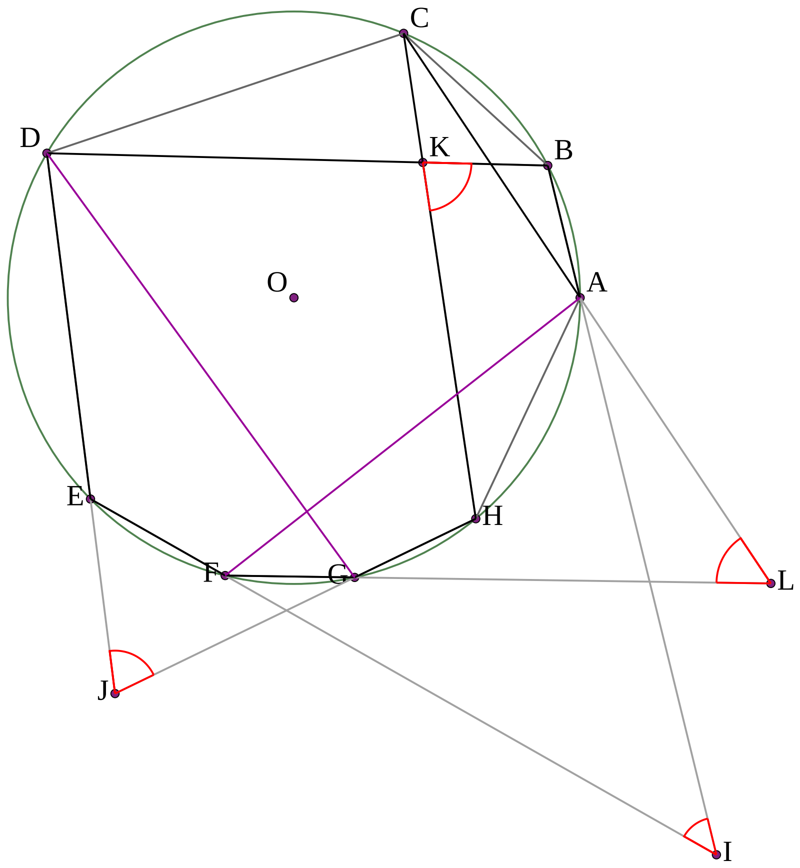
Solution to example 193



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of CH and AB. Let J be the intersection of HE and FG. Let K be the intersection of EA and DF. Let L be the intersection of BD and GC. Prove that AIH+GJH+AKF+CLD = 180

Draw line AC.
Let AIH=x. Let GJH=y. Let AKF=z. Let CLD=w.
Let GHJ=u.
As GHJ=u, GHE=180-u.
As EHGF is a cyclic quadrilateral, EFG=180-EHG, so EFG=u.
As AKF=z, FKE=180-z.
Let FEK=v.
As EKF=180-z, EFK=z-v.
As EFG=u, GFK=u+v-z.
As DFGC is a cyclic quadrilateral, DCG=180-DFG, so DCG=z-u-v+180.
As DCL=z-u-v+180, CDL=u+v-z-w.
As GJH=y, HGJ=180-y-u.
As HGJ=180-y-u, HGF=y+u.
As FGHE is a cyclic quadrilateral, FEH=180-FGH, so FEH=180-y-u.
As FEH=180-y-u, HEA=y+u+v-180.
As AEH and ACH stand on the same chord, ACH=AEH, so ACH=y+u+v-180.
As ACI=y+u+v-180, CAI=360-x-y-u-v.
As CAI=360-x-y-u-v, CAB=x+y+u+v-180.
As BAC and BDC stand on the same chord, BDC=BAC, so BDC=x+y+u+v-180.
But BDC=u+v-z-w, so x+y+u+v-180=u+v-z-w, or x+y+z+w=180, or AIH+GJH+AKF+CLD=180.

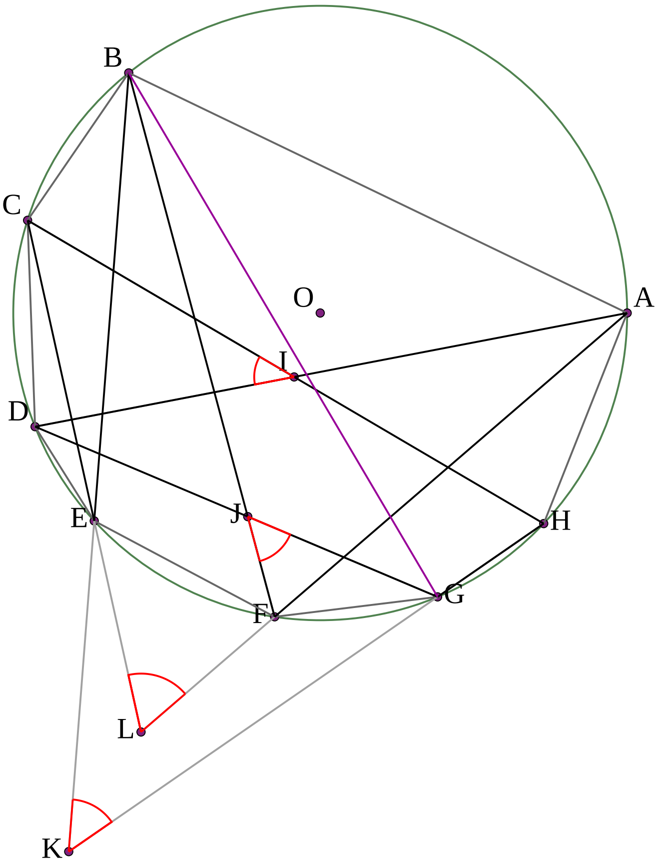
Solution to example 195



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of FE and BA. Let J be the intersection of ED and HG. Let K be the intersection of DB and CH. Let L be the intersection of AC and GF.
Angle EJG = 71°. Angle ALG = 55°. Angle FIA = 47°.
Find angle BKH.

Draw lines DG and AF.
Let FAL=v.
As ALF=55, AFL=125-v.
As FAL=v, FAC=180-v.
Let CDK=u.
As BDC and BAC stand on the same chord,
BAC=BDC, so BAC=u.
As CAF=180-v, FAB=u-v+180.
As BAF=u-v+180, FAI=v-u.
As FAI=v-u, AFI=u-v+133.
As AFG=125-v, GFI=u+8.
As GFI=u+8, GFE=172-u.
As EFGD is a cyclic quadrilateral, EDG=180-EFG,
so EDG=u+8.
As GDJ=u+8, DGJ=101-u.
As DGJ=101-u, DGH=u+79.
As DGHC is a cyclic quadrilateral,
DCH=180-DGH, so DCH=101-u.
As DCK=101-u, CKD=79.
As CKD=79, CKB=101.
As BKC=101, BKH=79.

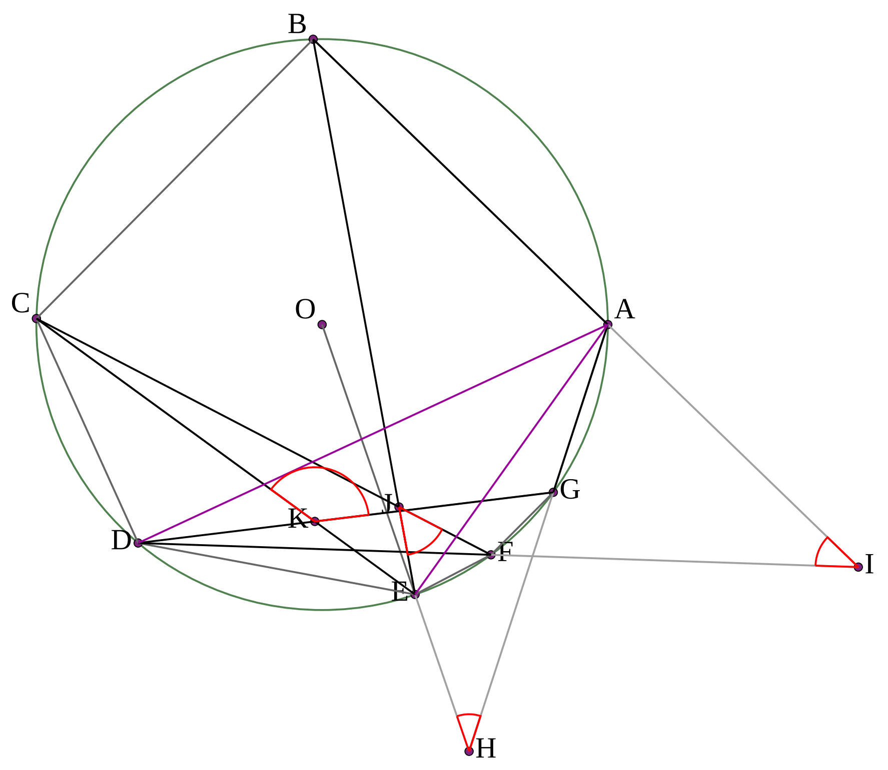
Solution to example 197



Let ABCDEFGH be a cyclic octagon with center O. Let I be the intersection of AD and HC. Let J be the intersection of DG and BF. Let K be the intersection of GH and EB. Let L be the intersection of CE and FA. Prove that CID+ELF = FJG+EKG

Draw line BG.
Let CID=x. Let FJG=y. Let EKG=z. Let ELF=w.
Let BGK=v.
As BKG=z, GBK=180-z-v.
As EBG and EDG stand on the same chord,
EDG=EBG, so EDG=180-z-v.
Let FEL=r.
As FEL=r, FEC=180-r.
As CEFB is a cyclic quadrilateral, CBF=180-CEF,
so CBF=r.
As FJG=y, GJB=180-y.
Let BGJ=u.
As BJG=180-y, GBJ=y-u.
As CBJ=r, CBG=y+r-u.
As CBGD is a cyclic quadrilateral,
CDG=180-CBG, so CDG=u-y-r+180.
As EDG=180-z-v, EDC=u-y-z-v-r+360.
As ELF=w, EFL=180-w-r.
As EFL=180-w-r, EFA=w+r.
As AFED is a cyclic quadrilateral, ADE=180-AFE,
so ADE=180-w-r.
As BGK=v, BGH=180-v.
As BGD=u, DGH=u-v+180.
As DGHC is a cyclic quadrilateral,
DCH=180-DGH, so DCH=v-u.
As DCI=v-u, CDI=u-x-v+180.
As EDI=180-w-r, EDC=u-x-w-v-r+360.
But CDE=u-y-z-v-r+360, so
u-x-w-v-r+360=u-y-z-v-r+360, or y+z=x+w, or
FJG+EKG=CID+ELF.

Solution to example 199



Let ABCDEFG be a cyclic heptagon with center O. Let H be the intersection of GA and EO. Let I be the intersection of AB and FD. Let J be the intersection of BE and CF. Let K be the intersection of EC and DG. Prove that AIF+CKG = EHG+EJF+90

Draw lines AE and AD.
Let EHG=x. Let AIF=y. Let EJF=z. Let CKG=w.
Let CEJ=u.
Let DEK=v.
As BEC=u, BED=u+v.
As BED and BAD stand on the same chord,
BAD=BED, so BAD=u+v.
As BAD=u+v, DAI=180-u-v.
As DAI=180-u-v, ADI=u+v-y.
As EJF=z, EJC=180-z.
As CJE=180-z, ECJ=z-u.
As ECF and EDF stand on the same chord,
EDF=ECF, so EDF=z-u.
As ADI=u+v-y, ADE=z+v-y.
As ADE and ABE stand on the same chord,
ABE=ADE, so ABE=z+v-y.
As CKG=w, GKE=180-w.
As EKG=180-w, EKD=w.
As DKE=w, EDK=180-w-v.
As EDG and EAG stand on the same chord,
EAG=EDG, so EAG=180-w-v.
As EAH=180-w-v, AEH=w+v-x.
As AEH=w+v-x, AEO=x-w-v+180.
As triangle AEO is isosceles,
AOE=2w+2v-2x-180.
As AOE is at the center of a circle on the same
chord as ABE, AOE=2ABE, so ABE=w+v-x-90.
But ABE=z+v-y, so w+v-x-90=z+v-y, or
y+w=x+z+90, or AIF+CKG=EHG+EJF+90.